

Método de Parámetros Variables.

$$\frac{dy}{dx} + p(x)y = q(x)$$

$$y(x) = \underbrace{C_1 e^{-\int p(x) dx}}_{y_{g/h}} + \underbrace{e^{-\int p(x) dx} \int e^{\int p(x) dx} q(x) dx}_{y_{p/q}}$$

$$y(x) = \left(C_1 + \int e^{\int p(x) dx} q(x) dx \right) e^{-\int p(x) dx}$$

$$y_{g/h} = A(x) e^{-\int p(x) dx}$$

$$y_{g/h} = C_1 e^{-\int p(x) dx}$$

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 3e^{4x}$$

1º obtener la sol. gen. homog. asociada

$$m^2 - 5m + 6 = 0$$

$$(m-2)(m-3) = 0$$

$$y_{g/h} = c_1 e^{2x} + c_2 e^{3x}$$

$$\rightarrow y_{g/n-h} = A(x)e^{2x} + B(x)e^{3x}$$

$$\frac{dy}{dx} = 2A(x)e^{2x} + 3B(x)e^{3x} + \left[A'(x)e^{2x} + B'(x)e^{3x} \right]$$

$$\rightarrow \frac{dy}{dx} = 2A(x)e^{2x} + 3B(x)e^{3x} + (0)$$

$$\frac{d^2 y}{dx^2} = 4A(x)e^{2x} + 9B(x)e^{3x} + \left[2A'(x)e^{2x} + 3B'(x)e^{3x} \right]$$

$$\rightarrow \frac{d^2 y}{dx^2} = 4A(x)e^{2x} + 9B(x)e^{3x} + 3e^{4x} = Q(x) = 3e^{4x}$$

$$A'(x)e^{2x} + B'(x)e^{3x} = 0$$

$$2A'(x)e^{2x} + 3B'(x)e^{3x} = 3e^{4x}$$

$$\begin{bmatrix} e^{2x} & e^{3x} \\ 2e^{2x} & 3e^{3x} \end{bmatrix} \begin{bmatrix} A'(x) \\ B'(x) \end{bmatrix} = \begin{bmatrix} 0 \\ 3e^{4x} \end{bmatrix}$$

$$\begin{bmatrix} A'(x) \\ B'(x) \end{bmatrix} = \begin{bmatrix} 0 \\ 3e^{4x} \end{bmatrix} \begin{bmatrix} e^{2x} & e^{3x} \\ 2e^{2x} & 3e^{3x} \end{bmatrix}^{-1}$$

$$\begin{aligned} & A'(x)e^{2x} + B'(x)e^{3x} = 0 \\ * -2 \left(\begin{aligned} & 2A'(x)e^{2x} + 3B'(x)e^{3x} = 3e^{4x} \\ & -2A'(x)e^{2x} - 2B'(x)e^{3x} = 0 \end{aligned} \right. + \\ & \hline (0)e^{2x} + B'(x)e^{3x} = 3e^{4x} \end{aligned}$$

$$A'(x)e^{2x} + B'(x)e^{3x} = 0 \quad B'(x) = \frac{3e^{4x}}{e^{3x}} \Rightarrow 3e^x$$

$$A'(x)e^{2x} = -3e^{4x}$$

$$A'(x) = -3 \frac{e^{4x}}{e^{2x}} \Rightarrow -3e^{2x}$$

$$A(x) = -\frac{3}{2} \int e^{2x} dx \Rightarrow -\frac{3}{2}e^{2x} + C_1$$

$$B(x) = 3 \int e^x dx \Rightarrow 3e^x + C_2$$

$$y_{g/h} = \left(-\frac{3}{2}e^{2x} + C_1\right)e^{2x} + \left(3e^x + C_2\right)e^{3x}$$

$$= C_1e^{2x} + C_2e^{3x} + \left(-\frac{3}{2} + 3\right)e^{4x}$$

$$y_{g/h} = C_1e^{2x} + C_2e^{3x} + \frac{3}{2}e^{4x}$$

