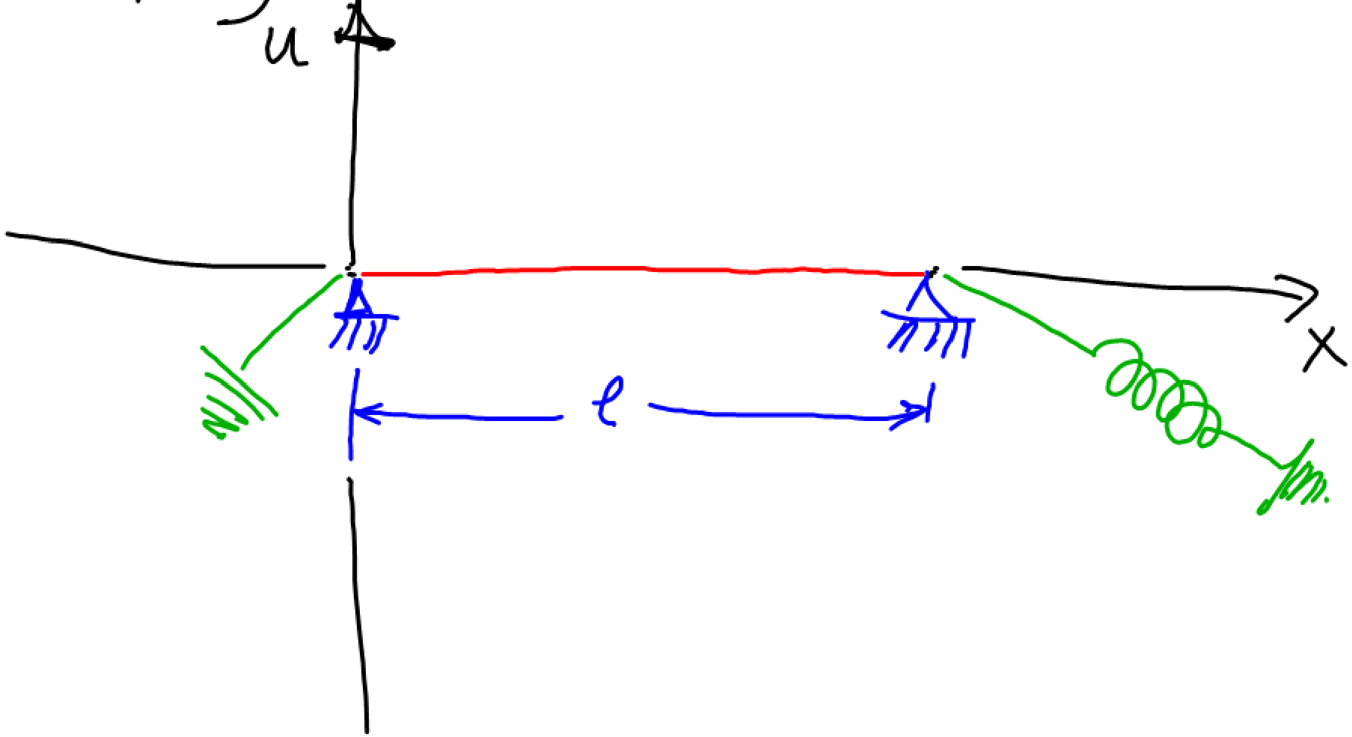
Temas III & IV

Examen parcial segundo

5 enero 2023 a las 11:00 h

Salón J205.

Resolver problema de la cuerda de guitarra.



$P(x_1, t_1)$. $\quad u(x,t)$

$$F_r = ma \qquad a = \frac{\partial^2 u(x,t)}{\partial t^2}$$

$$F_r = \rho \Delta s \frac{\partial^2 u(x,t)}{\partial t^2} \qquad m = \rho \Delta s$$

$$F_r = T_R - T_P \Rightarrow F_r = T_{V_R} - T_{V_P}$$

$$\operatorname{sen} \alpha \doteq \tan \alpha = \frac{\Delta u}{\Delta x}$$

$$\alpha < 4^\circ$$

$$T_{V_P} = T \frac{\Delta u}{\Delta x}$$

$$T_{V_P} = T \frac{\partial u}{\partial x} \qquad \Delta x \to 0$$

$$T_{V_R} = T \frac{\partial u}{\partial x} + \frac{\partial}{\partial x} T \left(\frac{\partial u}{\partial x} \right) \Delta x$$

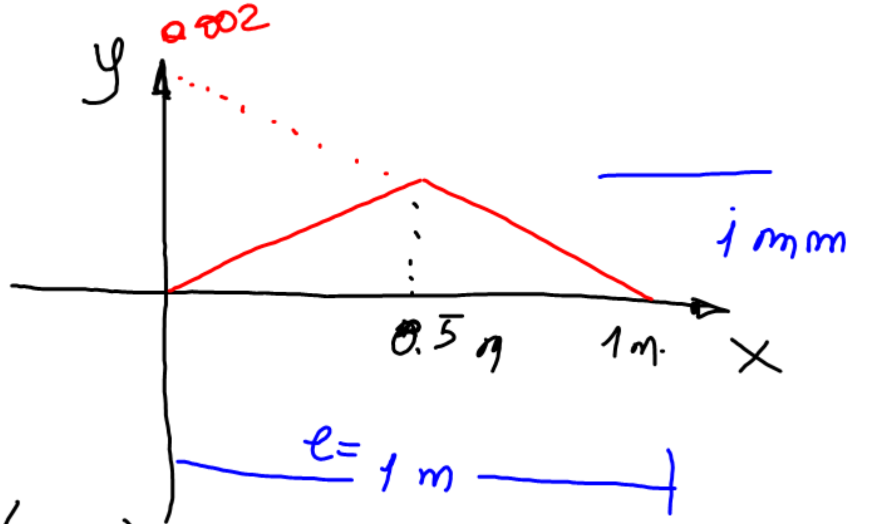
$$T_{V_R} = T \left(\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} \Delta x$$

$$F_R = T \frac{\partial^2 u}{\partial x^2} \Delta x$$

$$T \frac{\partial^2 u}{\partial x^2} \Delta x = \rho \Delta s \frac{\partial^2 u}{\partial t^2} \qquad \Delta x \to 0, \quad \Delta x = \Delta s$$

$$T \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2} \qquad \frac{T}{\rho} = c^2$$

$$\boxed{\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0}$$



Condiciones de frontera $y(0,t)=0$ $\qquad y(1,t)=0$

Condiciones iniciales

$$y(x,0)=f(x)\begin{cases}\dfrac{0{,}001}{0{,}5}x \; ; \; 0\leq x\leq 0{,}50 \text{ m}\\[2mm] 0{,}002-\dfrac{0{,}001}{0{,}5}x \; ; \; 0{,}50\leq x\leq 1 \text{ m.}\end{cases}$$

$$y'(x,0)=0$$

$$\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$$

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \qquad c^2 = 1.$$

CASO 1.

$$y(x,t) = F(x) \cdot G(t)$$

$$\frac{\partial^2 y}{\partial t^2} = F(x)\, G''(t) \qquad \frac{\partial^2 y}{\partial x^2} = F''(x) \cdot G(t)$$

$$F \cdot G'' = F'' \cdot G$$

$$\frac{G''}{G} = \frac{F''}{F} \qquad \frac{F''}{F} = \alpha \qquad \frac{G''}{G} = \alpha$$

para $\alpha = 0$

$$\frac{F''}{F} = 0 \rightarrow F'' = 0 \quad F \neq 0$$

$$\downarrow$$

$$F' = C_1$$

$$\downarrow$$

$$F(x) = C_1 x + C_2$$

$$y(0,t) = 0$$
$$y(1,t) = 0$$

$$F(0) \cdot G(t) = 0 \qquad F(0) = 0$$
$$F(1) \cdot G(t) = 0 \qquad F(1) = 0$$

$$\left.\begin{aligned} F(0) &= C_1 \cdot (0) + C_2 = 0 \qquad C_2 = 0 \\ F(1) &= C_1 \cdot (1) + (0) = 0 \qquad C_1 = 0 \end{aligned}\right\} F(x) = 0 \quad \underset{\alpha=0}{} \ \forall x$$

$$y(x,t)_{\alpha=0} = 0$$

para $\alpha > 0$ $\quad \alpha = \beta^2 \ \forall \beta \neq 0.$

$$y(x,t) = F(x) \cdot G(t)$$

$$\frac{F''(x)}{F(x)} = \beta^2 \qquad F''(x) = \beta^2 F(x)$$

$$F''(x) - \beta^2 F(x) = 0$$

$$(D^2 - \beta^2) F(x) = 0 \qquad m^2 - \beta^2 = 0$$

$$(m-\beta)(m+\beta) = 0$$

$$F(x) = C_1 e^{\beta x} + C_2 e^{-\beta x} \qquad m_1 = \beta \quad m_2 = -\beta$$

$$y(0,t) = F(0) \cdot G(t) = 0 \qquad F(0) = 0$$

$$y(1,t) = F(1) \cdot G(t) = 0 \qquad F(1) = 0$$

$$F(0) = C_1 e^{\beta(0)} + C_2 e^{-\beta(0)} = 0$$

$$C_1 + C_2 = 0 \qquad C_1 = -C_2$$

$$F(1) = C_1 e^{\beta} + C_2 e^{-\beta} = 0$$

$$C_1 e^{\beta} = -\frac{C_2}{e^{\beta}}$$

$$C_1 e^{2\beta} = -C_2$$

$$C_1 e^{2\beta} = C_1$$

$$e^{2\beta} = 1 \qquad \beta = 0$$

para $\alpha < 0$ $\quad \alpha = -\beta^2$

$$\frac{F''(x)}{F(x)} = -\beta^2 \qquad F''(x) = -\beta^2 F(x)$$

$$F''(x) + \beta^2 F(x) = 0$$

$$(D^2 + \beta^2) F(x) = 0$$

$$m^2 + \beta^2 = 0$$

$$m = \pm \beta i$$

$$F(x) = C_1 \cos(\beta x) + C_2 \operatorname{sen}(\beta x).$$

$$F(0) = C_1 \cos(0) + C_2 \operatorname{sen}(0) = 0$$

$$C_1 + (0) = 0 \qquad C_1 = 0$$

$$F(1) = C_2 \operatorname{sen}(\beta) = 0 \qquad \beta = n\pi$$

$$F(x) = C_2 \operatorname{sen}(n\pi) = 0$$

$$y(x,t) = C_2 \operatorname{sen}(n\pi x) \cdot G(t)$$

$$\frac{G(t)''}{G(t)} = -n\pi \qquad G''(t) = -n\pi\, G(t)$$

$$G''(t) + n\pi\, G(t) = 0$$

$$G(t) = C_{10} \operatorname{sen}(n\pi t) + \qquad (D^2 + n\pi) G(t) = 0$$

$$+ C_{20} \cos(n\pi t). \qquad m^2 + n\pi = 0 \qquad m_{1,2} = \pm n\pi i$$

$$y(x,t) = C_2 \operatorname{sen}(n\pi x)\left(C_{10} \operatorname{sen}(n\pi t) + C_{20} \cos(n\pi t)\right).$$

$$y(0,t) = 0 \qquad y(1,t) = 0$$