> *restart*

Problema de la cuerda de guitarra de 1 mt largo y rasgando 1 mm

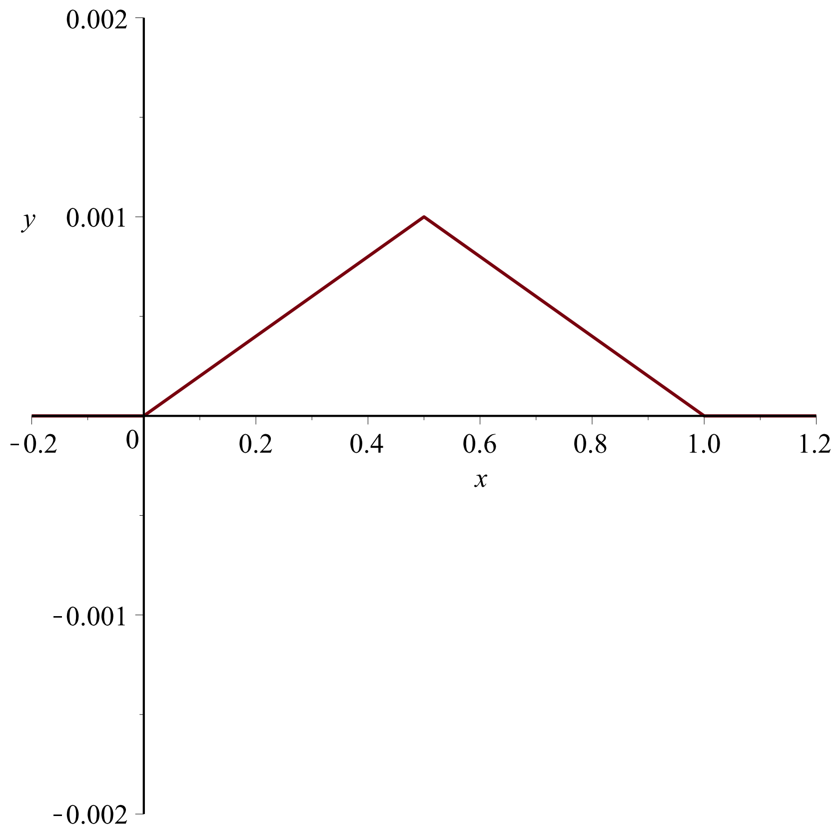
> $Ecua := diff(y(x,t), t\$2) = c^2 \cdot diff(y(x,t), x\$2)$

$$Ecua := \frac{\partial^2}{\partial t^2} y(x,t) = c^2 \left(\frac{\partial^2}{\partial x^2} y(x,t) \right) \quad \textbf{(1)}$$

> $CondIniTray := f = \frac{\left(\frac{1}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot x \cdot \mathrm{Heaviside}(x) - 2 \cdot \frac{\left(\frac{1}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot \left(x - \frac{5}{10}\right) \cdot \mathrm{Heaviside}\left(x - \frac{5}{10}\right) + \frac{\left(\frac{1}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot (x-1) \cdot \mathrm{Heaviside}(x-1)$

$$CondIniTray := f = \frac{1}{500}\, x\, \mathrm{Heaviside}(x) - \frac{1}{250} \left(x - \frac{1}{2}\right) \mathrm{Heaviside}\left(x - \frac{1}{2}\right) + \frac{1}{500} (x - 1)\, \mathrm{Heaviside}(x-1) \quad \textbf{(2)}$$

> $plot(rhs(CondIniTray), x = -0.2..1.2, y = -0.002..0.002)$



> $CondIniVel := 0$

(3)

$$CondIniVel := 0 \tag{3}$$

> $CondFrontera := F(0) = 0, F(1) = 0$

$$CondFrontera := F(0) = 0, F(1) = 0 \tag{4}$$

> $Hipotesis := y(x, t) = F(x) \cdot G(t)$

$$Hipotesis := y(x, t) = F(x)\, G(t) \tag{5}$$

> $EcuaSep := eval\left(subs\left(y(x, t) = rhs(Hipotesis), c^2 = 1, Ecua\right)\right)$

$$EcuaSep := F(x) \left(\frac{d^2}{dt^2}\, G(t) \right) = \left(\frac{d^2}{dx^2}\, F(x) \right) G(t) \tag{6}$$

> $EcuaSeparada := simplify\left(\frac{lhs(EcuaSep)}{F(x) \cdot G(t)} \right) = simplify\left(\frac{rhs(EcuaSep)}{F(x) \cdot G(t)} \right)$

$$EcuaSeparada := \frac{\frac{d^2}{dt^2}\, G(t)}{G(t)} = \frac{\frac{d^2}{dx^2}\, F(x)}{F(x)} \tag{7}$$

> $EcuaX := rhs(EcuaSeparada) = \text{alpha}$

$$EcuaX := \frac{\frac{d^2}{dx^2}\, F(x)}{F(x)} = \alpha \tag{8}$$

> $EcuaT := lhs(EcuaSeparada) = \text{alpha}$

$$EcuaT := \frac{\frac{d^2}{dt^2}\, G(t)}{G(t)} = \alpha \tag{9}$$

> $EcuaXneg := subs\left(\text{alpha} = -\beta^2, EcuaX\right)$

$$EcuaXneg := \frac{\frac{d^2}{dx^2}\, F(x)}{F(x)} = -\beta^2 \tag{10}$$

> $SolXneg := dsolve(EcuaXneg)$

$$SolXneg := F(x) = \_C1 \sin(\beta x) + \_C2 \cos(\beta x) \tag{11}$$

> $ParaDos := simplify(subs(x = 0, rhs(SolXneg) = 0))$

$$ParaDos := \_C2 = 0 \tag{12}$$

> $SolXnegBis := subs(\_C2 = rhs(ParaDos), SolXneg)$

$$SolXnegBis := F(x) = \_C1 \sin(\beta x) \tag{13}$$

> $\text{beta} := n \cdot \text{Pi}$

$$\beta := n\, \pi \tag{14}$$

> $SolXnegPart := SolXnegBis$

$$SolXnegPart := F(x) = \_C1 \sin(n\, \pi\, x) \tag{15}$$

> $EcuaTneg := subs\left(\text{alpha} = -\beta^2, EcuaT\right)$

$$EcuaTneg := \frac{\frac{d^2}{dt^2}\, G(t)}{G(t)} = -n^2\, \pi^2 \tag{16}$$

> $SolTneg := dsolve(EcuaTneg)$

$$SolTneg := G(t) = \_C1 \sin(n \pi t) + \_C2 \cos(n \pi t) \quad (17)$$

> $SolUno := y(x,t) = subs(\_C1 = 1, rhs(SolXnegPart)) \cdot rhs(SolTneg)$

$$SolUno := y(x,t) = \sin(n \pi x) \left(\_C1 \sin(n \pi t) + \_C2 \cos(n \pi t)\right) \quad (18)$$

HASTA LA CLASE DEL JUEVES

> $ComprobarUno := eval(subs(x = 0, SolUno))$

$$ComprobarUno := y(0,t) = 0 \quad (19)$$

> $ComprobarDos := subs(\sin(n \cdot \mathrm{Pi}) = 0, (subs(x = 1, SolUno)))$

$$ComprobarDos := y(1,t) = 0 \quad (20)$$

> $SolGeneral := y(x,t) = Sum(subs(\_C2 = b[n], \_C1 = a[n], rhs(SolUno)), n = 1 ..infinity)$

$$SolGeneral := y(x,t) = \sum_{n=1}^{\infty} \sin(n \pi x) \left(a_n \sin(n \pi t) + b_n \cos(n \pi t)\right) \quad (21)$$

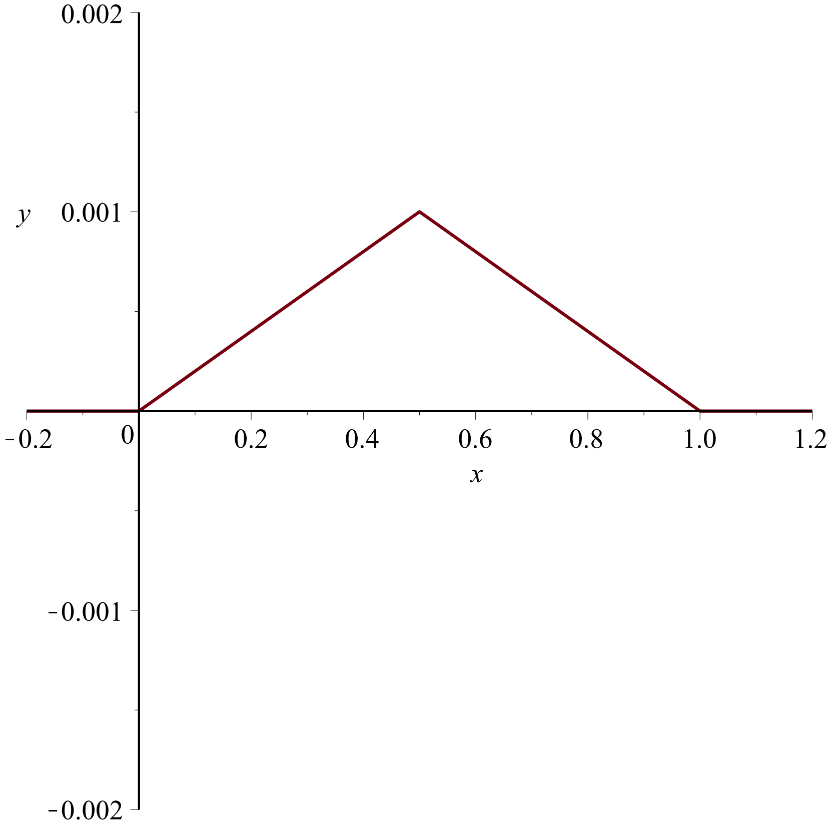
> $SolPartIni := F(x) = eval(subs(t = 0, rhs(SolGeneral)))$

$$SolPartIni := F(x) = \sum_{n=1}^{\infty} \sin(n \pi x)\, b_n \quad (22)$$

> $L := \frac{5}{10}$

$$L := \frac{1}{2} \quad (23)$$

> $plot(rhs(CondIniTray), x = -0.2 ..1.2, y = -0.002 ..0.002)$



> $b[n] := subs\left(\sin(n\cdot\text{Pi}) = 0, \left(\frac{1}{L}\right)\cdot int(rhs(CondIniTray)\cdot\sin(n\cdot\text{Pi}\cdot x), x = 0..1)\right)$

$$b_n := \frac{1}{125}\,\frac{\sin\left(\frac{1}{2}\,n\,\pi\right)}{n^2\,\pi^2} \tag{24}$$

> $a[n] := 0$

$$a_n := 0 \tag{25}$$

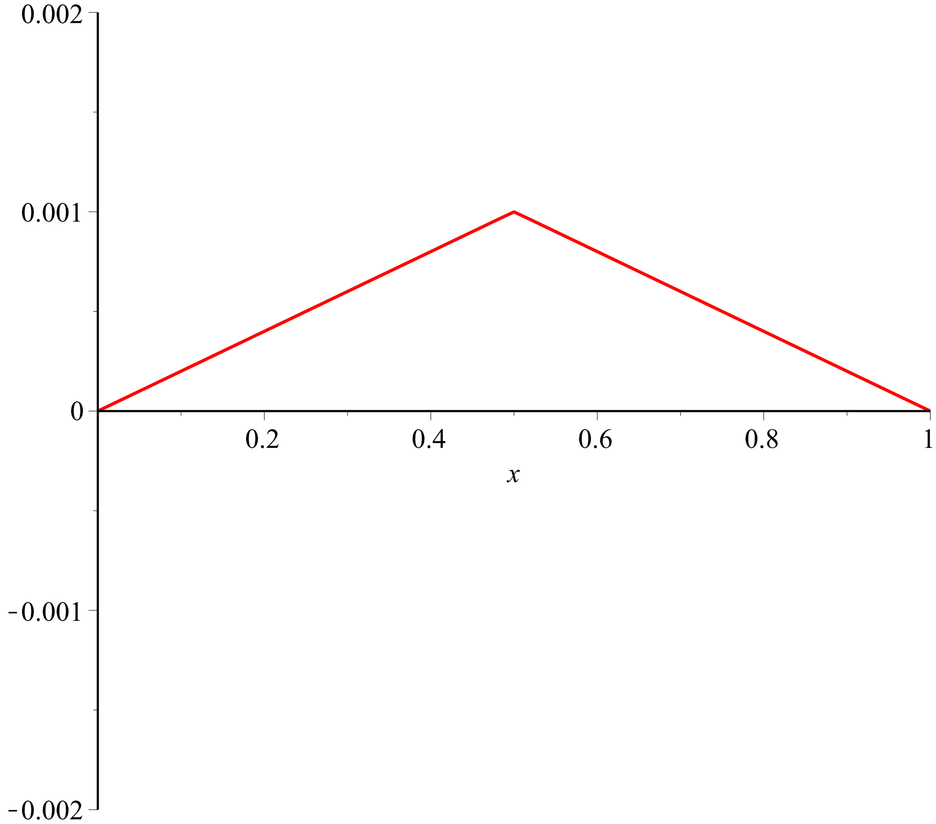
> $SolParticular := y(x,t) = Sum(subs(\_C2 = b[n], \_C1 = a[n], rhs(SolUno)), n = 1..infinity)$

$$SolParticular := y(x,t) = \sum_{n=1}^{\infty} \frac{1}{125}\,\frac{\sin(n\,\pi\,x)\,\sin\left(\frac{1}{2}\,n\,\pi\right)\,\cos(n\,\pi\,t)}{n^2\,\pi^2} \tag{26}$$

> $SolPart500 := y(x,t) = Sum(subs(\_C2 = b[n], \_C1 = a[n], rhs(SolUno)), n = 1..500)$:

> $with(plots)$:

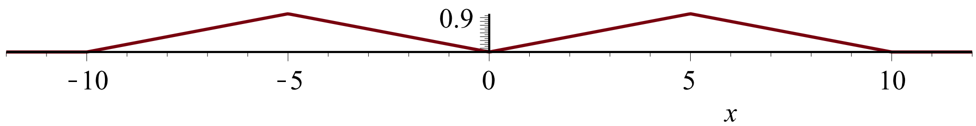
> $animate(rhs(SolPart500), x = 0..1, t = 0..2, frames = 150, view = [0..1, -0.002..0.002])$



> *restart*

> $f := \left(\frac{2}{10}\right)\cdot(x+10)\cdot\mathrm{Heaviside}(x+10) - 2\cdot\left(\frac{2}{10}\right)\cdot(x+5)\cdot\mathrm{Heaviside}(x+5) + \left(\frac{2}{10}\right)\cdot(x)\cdot\mathrm{Heaviside}(x) + \frac{2}{10}\cdot x\cdot\mathrm{Heaviside}(x) - 2\cdot\left(\frac{2}{10}\right)\cdot(x-5)\cdot\mathrm{Heaviside}(x-5) + \left(\frac{2}{10}\right)\cdot(x-10)\cdot\mathrm{Heaviside}(x-10);$ *plot*$(f, x=-12..12, scaling=CONSTRAINED)$

$$f := \frac{1}{5}\,(x+10)\,\mathrm{Heaviside}(x+10) - \frac{2}{5}\,(x+5)\,\mathrm{Heaviside}(x+5) + \frac{2}{5}\,x\,\mathrm{Heaviside}(x) - \frac{2}{5}\,(x-5)\,\mathrm{Heaviside}(x-5) + \frac{1}{5}\,(x-10)\,\mathrm{Heaviside}(x-10)$$



COMO LA FUNCIÓN f ES PAR

> $L := 10$

$$L := 10 \tag{27}$$

> $a[n] := \left(\frac{1}{L}\right) \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), x = -L..L\right)$

$$a_n := \frac{1}{10} \frac{80 \cos\left(\frac{1}{2} n \pi\right) - 40 \cos(n \pi) - 40}{n^2 \pi^2} \tag{28}$$

> $a[0] := \left(\frac{1}{L}\right) \cdot int(f, x = -L..L)$

$$a_0 := 1 \tag{29}$$

> $b[n] := \left(\frac{1}{L}\right) \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), x = -L..L\right)$

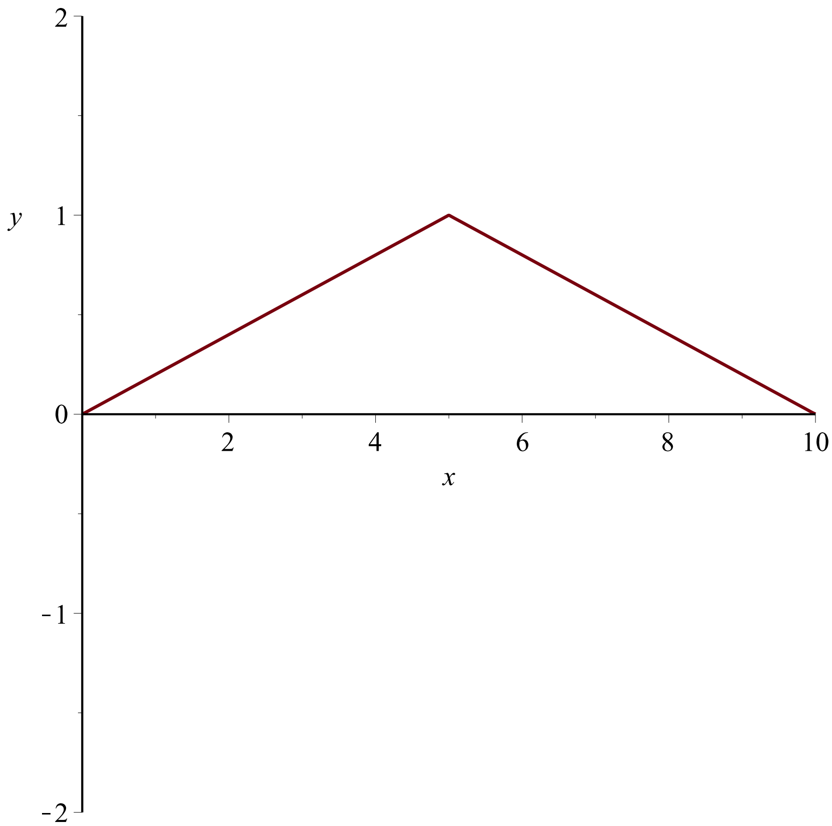
$$b_n := 0 \tag{30}$$

> $SolGeneralPar := \frac{a[0]}{2} + Sum\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), n = 1..infinity\right)$

$$SolGeneralPar := \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{10} \frac{\left(80\cos\left(\frac{1}{2}\, n\,\pi\right) - 40\cos(n\,\pi) - 40\right)\cos\left(\frac{1}{10}\, n\,\pi\, x\right)}{n^2\,\pi^2} \tag{31}$$

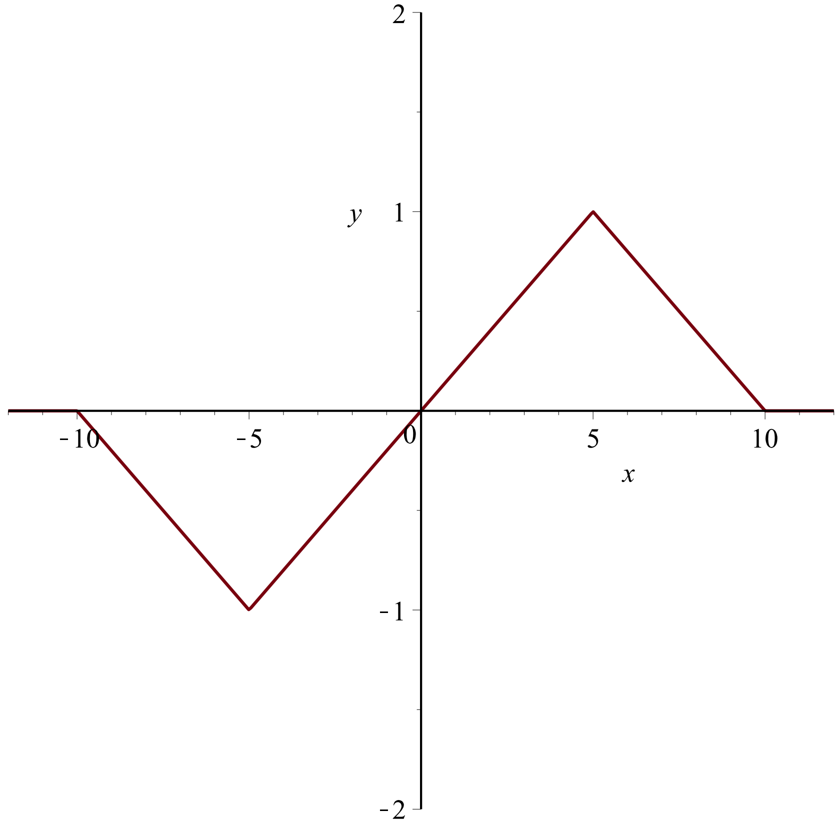
> $SolGral500 := \frac{a[0]}{2} + Sum\left(a[n]\cdot\cos\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), n = 1..500\right):$

> $plot(SolGral500, x = 0..10, y = -2..2)$



> $g := -\left(\frac{2}{10}\right)\cdot(x+10)\cdot\mathrm{Heaviside}(x+10) + 2\cdot\left(\frac{2}{10}\right)\cdot(x+5)\cdot\mathrm{Heaviside}(x+5) - \left(\frac{2}{10}\right)\cdot(x)\cdot\mathrm{Heaviside}(x) + \frac{2}{10}\cdot x\cdot\mathrm{Heaviside}(x) - 2\cdot\left(\frac{2}{10}\right)\cdot(x-5)\cdot\mathrm{Heaviside}(x-5) + \left(\frac{2}{10}\right)\cdot(x-10)\cdot\mathrm{Heaviside}(x-10);\ plot(g, x = -12..12, y = -2..2)$

$$g := -\frac{1}{5}\,(x+10)\,\mathrm{Heaviside}(x+10) + \frac{2}{5}\,(x+5)\,\mathrm{Heaviside}(x+5) - \frac{2}{5}\,(x-5)\,\mathrm{Heaviside}(x-5) + \frac{1}{5}\,(x-10)\,\mathrm{Heaviside}(x-10)$$



> a[0] := $\left(\frac{1}{L}\right)\cdot int(g, x=-L..L)$

$$a_0 := 0 \tag{32}$$

> a[n] := $\left(\frac{1}{L}\right)\cdot int\left(g\cdot\cos\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), x=-L..L\right)$

$$a_n := 0 \tag{33}$$

> b[n] := $\left(\frac{1}{L}\right)\cdot int\left(g\cdot\sin\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), x=-L..L\right)$

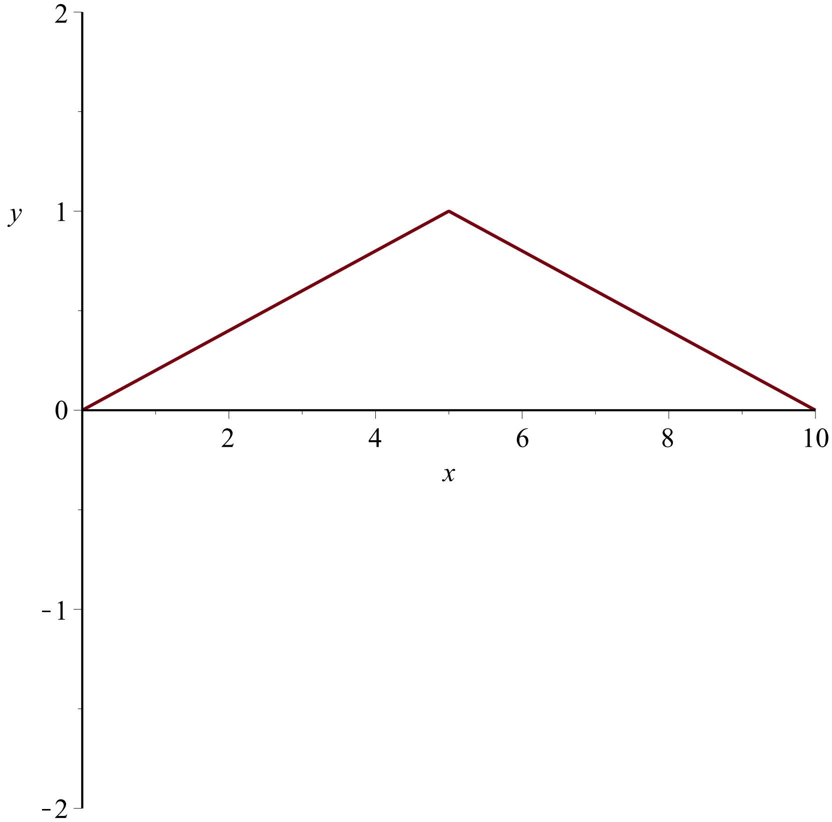
$$b_n := \frac{1}{10}\,\frac{-40\sin(n\pi) + 80\sin\left(\frac{1}{2}\,n\,\pi\right)}{n^2\,\pi^2} \tag{34}$$

> SolGralImpar := $Sum\left(b[n]\cdot\sin\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), n=1..infinity\right)$

$$SolGralImpar := \sum_{n=1}^{\infty} \frac{1}{10}\,\frac{\left(-40\sin(n\pi) + 80\sin\left(\frac{1}{2}\,n\,\pi\right)\right)\sin\left(\frac{1}{10}\,n\,\pi\,x\right)}{n^2\,\pi^2} \tag{35}$$

> $SolGralImpar500 := Sum\left(b[n]\cdot\sin\left(\frac{n\cdot\mathrm{Pi}}{L}\cdot x\right), n = 1..500\right):$

> $plot(SolGralImpar500, x = 0..10, y = -2..2)$



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