> *restart*

> $Sist := diff(x[1](t), t) = 3 \cdot x[1](t) + 4 \cdot x[2](t), diff(x[2](t), t) = 6 \cdot x[1](t) + 8 \cdot x[2](t) :$
> $Sist[1]; Sist[2]$

$$\frac{d}{dt} x_1(t) = 3\, x_1(t) + 4\, x_2(t)$$

$$\frac{d}{dt} x_2(t) = 6\, x_1(t) + 8\, x_2(t) \tag{1}$$

> $Sol := dsolve(\{Sist\}) : Sol[1]; Sol[2]$

$$x_1(t) = \_C1 + \_C2\, e^{11\,t}$$

$$x_2(t) = 2\,\_C2\, e^{11\,t} - \frac{3}{4}\,\_C1 \tag{2}$$

> $AA := array([[3, 4], [6, 8]])$

$$AA := \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix} \tag{3}$$

> $with(linalg) :$

> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} \dfrac{8}{11} + \dfrac{3}{11}\, e^{11\,t} & \dfrac{4}{11}\, e^{11\,t} - \dfrac{4}{11} \\[2ex] \dfrac{6}{11}\, e^{11\,t} - \dfrac{6}{11} & \dfrac{3}{11} + \dfrac{8}{11}\, e^{11\,t} \end{bmatrix} \tag{4}$$

> $det(AA)$

$$0 \tag{5}$$

> $Xcero := array([\_C10, \_C20])$

$$Xcero := \begin{bmatrix} \_C10 & \_C20 \end{bmatrix} \tag{6}$$

> $SolGral := evalm(MatExp \,\&*\, Xcero) : SolGral[1]; SolGral[2]$

$$\left(\frac{8}{11} + \frac{3}{11}\, e^{11\,t}\right)\_C10 + \left(\frac{4}{11}\, e^{11\,t} - \frac{4}{11}\right)\_C20$$

$$\left(\frac{6}{11}\, e^{11\,t} - \frac{6}{11}\right)\_C10 + \left(\frac{3}{11} + \frac{8}{11}\, e^{11\,t}\right)\_C20 \tag{7}$$

> $Sol[1]; Sol[2]$

$$x_1(t) = \_C1 + \_C2\, e^{11\,t}$$

$$x_2(t) = 2\,\_C2\, e^{11\,t} - \frac{3}{4}\,\_C1 \tag{8}$$

> *restart*

> $SistNoHom := diff(x[1](t), t) = 2 \cdot x[1](t) + 3 \cdot x[2](t) + \exp(2\,t) + t^2, diff(x[2](t), t)$
> $= x[1](t) + 4 \cdot x[2](t) + \cos(2\,t) : SistNoHom[1]; SistNoHom[2];$

$$\frac{d}{dt} x_1(t) = 2\, x_1(t) + 3\, x_2(t) + e^{2\,t} + t^2$$

$$\frac{d}{dt} x_2(t) = x_1(t) + 4\, x_2(t) + \cos(2\,t) \tag{9}$$

> $CondIni := x[1](0) = -3, x[2](0) = 5$

$$\tag{10}$$

$$CondIni := x_1(0) = -3, \ x_2(0) = 5 \tag{10}$$

> $AA := array([\,[2, 3], [1, 4]\,])$

$$AA := \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \tag{11}$$

> $Xcero := array([-3, 5])$

$$Xcero := \begin{bmatrix} -3 & 5 \end{bmatrix} \tag{12}$$

> $BB := array([\,[e^{2t} + t^2, \cos(2\,t)]\,])$

$$BB := \begin{bmatrix} e^{2t} + t^2 & \cos(2\,t) \end{bmatrix} \tag{13}$$

> $with(linalg):$
> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} \dfrac{3}{4}\,e^{t} + \dfrac{1}{4}\,e^{5t} & \dfrac{3}{4}\,e^{5t} - \dfrac{3}{4}\,e^{t} \\[2mm] \dfrac{1}{4}\,e^{5t} - \dfrac{1}{4}\,e^{t} & \dfrac{1}{4}\,e^{t} + \dfrac{3}{4}\,e^{5t} \end{bmatrix} \tag{14}$$

> $SolHom := evalm(MatExp \,\&*\, Xcero) : SolHom[1]; SolHom[2]$

$$-6\,e^{t} + 3\,e^{5t}$$
$$3\,e^{5t} + 2\,e^{t} \tag{15}$$

> $MatExpTau := map(rcurry(eval, t = t - \tau), MatExp)$

$$MatExpTau := \begin{bmatrix} \dfrac{3}{4}\,e^{t-\tau} + \dfrac{1}{4}\,e^{5t-5\tau} & \dfrac{3}{4}\,e^{5t-5\tau} - \dfrac{3}{4}\,e^{t-\tau} \\[2mm] \dfrac{1}{4}\,e^{5t-5\tau} - \dfrac{1}{4}\,e^{t-\tau} & \dfrac{1}{4}\,e^{t-\tau} + \dfrac{3}{4}\,e^{5t-5\tau} \end{bmatrix} \tag{16}$$

> $evalm(BB)$

$$\begin{bmatrix} e^{2t} + t^2 & \cos(2\,t) \end{bmatrix} \tag{17}$$

> $Btau := map(rcurry(eval, t = \tau), BB)$

$$Btau := \begin{bmatrix} e^{2\tau} + \tau^2 & \cos(2\,\tau) \end{bmatrix} \tag{18}$$

> $ProdTau := evalm(MatExpTau \,\&*\, Btau) : ProdTau[1]; ProdTau[2]$

$$\left(\dfrac{3}{4}\,e^{t-\tau} + \dfrac{1}{4}\,e^{5t-5\tau}\right)\left(e^{2\tau} + \tau^2\right) + \left(\dfrac{3}{4}\,e^{5t-5\tau} - \dfrac{3}{4}\,e^{t-\tau}\right)\cos(2\,\tau)$$
$$\left(\dfrac{1}{4}\,e^{5t-5\tau} - \dfrac{1}{4}\,e^{t-\tau}\right)\left(e^{2\tau} + \tau^2\right) + \left(\dfrac{1}{4}\,e^{t-\tau} + \dfrac{3}{4}\,e^{5t-5\tau}\right)\cos(2\,\tau) \tag{19}$$

> $SolNoHom := map(int, ProdTau, \tau = 0 ..t) : SolNoHom[1]; SolNoHom[2];$

$$\dfrac{2356}{10875}\,e^{5t} + \dfrac{3}{5}\,e^{t} + \dfrac{6}{145}\,\cos(t)^2 + \dfrac{2}{3}\,e^{2t} - \dfrac{72}{145}\,\cos(t)\sin(t) - \dfrac{38}{25}\,t - \dfrac{4}{5}\,t^2 - \dfrac{5527}{3625}$$
$$\dfrac{2356}{10875}\,e^{5t} - \dfrac{1}{5}\,e^{t} - \dfrac{52}{145}\,\cos(t)^2 - \dfrac{1}{3}\,e^{2t} + \dfrac{44}{145}\,\cos(t)\sin(t) + \dfrac{12}{25}\,t + \dfrac{1}{5}\,t^2 + \dfrac{2448}{3625} \tag{20}$$

> $SolFinal := evalm(SolHom + SolNoHom) : x[1](t) = simplify(SolFinal[1]); x[2](t)$
> $= simplify(SolFinal[2])$

$$x_1(t) = -\frac{27}{5} e^t + \frac{34981}{10875} e^{5t} + \frac{6}{145} \cos(t)^2 + \frac{2}{3} e^{2t} - \frac{72}{145} \cos(t) \sin(t) - \frac{38}{25} t - \frac{4}{5} t^2$$
$$\quad - \frac{5527}{3625}$$

$$x_2(t) = \frac{34981}{10875} e^{5t} + \frac{9}{5} e^t - \frac{52}{145} \cos(t)^2 - \frac{1}{3} e^{2t} + \frac{44}{145} \cos(t) \sin(t) + \frac{12}{25} t + \frac{1}{5} t^2 \qquad \textbf{(21)}$$
$$\quad + \frac{2448}{3625}$$

> $x[1](t) = evalf(SolFinal[1], 2); x[2](t) = evalf(SolFinal[2], 2)$

$$x_1(t) = -5.4 e^t + 3.2 e^{5.t} + 0.041 \cos(t)^2 + 0.67 e^{2.t} - 0.50 \cos(t) \sin(t) - 1.5 t - 0.80 t^2 - 1.5$$

$$x_2(t) = 3.2 e^{5.t} + 1.8 e^t - 0.36 \cos(t)^2 - 0.33 e^{2.t} + 0.30 \cos(t) \sin(t) + 0.48 t + 0.20 t^2 + 0.68 \qquad \textbf{(22)}$$

> $SistNoHom[1]$

$$\frac{d}{dt} x_1(t) = 2 x_1(t) + 3 x_2(t) + e^{2t} + t^2 \qquad \textbf{(23)}$$

> $ComprobarUno := simplify(eval(subs(x[1](t) = SolFinal[1], x[2](t) = SolFinal[2],$
> $\quad lhs(SistNoHom[1]) - rhs(SistNoHom[1]) = 0)))$

$$ComprobarUno := 0 = 0 \qquad \textbf{(24)}$$

> $SistNoHom[2]$

$$\frac{d}{dt} x_2(t) = x_1(t) + 4 x_2(t) + \cos(2 t) \qquad \textbf{(25)}$$

> $ComprobarDos := simplify(eval(subs(x[1](t) = SolFinal[1], x[2](t) = SolFinal[2],$
> $\quad lhs(SistNoHom[2]) - rhs(SistNoHom[2]) = 0)))$

$$ComprobarDos := 0 = 0 \qquad \textbf{(26)}$$

>