> restart

> $EcuacionOriginal := diff(y(x,t), t\$2) = c^2 \cdot diff(y(x,t), x\$2)$

$$EcuacionOriginal := \frac{\partial^2}{\partial t^2}\, y(x,t) = c^2 \left(\frac{\partial^2}{\partial x^2}\, y(x,t)\right) \tag{1}$$

> $Ecuacion := subs(c^2 = 1, EcuacionOriginal)$

$$Ecuacion := \frac{\partial^2}{\partial t^2}\, y(x,t) = \frac{\partial^2}{\partial x^2}\, y(x,t) \tag{2}$$

> $EcuacionSeparable := eval(subs(y(x,t) = F(x)\cdot G(t), Ecuacion))$

$$EcuacionSeparable := F(x)\left(\frac{d^2}{dt^2}\, G(t)\right) = \left(\frac{d^2}{dx^2}\, F(x)\right) G(t) \tag{3}$$

> $EcuacionSeparada := \frac{lhs(EcuacionSeparable)}{F(x)\cdot G(t)} = \frac{rhs(EcuacionSeparable)}{F(x)\cdot G(t)}$

$$EcuacionSeparada := \frac{\frac{d^2}{dt^2}\, G(t)}{G(t)} = \frac{\frac{d^2}{dx^2}\, F(x)}{F(x)} \tag{4}$$

> $Ecuacion[x] := rhs(EcuacionSeparada) = \mathrm{alpha}; Ecuacion[t] := lhs(EcuacionSeparada) = \mathrm{alpha}$

$$Ecuacion_x := \frac{\frac{d^2}{dx^2}\, F(x)}{F(x)} = \alpha$$

$$Ecuacion_t := \frac{\frac{d^2}{dt^2}\, G(t)}{G(t)} = \alpha \tag{5}$$

> $SolucionCero[x] := dsolve(subs(\mathrm{alpha} = 0, Ecuacion[x]))$

$$SolucionCero_x := F(x) = c_1\, x + c_2 \tag{6}$$

> $Sistema := subs(x = 0, rhs(SolucionCero[x]) = 0), subs(x = 1, rhs(SolucionCero[x]) = 0) : Sistema[1]; Sistema[2]$

$$c_2 = 0$$

$$c_1 + c_2 = 0 \tag{7}$$

> $Parametros := solve(\{Sistema\}, \{c_1, c_2\})$

$$Parametros := \{c_1 = 0, c_2 = 0\} \tag{8}$$

> $SolucionPos[x] := dsolve(subs(\mathrm{alpha} = \beta^2, Ecuacion[x]))$

$$SolucionPos_x := F(x) = c_1\, e^{\beta x} + c_2\, e^{-\beta x} \tag{9}$$

> $SistemaPos := eval(subs(x = 0, rhs(SolucionPos[x]) = 0)), subs(x = 1, rhs(SolucionPos[x]) = 0) : SistemaPos[1]; SistemaPos[2]$

$$c_1 + c_2 = 0$$

$$c_1\, e^{\beta} + c_2\, e^{-\beta} = 0 \tag{10}$$

> $ParametroPos := solve(\{SistemaPos\})$

$$ParametroPos := \{\beta = \beta, c_1 = 0, c_2 = 0\}, \{\beta = 0, c_1 = -c_2, c_2 = c_2\} \quad \textbf{(11)}$$

> $SolucionNeg[x] := dsolve\left(subs\left(\text{alpha} = -\beta^2, Ecuacion[x]\right)\right)$

$$SolucionNeg_x := F(x) = c_1 \sin(\beta x) + c_2 \cos(\beta x) \quad \textbf{(12)}$$

> $SistemaNeg := eval(subs(x = 0, rhs(SolucionNeg[x]) = 0)), subs(x = 1, rhs(SolucionNeg[x]) = 0) : SistemaNeg[1]; SistemaNeg[2]$

$$c_2 = 0$$

$$c_1 \sin(\beta) + c_2 \cos(\beta) = 0 \quad \textbf{(13)}$$

> $SolucionNegMod[x] := subs\left(\text{beta} = n \cdot \text{Pi}, c_2 = 0, SolucionNeg[x]\right)$

$$SolucionNegMod_x := F(x) = c_1 \sin(n \pi x) \quad \textbf{(14)}$$

> $SolucionNegMod[t] := dsolve\left(subs\left(\text{alpha} = -\beta^2, \text{beta} = n \cdot \text{Pi}, Ecuacion[t]\right)\right)$

$$SolucionNegMod_t := G(t) = c_1 \sin(n \pi t) + c_2 \cos(n \pi t) \quad \textbf{(15)}$$

> $SolucionNeg := y(x, t) = subs\left(c_1 = 1, rhs(SolucionNegMod[x])\right) \cdot rhs(SolucionNegMod[t])$

$$SolucionNeg := y(x, t) = \sin(n \pi x) \left(c_1 \sin(n \pi t) + c_2 \cos(n \pi t)\right) \quad \textbf{(16)}$$

> $SolucionGeneral := y(x, t) = Sum\left(rhs(SolucionNegMod[x]) \cdot subs\left(c_1 = a[n], c_2 = b[n], rhs(SolucionNegMod[t])\right), n = 1 ..\text{infinity}\right)$

$$SolucionGeneral := y(x, t) = \sum_{n=1}^{\infty} c_1 \sin(n \pi x) \left(a_n \sin(n \pi t) + b_n \cos(n \pi t)\right) \quad \textbf{(17)}$$

> $eval(subs(t = 0, SolucionGeneral))$

$$y(x, 0) = \sum_{n=1}^{\infty} c_1 \sin(n \pi x)\, b_n \quad \textbf{(18)}$$

> $b[n] := subs\left(\sin(n \cdot \text{Pi}) = 0, \cos(n \cdot \text{Pi}) = (-1)^n, simplify\left(\left(\frac{1}{\left(\frac{5}{10}\right)}\right) \cdot int\left(\left(\frac{\frac{5}{1000}}{\frac{5}{10}} \cdot x \cdot \sin(n \cdot \text{Pi} \cdot x), x = 0 .. \frac{5}{10}\right)\right) + \left(\frac{1}{\left(\frac{5}{10}\right)}\right) \cdot int\left(\left(-\frac{\left(\frac{5}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot x + \frac{1}{100}\right) \cdot \sin(n \cdot \text{Pi} \cdot x), x = \frac{5}{10} .. 1\right)\right)\right)$

$$b_n := \frac{\sin\left(\frac{n \pi}{2}\right)}{25\, n^2 \pi^2} \quad \textbf{(19)}$$

> $a[n] := 0$

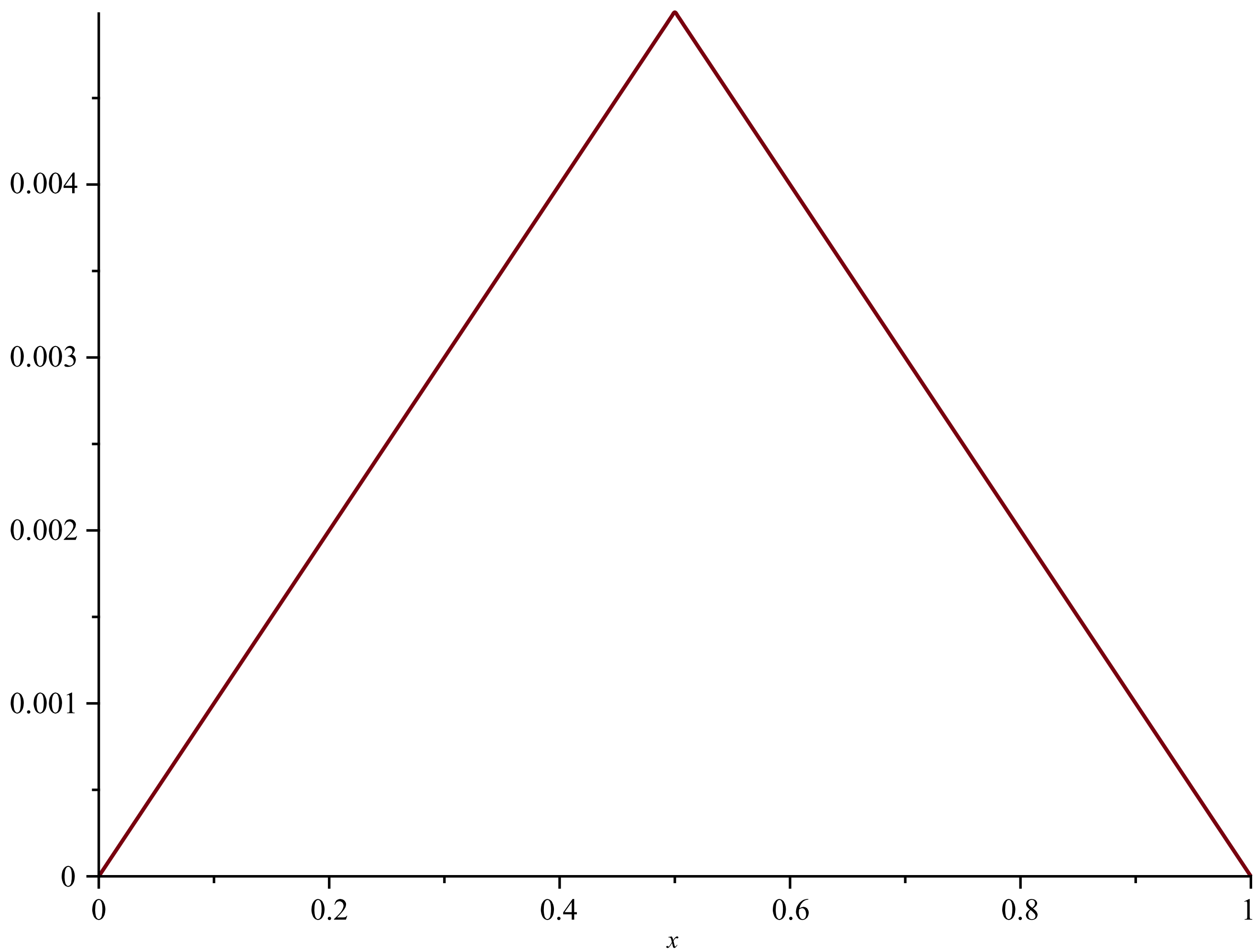
$$a_n := 0 \quad \textbf{(20)}$$

> $SolucionParticular := subs(c_1 = 1, SolucionGeneral)$

$$SolucionParticular := y(x,t) = \sum_{n=1}^{\infty} \frac{\sin(n\,\pi\,x)\,\sin\left(\frac{n\,\pi}{2}\right)\cos(n\,\pi\,t)}{25\,n^2\,\pi^2} \tag{21}$$

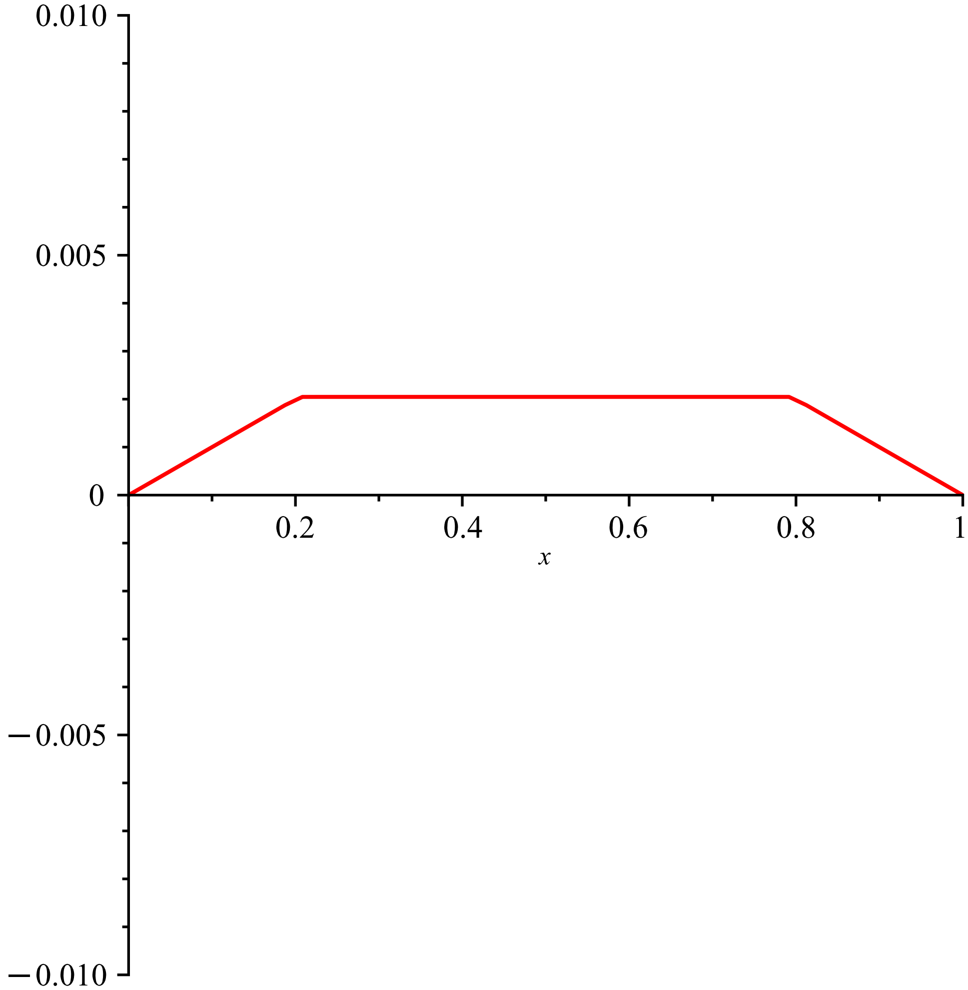
> $SolucionParticular500 := y(x,t) = sum\left(\frac{\sin(n\,\pi\,x)\,\sin\left(\frac{n\,\pi}{2}\right)\cos(n\,\pi\,t)}{25\,n^2\,\pi^2}, n = 1..500\right):$

> $plot(subs(t = 0, rhs(SolucionParticular500)), x = 0..1)$



> $with(plots):$

> $animate(rhs(SolucionParticular500), x = 0..1, t = 0..4, frames = 150, view = [0..1, -0.01..0.01])$



0.010
0.005
0
−0.005
−0.010
0.2
0.4
0.6
0.8
1
x

>
>
>
>