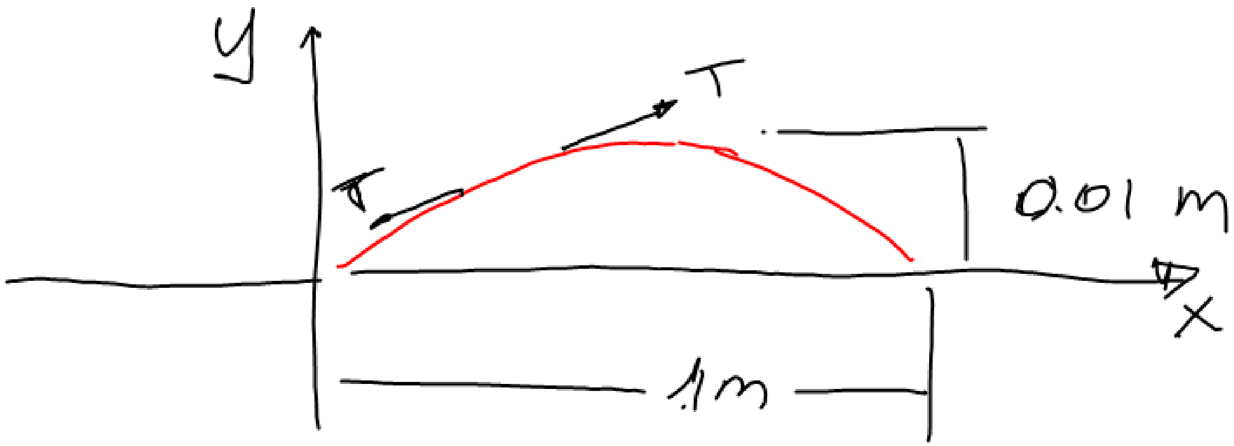


T = tensión relativa

$$T \frac{\partial^2 y(x,t)}{\partial x^2} = \rho \frac{\partial^2 y(x,t)}{\partial t^2}$$

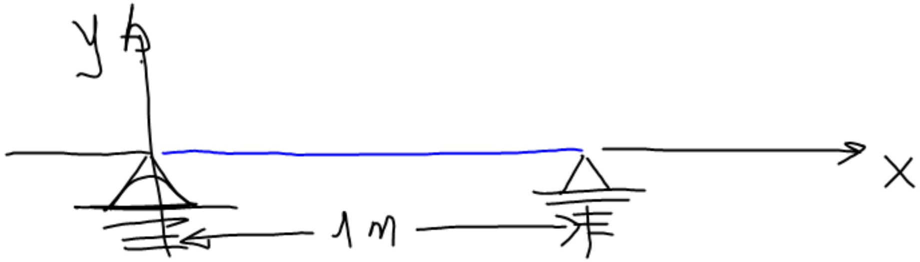
ρ = densidad relativa.

$$\frac{T}{\rho} = c^2 \qquad c \neq 0 \in \mathbb{R}$$

$$\boxed{\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}}$$

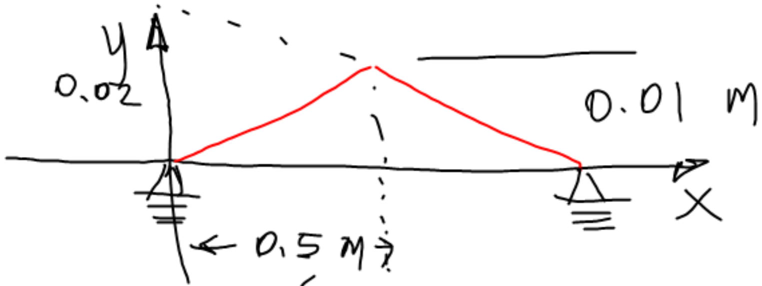
EenDP(2)Lcc.

Variables separables.



$$y(0,t)=0 \qquad y(1,t)=0$$

condición de frontera



$$y(x,0)=\begin{cases} \dfrac{0.01}{0.5}x & ;\ 0\le x\le 0.5 \\[2ex] 0{,}02-\dfrac{0{,}01}{0{,}5}x & ;\ 0.5\le x\le 1 \end{cases}$$

$$\frac{\partial y(x,0)}{\partial t}=0$$

condiciones iniciales

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \qquad c^2 > 0$$

$$y(x,t) = F(x) \cdot G(t)$$

$$\frac{\partial y}{\partial t} = F \cdot G' \qquad \frac{\partial y}{\partial x} = F' \cdot G$$

$$\frac{\partial^2 y}{\partial t^2} = F \cdot G'' \qquad \frac{\partial^2 y}{\partial x^2} = F'' G$$

$$F \cdot G'' = c^2 F'' \cdot G$$

$$\frac{G''}{c^2 G} = \frac{F''}{F} \qquad \frac{F''}{F} = \alpha \qquad \frac{G''}{c^2 G} = \alpha$$

CASO I: $\alpha = 0$

$$\frac{F''}{F} = 0 \qquad F \neq 0 \qquad F'' = 0 \qquad F' = C_1 \qquad \boxed{F = C_1 x + C_2}$$

C.F.: $y(0,t) = 0 \qquad G(t) \neq 0 \qquad F(0) = 0 \qquad F(0) = C_1(0) + C_2 = 0$

$y(1,t) = 0 \qquad G(t) \neq 0 \qquad F(1) = 0 \qquad \boxed{C_2 = 0}$

$$F(1) = C_1(1) + (0) = 0$$

$$\boxed{C_1 = 0}$$

$F(x) = 0 \quad \forall x.$

$$y(x,t)\big|_{\alpha=0} = 0$$

CASO II $\alpha > 0$ $\quad \alpha = \beta^2$ $\quad \forall \beta \neq 0 \in \mathbb{R}$

$$\frac{F''}{F} = \beta^2 \qquad F'' = \beta^2 F \qquad F'' - \beta^2 F = 0$$

$$m^2 - \beta^2 = 0 \qquad (m-\beta)(m+\beta) = 0$$

$$m_1 = \beta \qquad m_2 = -\beta$$

$$\boxed{F(x) = c_1 e^{\beta x} + c_2 e^{-\beta x}}$$

$F(0) = 0$ $\quad F(0) = c_1 e^{(0)} + c_2 e^{(0)} = 0$ $\quad c_1 + c_2 = 0$

$F(1) = 0$ $\quad F(1) = c_1 e^{\beta} + c_2 e^{-\beta} = 0$ $\quad c_1 = -c_2$

$$c_1 e^{\beta} = -\frac{c_2}{e^{\beta}}$$

$$c_1 + c_2 = 0$$

$$c_1 e^{\beta} = \frac{c_1}{e^{\beta}}$$

$$c_1 + c_2 = 0$$

$$c_1 (e^{\beta})^2 = c_1$$

$$c_1 = c_2 = 0$$

$$e^{2\beta} = 1$$

$$\beta = 0$$

CASO III $\alpha < 0$ $\alpha = -\beta^2$ $\forall \beta \neq 0 \in \mathbb{R}$

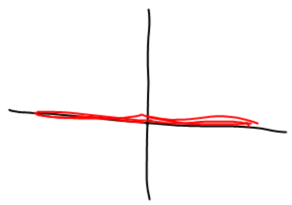
$$\frac{F''}{F} = -\beta^2 \qquad F'' = -\beta^2 F \qquad F'' + \beta^2 F = 0$$

$$m^2 + \beta^2 = 0 \qquad m_{1,2} = \pm \beta i$$

$$\boxed{F(x) = C_1 \cos(\beta x) + C_2 \operatorname{sen}(\beta x)}$$

$$F(0) = C_1 \cos(0) + C_2 \operatorname{sen}(0) = 0 \qquad C_1 = 0$$

$$F(1) = C_2 \operatorname{sen}(\beta) = 0 \qquad \beta = n\pi \qquad C_2 \neq 0$$



$$F(x) = C_2 \operatorname{sen}(n\pi x)$$

$$\frac{G''(t)}{c^2 G(t)} = -n^2\pi^2$$

$$G''(t) = -c^2 n^2 \pi^2 G(t)$$

$$G''(t) + c^2 n^2 \pi^2 G(t) = 0$$

$$m^2 + c^2 n^2 \pi^2 = 0$$

$$G(t) = k_1 \cos(cn\pi t) + k_2 \operatorname{sen}(cn\pi t).$$

$$\boxed{y(x,t) = \operatorname{sen}(n\pi x)\left(k_{10} \cos(cn\pi t) + k_{20} \cos(cn\pi t)\right).}$$