$$3X_1 + 4X_2 = 20$$
$$6X_1 + 8X_2 = 30.$$

$$\begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$$

$$A$$

$$|A| = 0$$

$$\frac{dx_1}{dt} = 3X + 4X_2$$

$$\frac{dx_2}{dt} = 6X_1 + 8X_2$$

$$\frac{d}{dt} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix}$$

$$\frac{d}{dt}\bar{X} = A\bar{X}$$

$$\bar{X} = e^{At}\bar{X}(0) \qquad \bar{X}(0) = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$X_1 = \left(\frac{8}{11} + \frac{3}{11}e^{11t}\right)C_{10} + \left(\frac{4}{11}e^{11t} - \frac{4}{11}\right)C_{20}$$

$$X_2 = \left(\frac{6}{11}e^{11t} - \frac{6}{11}\right)C_{10} + \left(\frac{3}{11} + \frac{8}{11}e^{11t}\right)C_{20}$$

$$X_1 = C_1 + C_2 e^{11t} = \left(\frac{8}{11} + \frac{3}{11}e^{11t}\right)C_{10} + \left(\frac{4}{11}e^{11t} - \frac{4}{11}\right)C_{20}$$

$$X_2 = 2C_2 e^{11t} - \frac{3}{4}C_1 = \left(\frac{6}{11}e^{11t} - \frac{6}{11}\right)C_{10} + \left(\frac{3}{11} + \frac{8}{11}e^{11t}\right)C_{20}$$

$$\frac{8}{11}C_{10} - \frac{4}{11}C_{20} = C_1 \qquad -\frac{3}{4}C_1 = -\frac{6}{11}C_{10} + \frac{3}{11}C_{20}$$

$$\frac{3}{11}C_{10} + \frac{4}{11}C_{20} = C_2 \qquad 2C_2 = \frac{6}{11}C_{10} + \frac{8}{11}C_{20}$$

$$= \left\{ \begin{array}{l} \left| \begin{array}{l} 8C_{10} - 4C_{20} = 11C_1 \\[2mm] 3C_{10} + 4C_{20} = 11C_2 \end{array} \right. \\[6mm] \left| \begin{array}{l} 33C_1 = +24C_{10} - 12C_{20} \\[2mm] 22C_2 = 6C_{10} + 8C_{20} \end{array} \right. \end{array} \right.$$

$$\frac{dx_1(t)}{dt} = 2x_1 + 3x_2 + e^{2t} + t^2$$

$$\frac{dx_2(t)}{dt} = x_1 + 4x_2 + \cos(2t)$$

$$\frac{d\bar{x}}{dt} = A\bar{x} + \bar{b}(t)$$

$$\bar{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \quad \bar{b}(t) = \begin{bmatrix} e^{2t} + t^2 \\ \cos(2t) \end{bmatrix}$$

$$\bar{x} = e^{A(t)}\bar{x}(0) + \int_0^t e^{A(t-\tau)} \bar{b}(\tau) \, d\tau.$$