

$f(t)$	$F(s)$
1	$\frac{1}{s}$
$t$	$\frac{1}{s^2}$
$t^n$	$\frac{n!}{s^{n+1}}$
$e^{at}$	$\frac{1}{s-a}$
$\cos(bt)$	$\frac{s}{s^2+b^2}$
$\sin(bt)$	$\frac{b}{s^2+b^2}$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt = F(s)$$

$$\mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} e^{st} F(s) ds = f(t)$$

$$s \in \mathbb{C}$$

$$F \in \mathbb{R}$$

① "Linealidad"

$$\mathcal{L}\{af(t) + bg(t)\} = aF(s) + bG(s)$$

$a, b \in \mathbb{R}$

$$\mathcal{L}\{5t\} = 5 \mathcal{L}\{t\} = \frac{5}{s^2}$$

②  $\mathcal{L}\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$

"Semejanza"

$$\mathcal{L}\{e^t\} = \frac{1}{s-1}$$

$$\mathcal{L}\{e^{4t}\} = \frac{1}{4} \left( \frac{1}{\frac{s}{4} - 1} \right)$$

$$= \left( \frac{\cancel{4}}{s-4} \right) \frac{1}{\cancel{4}}$$

$$\mathcal{L}\{e^{4t}\} = \frac{1}{s-4}$$

### ③ Derivada

$$\rightarrow \mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\rightarrow \mathcal{L}\{f''(t)\} = s^2 F(s) - sf(0) - f'(0)$$

$$\vdots \quad \mathcal{L}\{f'''(t)\} = s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$$

$$\begin{cases} y'' - 4y' + 3y = 8e^{2t} \\ \rightarrow y(0) = 4 \quad y'(0) = -3 \end{cases}$$

$$\mathcal{L}\{y'' - 4y' + 3y\} = \mathcal{L}\{8e^{2t}\}$$

$$\mathcal{L}\{y''\} - 4\mathcal{L}\{y'\} + 3\mathcal{L}\{y\} = 8\mathcal{L}\{e^{2t}\}$$

$$[s^2\mathcal{L}\{y\} - s(4) - (-3)] - 4[s\mathcal{L}\{y\} - (4)] + 3\mathcal{L}\{y\} = 8\left(\frac{1}{s-2}\right)$$

$$(s^2 - 4s + 3)\mathcal{L}\{y\} - 4s + 19 = \frac{8}{s-2}$$

$$\begin{aligned} (s^2 - 4s + 3)\mathcal{L}\{y\} &= \frac{8}{(s-2)} + 4s - 19 \\ &= \frac{8 + (4s - 19)(s-2)}{(s-2)} \\ &= \frac{4s^2 - 27s + 38 + 8}{(s-2)} \end{aligned}$$

$$(s^2 - 4s + 3)\mathcal{L}\{y\} = \frac{4s^2 - 27s + 46}{(s-2)}$$

$$\mathcal{L}\{y\} = \frac{4s^2 - 27s + 46}{(s-2)(s^2 - 4s + 3)}$$

$$\mathcal{L}\{y\} = \frac{4s^2 - 27s + 46}{(s-2)(s-3)(s-1)}$$

$$\frac{4s^2 - 27s + 46}{(s-2)(s-3)(s-1)} = \frac{A}{(s-2)} + \frac{B}{(s-3)} + \frac{C}{(s-1)}$$

$$4s^2 - 27s + 46 = A(s-3)(s-1) + B(s-2)(s-1) + C(s-2)(s-3)$$

$$= A(s^2 - 4s + 3) + B(s^2 - 3s + 2) + C(s^2 - 5s + 6)$$

$$4s^2 - 27s + 46 = (A+B+C)s^2 + (-4A-3B-5C)s + (3A+2B+6C)$$

$$\begin{array}{l} B - C = -11 \\ -3A + 3C = 34 \\ \hline 2C = 23 \\ \underline{C = \frac{23}{2}} \end{array} \quad \begin{array}{l} A + B + C = 4 \\ -4A - 3B - 5C = -27 \\ 3A + 2B + 6C = 46 \end{array}$$

$$B = -11 + C \quad B = -11 + \frac{23}{2}$$

$$B = \frac{1}{2}$$

$$A = -B - C + 4$$

$$A = -\frac{1}{2} - \frac{23}{2} + 4$$

$$A = -8$$

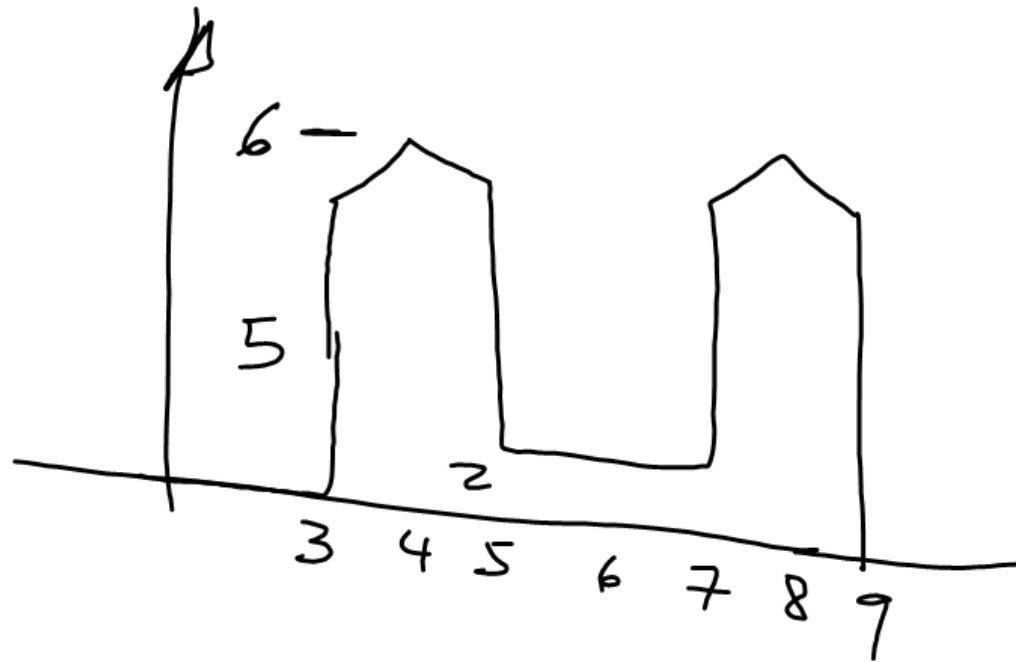
$$\hookrightarrow y = \frac{-8}{s-2} + \frac{1/2}{(s-3)} + \frac{23/2}{(s-1)}$$

$$y = -8 \mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} + \frac{1}{2} \mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} + \frac{23}{2} \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}$$

$$y = -8e^{2t} + \frac{1}{2}e^{3t} + \frac{23}{2}e^t$$

$$\frac{dy}{dt} = -16e^{2t} + \frac{3}{2}e^{3t} + \frac{23}{2}e^t$$

$$\frac{dy}{dt}\bigg|_{t=0} = -16 + \frac{26}{2}$$



$$4.- \mathcal{L}^{-1}\{F'(s)\} = -tf(t)$$

$$\mathcal{L}^{-1}\{F^{(n)}(s)\} = (-1)^n t^n f(t)$$

$$5.- \mathcal{L}\left\{\int_0^t f(t) dt\right\} = \frac{F(s)}{s}$$

$$6.- \mathcal{L}^{-1}\left\{\int_s^\infty F(\sigma) d\sigma\right\} = \frac{f(t)}{t}$$

$$7.- \mathcal{L}\{f(t-z)\} = e^{-sz} F(s)$$

$$8.- \mathcal{L}\{e^{at} f(t)\} = F(s-a)$$

9.- Convolution

$$\mathcal{L}^{-1}\{F(s)G(s)\} = f(t)*g(t)$$

$$f(t)*g(t) = \int_0^t f(\tau)g(t-\tau) d\tau$$