

> restart

> AA := array([[2, -7], [3, 6]])

$$AA := \begin{bmatrix} 2 & -7 \\ 3 & 6 \end{bmatrix} \quad (1)$$

> Xcero := array([-1, 6])

$$Xcero := \begin{bmatrix} -1 & 6 \end{bmatrix} \quad (2)$$

> with(linalg) :

> MatExp := exponential(AA, t) : evalf(MatExp[1, 1], 3); evalf(MatExp[1, 2], 3);
evalf(MatExp[2, 1], 3); evalf(MatExp[2, 2], 3)

$$\begin{aligned} & e^{4.1t} \cos(4.12 t) - 0.486 e^{4.1t} \sin(4.12 t) \\ & - 1.70 e^{4.1t} \sin(4.12 t) \\ & 0.725 e^{4.1t} \sin(4.12 t) \\ & e^{4.1t} \cos(4.12 t) + 0.486 e^{4.1t} \sin(4.12 t) \end{aligned} \quad (3)$$

> IdentidadUno := map(rcurry(eval, t=0'), MatExp)

$$IdentidadUno := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (4)$$

> InvMatExp := map(rcurry(eval, t=-t'), MatExp) :

> IdentidadDos := simplify(evalm(MatExp &* InvMatExp))

$$IdentidadDos := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (5)$$

> DerMatExp := map(diff, MatExp, t) :

> ProdAAMatExp := evalm(AA &* MatExp) :

> Comprobar := simplify(evalm(DerMatExp - ProdAAMatExp))

$$Comprobar := \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (6)$$

> SolPart := evalm(MatExp &* Xcero) : x[1](t) = SolPart[1]; x[2](t) = SolPart[2]

$$\begin{aligned} x_1(t) &= -e^{4t} \cos(\sqrt{17} t) - \frac{40 e^{4t} \sin(\sqrt{17} t) \sqrt{17}}{17} \\ x_2(t) &= \frac{9 e^{4t} \sin(\sqrt{17} t) \sqrt{17}}{17} + 6 e^{4t} \cos(\sqrt{17} t) \end{aligned} \quad (7)$$

> SolPartCero := simplify(map(rcurry(eval, t=0'), SolPart))

$$SolPartCero := \begin{bmatrix} -1 & 6 \end{bmatrix} \quad (8)$$

> Sistema := diff(x[1](t), t) = 2·x[1](t) - 7·x[2](t), diff(x[2](t), t) = 3·x[1](t) + 6·x[2](t) :
Sistema[1]; Sistema[2];

$$\frac{d}{dt} x_1(t) = 2 x_1(t) - 7 x_2(t)$$

$$\frac{d}{dt} x_2(t) = 3 x_1(t) + 6 x_2(t) \quad (9)$$

$$\begin{aligned} &> \text{ComprobarUno} := \text{simplify}(\text{eval}(\text{subs}(x[1](t) = \text{SolPart}[1], x[2](t) = \text{SolPart}[2], \\ &\quad \text{lhs}(\text{Sistema}[1]) - \text{rhs}(\text{Sistema}[1]) = 0))) \\ &\quad \text{ComprobarUno} := 0 = 0 \end{aligned} \quad (10)$$

$$\begin{aligned} &> \text{ComprobarDos} := \text{simplify}(\text{eval}(\text{subs}(x[1](t) = \text{SolPart}[1], x[2](t) = \text{SolPart}[2], \\ &\quad \text{lhs}(\text{Sistema}[2]) - \text{rhs}(\text{Sistema}[2]) = 0))) \\ &\quad \text{ComprobarDos} := 0 = 0 \end{aligned} \quad (11)$$

restart

$$\begin{aligned} &> AA := \text{array}([[0, 1, 0], [0, -2, -5], [0, 1, 2]]) \\ &\quad AA := \begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & -5 \\ 0 & 1 & 2 \end{bmatrix} \end{aligned} \quad (12)$$

$\text{with}(\text{linalg}) :$

$$\begin{aligned} &> \text{MatExp} := \text{exponential}(AA, t) \\ &\quad \text{MatExp} := \begin{bmatrix} 1 & \sin(t) + 2 \cos(t) - 2 & 5 \cos(t) - 5 \\ 0 & -2 \sin(t) + \cos(t) & -5 \sin(t) \\ 0 & \sin(t) & 2 \sin(t) + \cos(t) \end{bmatrix} \end{aligned} \quad (13)$$

$$\begin{aligned} &> Xcero := \text{array}([0, 0, 1]) \\ &\quad Xcero := \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (14)$$

$$\begin{aligned} &> BB := \text{array}([0, 3, 0]) \\ &\quad BB := \begin{bmatrix} 0 & 3 & 0 \end{bmatrix} \end{aligned} \quad (15)$$

$$\begin{aligned} &> \text{ParteUno} := \text{evalm}(\text{MatExp} \& Xcero) : \text{ParteUno}[1]; \text{ParteUno}[2]; \text{ParteUno}[3] \\ &\quad 5 \cos(t) - 5 \\ &\quad -5 \sin(t) \\ &\quad 2 \sin(t) + \cos(t) \end{aligned} \quad (16)$$

$$\begin{aligned} &> \text{MatExpTau} := \text{map}(\text{rcurry}(\text{eval}, t \mapsto t - \text{tau}'), \text{MatExp}) \\ &\quad \text{MatExpTau} := \begin{bmatrix} 1 & \sin(t - \tau) + 2 \cos(t - \tau) - 2 & 5 \cos(t - \tau) - 5 \\ 0 & -2 \sin(t - \tau) + \cos(t - \tau) & -5 \sin(t - \tau) \\ 0 & \sin(t - \tau) & 2 \sin(t - \tau) + \cos(t - \tau) \end{bmatrix} \end{aligned} \quad (17)$$

$$\begin{aligned} &> \text{ProdTau} := \text{evalm}(\text{MatExpTau} \& BB) : \text{ProdTau}[1]; \text{ProdTau}[2]; \text{ProdTau}[3] \\ &\quad 3 \sin(t - \tau) + 6 \cos(t - \tau) - 6 \\ &\quad -6 \sin(t - \tau) + 3 \cos(t - \tau) \\ &\quad 3 \sin(t - \tau) \end{aligned} \quad (18)$$

$$\begin{aligned} &> \text{ParteDos} := \text{map}(\text{int}, \text{ProdTau}, \text{tau} = 0 .. t) : \text{ParteDos}[1]; \text{ParteDos}[2]; \text{ParteDos}[3]; \\ &\quad 3 + 6 \sin(t) - 3 \cos(t) - 6 t \\ &\quad -6 + 6 \cos(t) + 3 \sin(t) \end{aligned}$$

$$3 - 3 \cos(t) \quad (19)$$

> *ComprobarTres* := map(rcurry(eval, t='0'), ParteDos)

$$\text{ComprobarTres} := \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \quad (20)$$

> restart

> *AA* := array([[1, 2], [2, 4]])

$$AA := \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \quad (21)$$

> *Xcero* := array([_C1, _C2])

$$Xcero := \begin{bmatrix} _C1 & _C2 \end{bmatrix} \quad (22)$$

> *BB* := array([3·exp(t), 4·exp(-t)])

$$BB := \begin{bmatrix} 3 e^t & 4 e^{-t} \end{bmatrix} \quad (23)$$

> with(linalg) :

> *MatExp* := exponential(*AA*, *t*)

$$MatExp := \begin{bmatrix} \frac{4}{5} + \frac{e^{5t}}{5} & \frac{2e^{5t}}{5} - \frac{2}{5} \\ \frac{2e^{5t}}{5} - \frac{2}{5} & \frac{1}{5} + \frac{4e^{5t}}{5} \end{bmatrix} \quad (24)$$

> *ComprobarUno* := det(*AA*)

$$\text{ComprobarUno} := 0 \quad (25)$$

> *ParteUno* := evalm(*MatExp* &* *Xcero*) : *ParteUno*[1]; *ParteUno*[2]

$$\begin{aligned} & \left(\frac{4}{5} + \frac{e^{5t}}{5} \right) _C1 + \left(\frac{2e^{5t}}{5} - \frac{2}{5} \right) _C2 \\ & \left(\frac{2e^{5t}}{5} - \frac{2}{5} \right) _C1 + \left(\frac{1}{5} + \frac{4e^{5t}}{5} \right) _C2 \end{aligned} \quad (26)$$

> *ComprobarDos* := map(rcurry(eval, t='0'), *ParteUno*)

$$\text{ComprobarDos} := \begin{bmatrix} _C1 & _C2 \end{bmatrix} \quad (27)$$

> *MatExpTau* := map(rcurry(eval, t='t - tau'), *MatExp*)

$$MatExpTau := \begin{bmatrix} \frac{4}{5} + \frac{e^{5t-5\tau}}{5} & \frac{2e^{5t-5\tau}}{5} - \frac{2}{5} \\ \frac{2e^{5t-5\tau}}{5} - \frac{2}{5} & \frac{1}{5} + \frac{4e^{5t-5\tau}}{5} \end{bmatrix} \quad (28)$$

> *BBtau* := map(rcurry(eval, t='tau'), *BB*)

$$BBtau := \begin{bmatrix} 3 e^\tau & 4 e^{-\tau} \end{bmatrix} \quad (29)$$

> *ProdTau* := evalm(*MatExpTau* &* *BBtau*) : *ProdTau*[1]; *ProdTau*[2]

$$3 \left(\frac{4}{5} + \frac{e^{5t-5\tau}}{5} \right) e^\tau + 4 \left(\frac{2e^{5t-5\tau}}{5} - \frac{2}{5} \right) e^{-\tau}$$

$$3 \left(\frac{2 e^{5t-5\tau}}{5} - \frac{2}{5} \right) e^\tau + 4 \left(\frac{1}{5} + \frac{4 e^{5t-5\tau}}{5} \right) e^{-\tau} \quad (30)$$

> *ParteDos* := map(int, ProdTau, tau=0..t) : *ParteDos*[1]; *ParteDos*[2]

$$\begin{aligned} & -4 + \frac{5 e^{5t}}{12} + \frac{9 e^t}{4} + \frac{4 e^{-t}}{3} \\ & 2 + \frac{5 e^{5t}}{6} - \frac{4 e^{-t}}{3} - \frac{3 e^t}{2} \end{aligned} \quad (31)$$

> *ComprobarTres* := map(rcurry(eval, t=0'), *ParteDos*)

$$\text{ComprobarTres} := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (32)$$

> *SolGral* := evalm(*ParteUno* + *ParteDos*) : *x*[1](*t*) = *SolGral*[1]; *x*[2](*t*) = *SolGral*[2]

$$\begin{aligned} x_1(t) &= \left(\frac{4}{5} + \frac{e^{5t}}{5} \right) _{C1} + \left(\frac{2 e^{5t}}{5} - \frac{2}{5} \right) _{C2} - 4 + \frac{5 e^{5t}}{12} + \frac{9 e^t}{4} + \frac{4 e^{-t}}{3} \\ x_2(t) &= \left(\frac{2 e^{5t}}{5} - \frac{2}{5} \right) _{C1} + \left(\frac{1}{5} + \frac{4 e^{5t}}{5} \right) _{C2} + 2 + \frac{5 e^{5t}}{6} - \frac{4 e^{-t}}{3} - \frac{3 e^t}{2} \end{aligned} \quad (33)$$

> *SolGralCero* := map(rcurry(eval, t=0'), *SolGral*)

$$\text{SolGralCero} := \begin{bmatrix} _{C1} & _{C2} \end{bmatrix} \quad (34)$$

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