Método de Variables Separables

Restricción:

1) Sólo para EDenDP cuya solución depende de 2. variables independ.
2) Parte de suponer una hipótesis para aplicar una prueba y error
3) Hipótesis funciona, entonces procedimos a separar las Variables y resolver la EDOrdinarias correspondientes.

$$\frac{\partial^2 z}{\partial x^2} - 6\frac{\partial z}{\partial y} = z \qquad z(x,y)$$

$$H_0: \; z(x,y) = F(x)\cdot G(y)$$

$$\frac{\partial z}{\partial x} = F'(x)\cdot G(y)$$

$$\frac{\partial z}{\partial y} = F(x)\cdot G'(y)$$

$$\frac{\partial^2 z}{\partial x^2} = F''(x)\cdot G(y)$$

$$\left(F''(x)\cdot G(y)\right) - 6\left(F(x)\cdot G'(y)\right) = F(x)\cdot G(y)$$

$$F''(x)\cdot G(y) = 6F(x)\cdot G'(y) + F(x)\cdot G(y)$$

$$= F(x)\cdot\left(6G'(y) + G(y)\right)$$

$$\frac{F''(x)}{F(x)} = \frac{6G'(y) + G(y)}{G(y)} \qquad SVS$$

$\alpha = \text{constante}.$

$$\frac{F''(x)}{F(x)} = \alpha \qquad \frac{6G'(y)+G(y)}{G(y)} = \alpha$$

$\alpha = 0 \; ; \; \alpha > 0 \; ; \; \alpha < 0$

CASO I: $\alpha = 0$

$$\frac{F''(x)}{F(x)} = 0 \qquad F(x) \neq 0 \qquad F''(x) = 0$$

$$F'(x) = C_1$$

$$\boxed{F(x) = C_1 x + C_2}$$

$$\frac{6G'(y)+G(y)}{G(y)} = 0 \qquad G(y) \neq 0$$

$$6G'(y) + G(y) = 0$$

$$6G'(y) = -G(y)$$

$$G'(y) = -\frac{G(y)}{6}$$

EDO(1) LCCH. $\quad G'(y) + \frac{G(y)}{6} = 0$

$$m + \frac{1}{6} = 0$$

$$m = -\frac{1}{6}$$

$$\boxed{G(y) = k_1 e^{-\frac{1}{6}y}}$$

$$Z(x,y) = (C_1 \cdot x + C_2)\, k_1 e^{-\frac{1}{6}y}$$

$$\boxed{Z_{\alpha=0}(x,y) = e^{-\frac{1}{6}y}\left(C_{10}\, x + C_{20}\right).}$$

CASO II:- $\alpha > 0$ $\quad \alpha = \beta^2 \quad \forall \beta \neq 0 \in \mathbb{R}$

$$\frac{F''(x)}{F(x)} = \beta^2 \qquad \frac{6G'(y)+G(y)}{G(y)} = \beta^2$$

$$F''(x) = \beta^2 F(x)$$

$$F''(x) - \beta^2 F(x) = 0$$

$$m^2 - \beta^2 = 0$$

$$(m+\beta)(m-\beta) = 0 \qquad m_1 = \beta \quad m_2 = -\beta$$

$$F(x) = c_1 e^{\beta x} + c_2 e^{-\beta x}$$

$$\frac{6G'(y)+G(y)}{G(y)} = \beta^2$$

$$6G'(y) + G(y) = \beta^2 G(y)$$

$$6G'(y) + (1-\beta^2) G(y) = 0$$

$$G'(y) + \left(\frac{1-\beta^2}{6}\right) G(y) = 0$$

$$m - \left(\frac{\beta^2-1}{6}\right) = 0 \qquad m = \frac{\beta^2-1}{6}$$

$$G(y) = k_1 e^{\left(\frac{\beta^2-1}{6} y\right)}$$

$$Z(x,y)_{\alpha>0} = \left(c_1 e^{\beta x} + c_2 e^{-\beta x}\right) k_1 e^{\left(\frac{\beta^2-1}{6} y\right)}$$

$$\boxed{Z(x,y)_{\alpha>0} = e^{\left(\frac{\beta^2-1}{6} y\right)} \left(c_{10} e^{\beta x} + c_{20} e^{-\beta x}\right)}$$

CASO 3.- $\alpha < 0$ $\quad \alpha = -\beta^2 \quad \forall \beta \neq 0 \in \mathbb{R}$

$$\frac{F''(x)}{F(x)} = -\beta^2 \qquad \frac{6G'(y)+G(y)}{G(y)} = -\beta^2$$

$$F''(x) = -\beta^2 F(x)$$

$$F''(x) + \beta^2 F(x) = 0$$

$$m^2 + \beta^2 = 0 \qquad m_1 = +\beta i$$
$$m_2 = -\beta i$$

$$F(x) = C_1 \cos(\beta x) + C_2 \operatorname{sen}(\beta x)$$

$$\frac{6G'(y)+G(y)}{G(y)} = -\beta^2$$

$$6G'(y) + G(y) = -\beta^2 G(y)$$

$$6G'(y) + G(y) + \beta^2 G(y) = 0$$

$$G'(y) + \left(\frac{1+\beta^2}{6}\right) G(y) = 0$$

$$m + \left(\frac{1+\beta^2}{6}\right) = 0 \qquad m = -\left(\frac{1+\beta^2}{6}\right)$$

$$G(y) = k_1 e^{-\left(\frac{1+\beta^2}{6} y\right)}$$

$$\boxed{Z(x,y)_{\alpha<0} = e^{-\left(\frac{1+\beta^2}{6} y\right)} \left(C_{10} \cos(\beta x) + C_{20} \operatorname{sen}(\beta x) \right)}$$

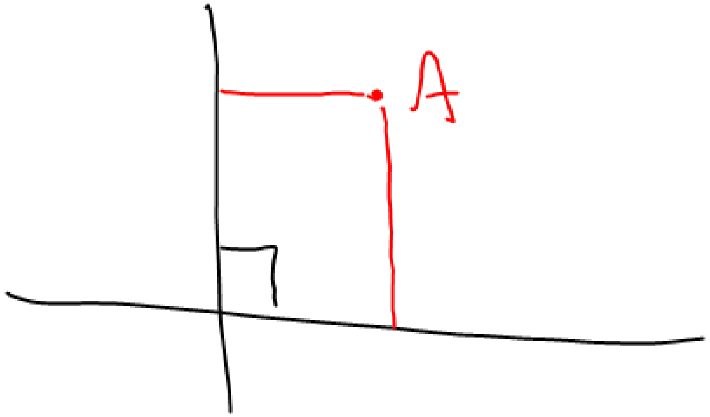
$$\frac{\partial^2 z}{\partial x^2} - 6\frac{\partial z}{\partial y} = z \qquad z = F(x)\cdot G(y)$$

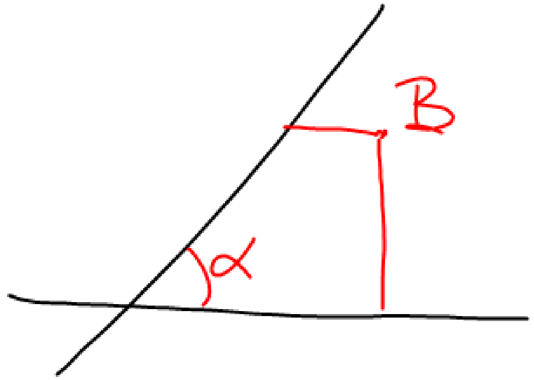
$$\left(F''(x)\cdot G(y)\right) - \left(F(x)\cdot G(y)\right) = 6\left(F(x)\cdot G'(y)\right)$$

$$G(y)\left(F''(x) - F(x)\right) = 6\left(F(x)\cdot G'(y)\right)$$

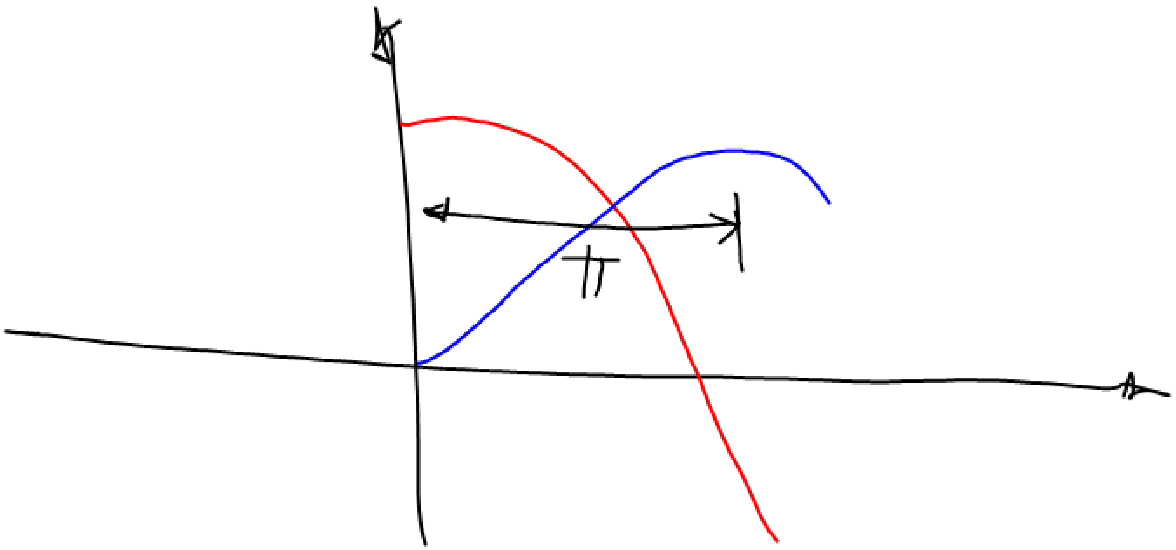
$$\frac{F''(x) - F(x)}{6F(x)} = \frac{G'(y)}{G(y)}$$

SERIE TRIGONOMÉTRICA FOURIER



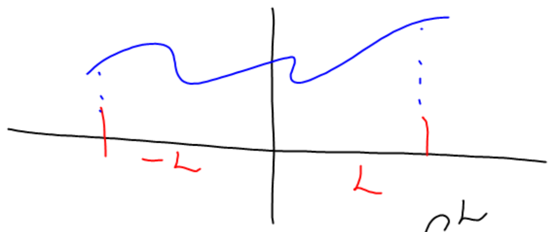


$A(x, y)$ $B(x, y, \alpha)$



$$f(x) = C + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \operatorname{sen}\left(\frac{n\pi}{L}x\right)\right)$$

SERIE TRIG. FOURIER.



$$C = \frac{a_0}{2} \qquad a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi}{L}x\right)dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\operatorname{sen}\left(\frac{n\pi}{L}x\right)dx.$$