TEMA 4

¿Qué es una Ecuación Diferencial en Derivadas Parciales?

EDO $\longrightarrow F\left(x, y, \frac{dy}{dx}, \dots\right) = 0 \quad y = f(x)$

V.i.

EDenDP $\longrightarrow F\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \dots\right) = 0$

$z(x, y)$

$F\left(x, y, z, \theta, \frac{\partial \theta}{\partial x}, \dots\right) = 0$

V.i.

$\theta = f(x, y, z)$.

V.i.

EDenDP → orden

$$\frac{\partial z}{\partial x} + 4\frac{\partial z}{\partial y} = z \qquad \text{orden} = 1$$

$$\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial z}{\partial y} = 10e^{5x} + 4y$$

orden = 2

$$\frac{\partial^3 \theta}{\partial x \partial y^2} + \frac{\partial^2 \theta}{\partial z^2} = 0$$

orden = 3

grado de EDenDP.

"potencia entera a la que está elevada la derivada parcial de mayor orden"

$$\left(\frac{\partial^3 z}{\partial x^3}\right)^2 + \left(\frac{\partial^2 z}{\partial y^2}\right)^3 + \left(\frac{\partial z}{\partial x}\right)^2 = 0$$

orden = 3

grado = 2

$$\frac{\partial^2 z}{\partial x^2} - 6\left(\frac{\partial^3 z}{\partial x \partial y}\right)^2 = z$$

orden = 2

grado = 2

linealidad

lineals
No Lineales
Cuasi Lineales

Lineal en Derivadas Parciales

"es aquella que todos los terminos de la incógnita y todas derivadas son lineales."

$$\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial z}{\partial y} = z$$

No Lineal

$$z\frac{\partial z}{\partial x} + 5z = 0$$

$$\left(\frac{\partial z}{\partial y}\right)^2 + \frac{\partial z}{\partial x} = 0$$

Cuasi Lineal

"Si la derivada de mayor orden es linea, aunque las demás derivadas y la incógnita no lo sean, será ~~es~~ EnDP Cuasi Lineal."

$$\frac{\partial^2 z}{\partial y^2} + 5\left(\frac{\partial z}{\partial x}\right)^2 = z^3$$

$$\frac{\partial z}{\partial x} + 8\frac{\partial z}{\partial y} = z^2$$

$$\frac{\partial u(x,y)}{\partial x} - 2x\,\frac{\partial u(x,y)}{\partial y} = 0$$

$$u(x,y) = f(ax^2 + ay + b)$$

g

$$\frac{\partial u}{\partial x} = f' \cdot [2ax + 0 + 0]$$

$$\frac{\partial u}{\partial x} = 2ax\,f'$$

$$\frac{\partial u}{\partial y} = f' \cdot [a]$$

$$= a\,f'$$

$$[2axf'] - 2x[af'] = 0$$

$$(2ax - 2ax)f' = 0$$

$$(0)f' = 0$$

$$0 \equiv 0$$

$$\frac{\partial u}{\partial x} - 2x\frac{\partial u}{\partial y} = 0$$

$$u_g = f(ax^2 + ay + b)$$

$$u_p = e^{(ax^2 + ay + b)}$$

$$= e^{ax^2} e^{ay} e^{b}$$

$$u_p = c_1 e^{ax^2} e^{ay}$$

$$\frac{\partial u}{\partial x} = c_1 e^{ax^2} \cdot (2ax) \cdot e^{ay}$$

$$\frac{\partial u}{\partial y} = c_1 e^{ax^2} \cdot (a) e^{ay}$$

$$\left(2axc_1 e^{ax^2} e^{ay}\right) - 2x\left(c_1 a e^{ax^2} e^{ay}\right) = 0$$

$$(2axc_1 - 2axc_1) e^{ax^2} e^{ay} = 0$$

$$(0) e^{ax^2} e^{ay} = 0$$

$$0 = 0$$

$$\frac{\partial^2 z(x,y)}{\partial x^2} - 3\frac{\partial^2 z(x,y)}{\partial x \partial y} + 2\frac{\partial^2 z(x,y)}{\partial y^2} = 0$$

$$z(x,y) = f\,(mx+y)$$

$$\frac{\partial z}{\partial x} = f' \cdot (m) \qquad \frac{\partial z}{\partial y} = f' \cdot (1)$$

$$\frac{\partial^2 z}{\partial x^2} = f'' \cdot (m^2) \qquad \frac{\partial^2 z}{\partial y \partial x} = f'' \cdot (m) \qquad \frac{\partial^2 z}{\partial y^2} = f''$$

$$m^2 f'' - 3mf'' + 2f'' = 0$$

$$(m^2 - 3m + 2)f'' = 0 \qquad f'' = 0 \ \ \underline{\text{inútil.}}$$

$$(m^2 - 3m + 2) = 0 \qquad m_1 = 1$$

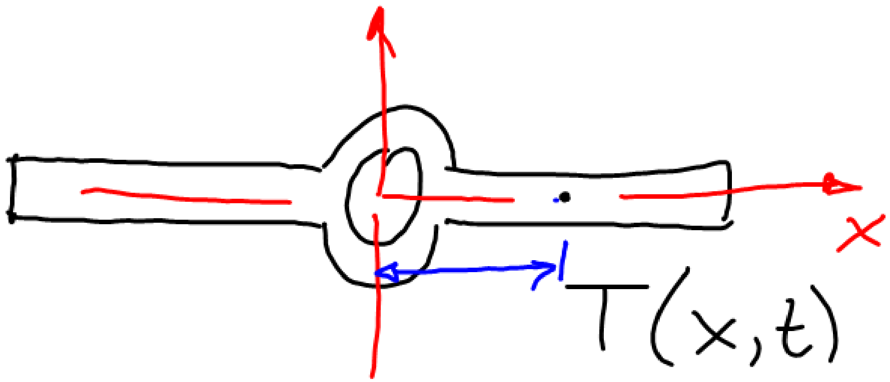
$$(m-1)(m-2) = 0 \qquad m_2 = 2$$

$$z(x,y) = f_1(x+y) + f_2(2x+y)$$

g

$$z_p = \cos(x+y) + 5e^{(2x+y)}$$

①

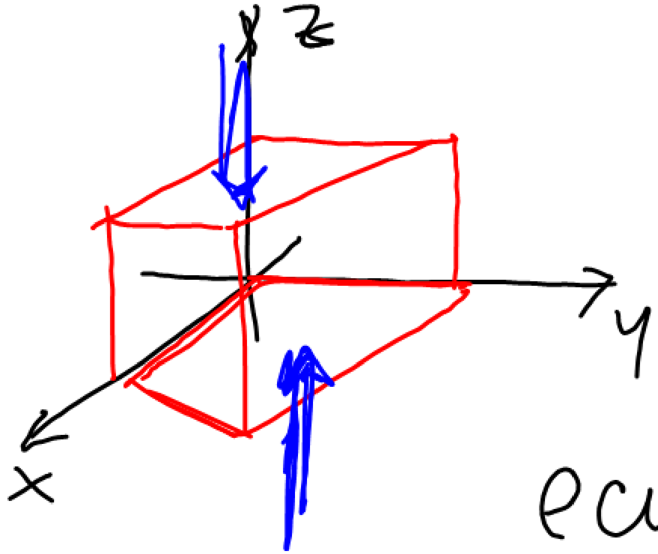


$$\frac{\partial^2 T(x,t)}{\partial x^2} = c\,\frac{\partial T(x,t)}{\partial t}$$

② Transmisión Eléctrica

$v(x,t) \qquad i(x,t)$

$$\begin{cases} -\dfrac{\partial v(x,t)}{\partial x} = L\,\dfrac{\partial i(x,t)}{\partial t} + R\,i(x,t) \\ -\dfrac{\partial i(x,t)}{\partial x} = K\,\dfrac{\partial v(x,t)}{\partial t} + S\,v(x,t) \end{cases}$$



Mecánica del
Medio Continuo

ecuación de Laplace

$$\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}=0$$

4