Transformada de Laplace.

$$\mathcal{L}\{f(t)\} = F(s)$$

$f, t \in \mathbb{R}$ $\quad F \in \mathbb{R}$ $\quad s \in \mathbb{C}$.

núcleo

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\, dt$$

operador

argumento

Teorema de existencia y unicidad de la Transformada de Laplace

Para que $f(t)$ tenga transformada de Laplace debe ser de clase "A"

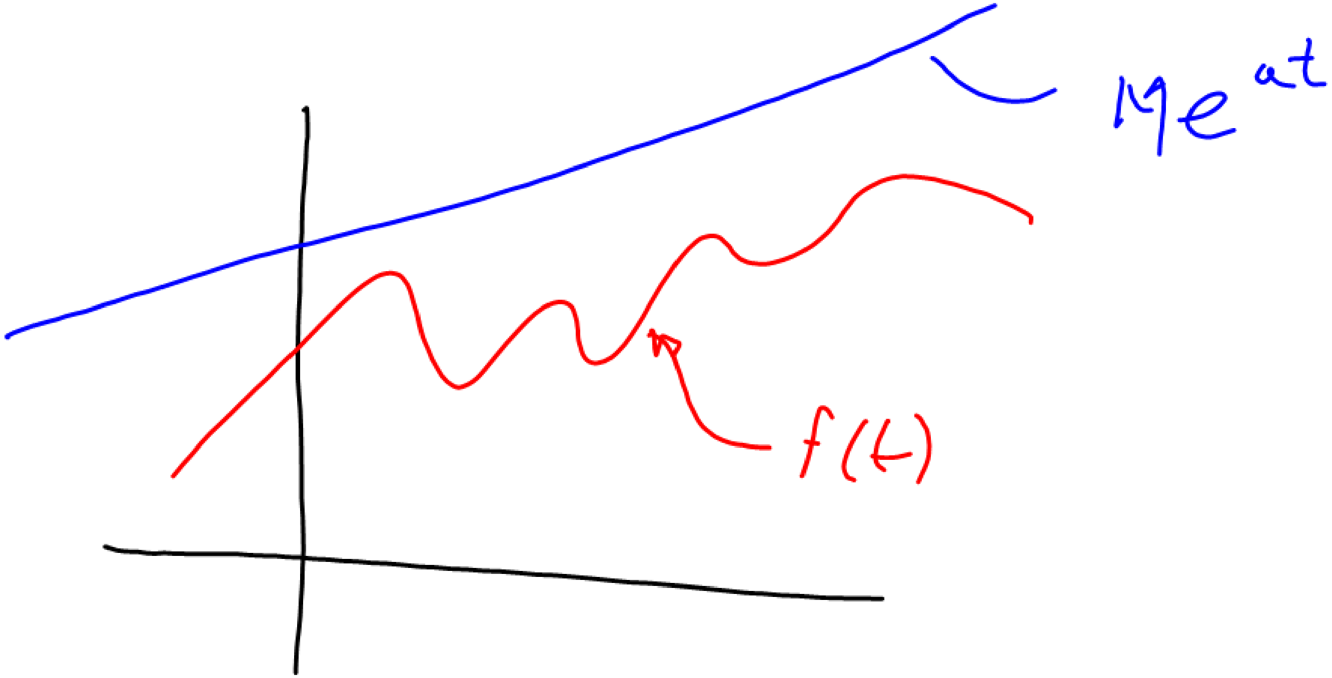
una función es de clase "A" cuando:

a) es de orden exponencial

b) es seccionalmente continua

a) es de orden exponencial si cumple

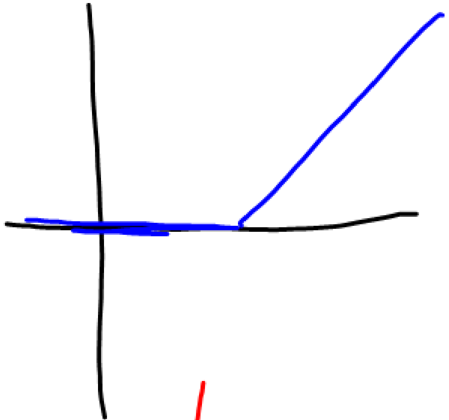
$$|f(t)| \le Me^{at} \quad \begin{matrix} M \in \mathbb{R} \\ a \in \mathbb{R} \end{matrix}$$

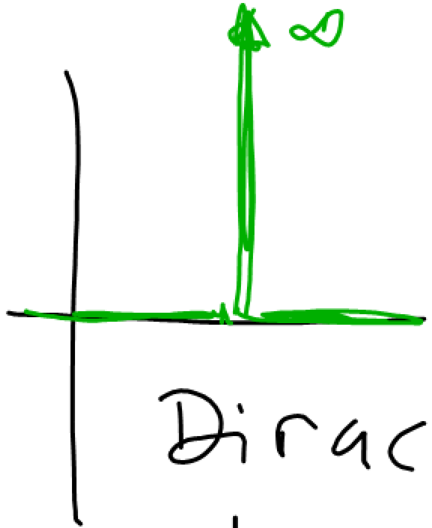


a) e^{t^n} $n \geq 2$

$$|e^{t^n}| \not\leq Me^{at}$$

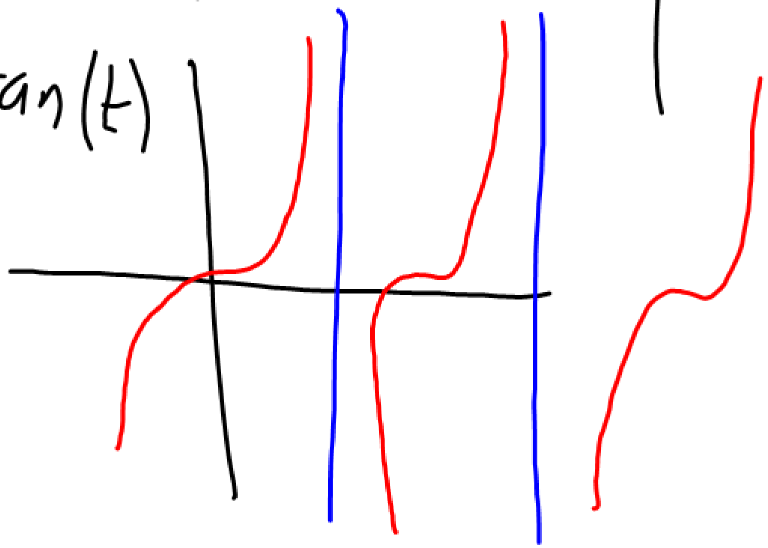
b) seccionalmente continua





Dirac.

tan(t)



$$\delta(t-a) = \begin{cases} 0 \; ; \; t \neq a \\ \displaystyle\int_{-\infty}^{-\infty} \delta(t-a)\,dt = 1. \end{cases}$$

Propiedades de la T$\mathcal{L}$.

a)

$$\mathcal{L}\{a\,f(t)+b\,g(t)\}=a\,F(s)+b\,G(s)$$

b)

$$\mathcal{L}\{f(at)\}=\frac{1}{a}F\left(\frac{s}{a}\right)$$

c)

$$\mathcal{L}\{f'(t)\}=sF(s)-f(0)$$

$$\mathcal{L}\{f''(t)\}=s^2F(s)-sf(0)-f'(0)$$

$$\mathcal{L}\{f'''(t)\}=s^3F(s)-s^2f(0)-sf'(0)-f''(0)$$

$$\mathcal{L}\{f^{(n)}(t)\}=s^nF(s)-\sum_{i=1}^{n}s^{n-i}f^{(i-1)}(0)$$

d) $\mathcal{L}^{-1}\left\{F'(s)\right\} = -t f(t)$

$\mathcal{L}^{-1}\left\{F^{(n)}(s)\right\} = (-1)^n t^n f(t)$

e) $\mathcal{L}\left\{\int_0^t f(t)\,dt\right\} = \frac{F(s)}{s}$

f) $\mathcal{L}^{-1}\left\{\int_s^\infty F(s)\,ds\right\} = \frac{f(t)}{t}$

Considerando

$$f(t-a)\mu(t-a) = \begin{cases} 0 & ; t < a \\ f(t-a) & ; t \geq a \end{cases}$$

g) $\mathcal{L}\{f(t-a)\mu(t-a)\} = e^{-as}F(s)$

h) $\mathcal{L}\{e^{at}f(t)\} = F(s-a)$

Ejemplo b)

$$\mathcal{L}\{1\} = \frac{1}{s} \qquad \mathcal{L}\{e^{5s}\} = \frac{1}{s-5}$$

por la propiedad:

$$\mathcal{L}\{e^{t}\} = \frac{1}{s-1}$$

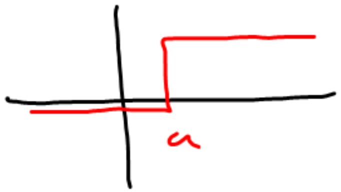
$$\mathcal{L}\{e^{5t}\} = \frac{1}{5}\left(\frac{1}{\left(\frac{s}{5}\right)-1}\right)$$

$$\mathcal{L}\{e^{5t}\} = \frac{1}{5}\left(\frac{1}{\left(\frac{s-5}{5}\right)}\right)$$

$$\mathcal{L}\{e^{5t}\} = \frac{1}{\cancel{5}}\left(\frac{\cancel{5}}{s-5}\right)$$

$$\mathcal{L}\{e^{5t}\} = \frac{1}{s-5}$$

Ejemplo de g) & c) $\mathcal{L}\{1\} = \frac{1}{s}$



$$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$$

$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

$$\mathcal{L}\{(t-a)\cdot u(t-a)\} = \frac{e^{-as}}{s^2}$$

$$\mathcal{L}\left\{\frac{d}{dt}\left((t-a)\,u(t-a)\right)\right\} = s\left(\frac{e^{-as}}{s^2}\right) - (0)$$

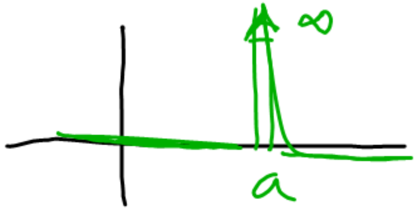
$$= \frac{e^{-as}}{s}$$

$$\mathcal{L}\left\{\frac{d}{dt}\left((t-a)\,u(t-a)\right)\right\} = \mathcal{L}\{u(t-a)\}$$

$$\frac{d}{dt}(t-a)u(t-a) = u(t-a)$$

rampa escalón unitario

continuando con g) 8 c)



$$\delta(t-a) = \begin{cases} 0 \;; & t \neq a \\ \displaystyle\int_{-\infty}^{\infty} \delta(t-a)\,dt = 1 \end{cases}$$

Dirac

$$\mathcal{L}\left\{ \frac{d}{dt}\left(u(t-a)\right)\right\} = s\left(\frac{e^{as}}{s}\right) - (0)$$

$$\mathcal{L}\left\{ \frac{d}{dt}\left(u(t-a)\right)\right\} = e^{-as}$$

$$\mathcal{L}\left\{ \frac{d}{dt}\left(u(t-a)\right)\right\} = \mathcal{L}\left\{\delta(t-a)\right\}$$

$$\frac{d}{dt}\left(u(t-a)\right) = \delta(t-a).$$

escalón unitario — Dirac.