CASO II:- $m_1 = m_2 = m_3$

$(m-2)^3 = 0 \quad m_1 = m_2 = m_3 = 2$

$m^3 - 6m^2 + 12m - 8 = 0$

$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 12\frac{dy}{dx} - 8y = 0$

$e^{mx} \xrightarrow{m=2} e^{2x}$

$\frac{d}{dm}$ $xe^{mx} \xrightarrow{m=2} xe^{2x}$

$\frac{d}{dm}$ $x^2e^{mx} \xrightarrow{m=2} x^2e^{2x}$

$$y_g = C_1e^{2x} + C_2xe^{2x} + C_3x^2e^{2x}$$

$y(0) = 4 \quad y'(0) = -3 \quad y''(0) = 2$

$x=0 \quad 4 = C_1e^{(0)} + (0) + (0) \quad C_1 = 4$

$\frac{dy}{dx} = 2(4)e^{2x} + C_2\left(2xe^{2x} + e^{2x}\right) + C_3\left(2x^2e^{2x} + 2xe^{2x}\right)$

$-3 = 8 + C_2(0+1) + C_3(0+0)$

$-3 = 8 + C_2 \quad C_2 = -11$

$\frac{d^2y}{dx^2} = 16e^{2x} - 11\left(4xe^{2x} + 2e^{2x} + 2e^{2x}\right) + C_3\left(4x^2e^{2x} + 4xe^{2x} + 4xe^{2x} + 2e^{2x}\right)$

$2 = 16 - 44 + 2C_3$

$30 = 2C_3 \quad C_3 = 15$

$$y_p = 4e^{2x} - 11xe^{2x} + 15x^2e^{2x}$$

CASO III: $m_{1,2} = a \pm bi$ $\quad a \in \mathbb{R}$, $b \in \mathbb{R}^+$

$m_1 \neq m_2$

$$y_g = C_1 e^{(a+bi)x} + C_2 e^{(a-bi)x}$$

$$y_g = e^{ax}\left(C_1 e^{(bx)i} + C_2 e^{(-bx)i}\right) \quad x \in \mathbb{R},\ y \in \mathbb{R}$$

EULER $\quad e^{\pi i} = -1$

$$re^{\theta i} = r\cos(\theta) + i\, r\,\text{sen}(\theta)$$

$$re^{-\theta i} = r\cos(\theta) - i\, r\,\text{sen}(\theta)$$

$$y_g = e^{ax}\left(C_1\left(\cos(bx) + i\,\text{sen}(bx)\right) + \right.$$

$$y_g = e^{ax}\left(\begin{array}{l} \left. + C_2\left(\cos(bx) - i\,\text{sen}(bx)\right)\right) \\ (C_1 + C_2)\cos(bx) + (C_1 i - C_2 i)\,\text{sen}(bx)\end{array}\right)$$

$$y_g = e^{ax}\left(C_{10}\cos(bx) + C_{20}\,\text{sen}(bx)\right)$$

$$y_g = C_{10}e^{ax}\cos(bx) + C_{20}e^{ax}\,\text{sen}(bx).$$

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

$$m^2 + m + 1 = 0$$

$$m_{1,2} = \frac{-1 \pm \sqrt{1^2 - 4(1)}}{2}$$

$$m_{1,2} = -\frac{1}{2} \pm \frac{\sqrt{-3}}{2}$$

$$m_{1,2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i \qquad a = -\frac{1}{2} \qquad b = \frac{\sqrt{3}}{2}$$

$$y_g = C_1 e^{-\frac{x}{2}} \cos\left(\frac{\sqrt{3}}{2} x\right) + C_2 e^{-\frac{x}{2}} \operatorname{sen}\left(\frac{\sqrt{3}}{2} x\right)$$

$$\frac{d^2y}{dx^2} + 4y = 0$$

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$m = \sqrt{-4} \qquad m_1 = 2i \qquad m_2 = -2i$$

$$y_g = C_1 \cos(2x) + C_2 \operatorname{sen}(2x)$$

$$\frac{dy}{dx} = -2C_1 \operatorname{sen}(2x) + 2C_2 \cos(2x)$$

$$\frac{d^2y}{dx^2} = -4C_1 \cos(2x) - 4C_2 \operatorname{sen}(2x)$$

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$$\frac{d^2y}{dx} = -4C_1 \cos(2x) - 4C_2 \operatorname{sen}(2x)$$

$$+$$

$$4y = 4C_1 \cos(2x) + 4C_2 \operatorname{sen}(2x)$$

$$\equiv$$

$$0 = (0)\cos(2x) + (0)\operatorname{sen}(2x)$$

$$y_g = C_1 e^{3x} + C_2 e^{2x} + C_3 x e^{2x}$$

$$(m-3)(m-2)^2 = 0$$

$$(m-3)(m^2 - 4m + 4) = 0$$

$$m^3 - 7m^2 + 16m - 12 = 0$$

$$\frac{d^3y}{dx^3} - 7\frac{d^2y}{dx^2} + 16\frac{dy}{dx} - 12y = 0$$