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> restart
> Ecua := y'' - 5·y' + 6·y = 2·exp(2·x)
      Ecua :=  $\frac{d^2}{dx^2} y(x) - 5 \frac{d}{dx} y(x) + 6 y(x) = 2 e^{2x}$  (1)
=
> CondIni := y(0) = 4, D(y)(0) = -3
      CondIni :=  $y(0) = 4, D(y)(0) = -3$  (2)
=
> with(inttrans) :
> EcuaTL := subs(CondIni, laplace(Ecua, x, s))
      EcuaTL :=  $s^2 \mathcal{L}(y(x), x, s) + 23 - 4s - 5s \mathcal{L}(y(x), x, s) + 6 \mathcal{L}(y(x), x, s) = \frac{2}{s-2}$  (3)
=
> SolPartTL := simplify(isolate(EcuaTL, laplace(y(x), x, s)))
      SolPartTL :=  $\mathcal{L}(y(x), x, s) = \frac{4s^2 - 31s + 48}{(s-2)^2 (s-3)}$  (4)
=
> SolPart := invlaplace(SolPartTL, s, x)
      SolPart :=  $y(x) = -9e^{3x} - e^{2x}(2x - 13)$  (5)
=
> CondicionUno := y(0) = simplify(subs(x=0, rhs(SolPart)))
      CondicionUno :=  $y(0) = 4$  (6)
=
> CondIni[1]
       $y(0) = 4$  (7)
=
> CondicionDos := D(y)(0) = simplify(subs(x=0, rhs(diff(SolPart, x))))
      CondicionDos :=  $D(y)(0) = -3$  (8)
=
> CondIni[2]
       $D(y)(0) = -3$  (9)
=
> Comprobacion := simplify(eval(subs(y(x) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))
      Comprobacion :=  $0 = 0$  (10)
=
> restart
> Sistema := diff(x[1](t), t) = 4·x[1](t) + 3·x[2](t) + 8·exp(t), diff(x[2](t), t) = 2·x[1](t)
      - 4·x[2](t) - t^2 : Sistema[1]; Sistema[2]
       $\frac{d}{dt} x_1(t) = 4x_1(t) + 3x_2(t) + 8e^t$ 
       $\frac{d}{dt} x_2(t) = 2x_1(t) - 4x_2(t) - t^2$  (11)
=
> AA := array([ [4, 3], [2, -4] ])
       $AA := \begin{bmatrix} 4 & 3 \\ 2 & -4 \end{bmatrix}$  (12)
=
> BB := array([ 8·exp(t), -t^2 ])
       $BB := \begin{bmatrix} 8e^t & -t^2 \end{bmatrix}$  (13)
=
> Xcero := array([_C1, _C2])
      (14)

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$$Xcero := \begin{bmatrix} -C1 & -C2 \end{bmatrix} \quad (14)$$

> with(linalg) :

> MatExp := exponential(AA, t)

$$MatExp := \begin{bmatrix} \frac{e^{\sqrt{22} t}}{2} + \frac{e^{-\sqrt{22} t}}{2} + \frac{\sqrt{22} e^{\sqrt{22} t}}{11} - \frac{\sqrt{22} e^{-\sqrt{22} t}}{11}, \frac{3\sqrt{22} e^{\sqrt{22} t}}{44} \\ - \frac{3\sqrt{22} e^{-\sqrt{22} t}}{44} \\ \left[ \frac{\sqrt{22} e^{\sqrt{22} t}}{22} - \frac{\sqrt{22} e^{-\sqrt{22} t}}{22}, \frac{e^{\sqrt{22} t}}{2} + \frac{e^{-\sqrt{22} t}}{2} - \frac{\sqrt{22} e^{\sqrt{22} t}}{11} + \frac{\sqrt{22} e^{-\sqrt{22} t}}{11} \right] \end{bmatrix} \quad (15)$$

> SolHom := evalm(MatExp &\* Xcero) : xh[1](t) = SolHom[1]; xh[2](t) = SolHom[2]

$$xh_1(t) = \left( \frac{e^{\sqrt{22} t}}{2} + \frac{e^{-\sqrt{22} t}}{2} + \frac{\sqrt{22} e^{\sqrt{22} t}}{11} - \frac{\sqrt{22} e^{-\sqrt{22} t}}{11} \right) -C1 + \left( \frac{3\sqrt{22} e^{\sqrt{22} t}}{44} - \frac{3\sqrt{22} e^{-\sqrt{22} t}}{44} \right) -C2$$

$$xh_2(t) = \left( \frac{\sqrt{22} e^{\sqrt{22} t}}{22} - \frac{\sqrt{22} e^{-\sqrt{22} t}}{22} \right) -C1 + \left( \frac{e^{\sqrt{22} t}}{2} + \frac{e^{-\sqrt{22} t}}{2} - \frac{\sqrt{22} e^{\sqrt{22} t}}{11} + \frac{\sqrt{22} e^{-\sqrt{22} t}}{11} \right) -C2 \quad (16)$$

> MatExpTau := map(rcurry(eval, t = t - tau'), MatExp)

$$MatExpTau := \begin{bmatrix} \frac{e^{\sqrt{22} (t-\tau)}}{2} + \frac{e^{-\sqrt{22} (t-\tau)}}{2} + \frac{\sqrt{22} e^{\sqrt{22} (t-\tau)}}{11} - \frac{\sqrt{22} e^{-\sqrt{22} (t-\tau)}}{11}, \\ \frac{3\sqrt{22} e^{\sqrt{22} (t-\tau)}}{44} - \frac{3\sqrt{22} e^{-\sqrt{22} (t-\tau)}}{44} \\ \left[ \frac{\sqrt{22} e^{\sqrt{22} (t-\tau)}}{22} - \frac{\sqrt{22} e^{-\sqrt{22} (t-\tau)}}{22}, \frac{e^{\sqrt{22} (t-\tau)}}{2} + \frac{e^{-\sqrt{22} (t-\tau)}}{2} - \frac{\sqrt{22} e^{\sqrt{22} (t-\tau)}}{11} + \frac{\sqrt{22} e^{-\sqrt{22} (t-\tau)}}{11} \right] \end{bmatrix} \quad (17)$$

> BBtau := map(rcurry(eval, t = tau'), BB)

$$BBtau := \begin{bmatrix} 8 e^{\tau} & -\tau^2 \end{bmatrix} \quad (18)$$

> ProdTau := evalm(MatExpTau &\* BBtau)

$$ProdTau := \begin{bmatrix} 8 \left( \frac{e^{\sqrt{22} (t-\tau)}}{2} + \frac{e^{-\sqrt{22} (t-\tau)}}{2} + \frac{\sqrt{22} e^{\sqrt{22} (t-\tau)}}{11} - \frac{\sqrt{22} e^{-\sqrt{22} (t-\tau)}}{11} \right) e^{\tau} \\ - \left( \frac{3\sqrt{22} e^{\sqrt{22} (t-\tau)}}{44} - \frac{3\sqrt{22} e^{-\sqrt{22} (t-\tau)}}{44} \right) \tau^2, 8 \left( \frac{\sqrt{22} e^{\sqrt{22} (t-\tau)}}{22} \right) \end{bmatrix} \quad (19)$$

$$\left. - \frac{\sqrt{22} e^{-\sqrt{22} (t-\tau)}}{22} \right) e^{\tau} - \left( \frac{e^{\sqrt{22} (t-\tau)}}{2} + \frac{e^{-\sqrt{22} (t-\tau)}}{2} - \frac{\sqrt{22} e^{\sqrt{22} (t-\tau)}}{11} + \frac{\sqrt{22} e^{-\sqrt{22} (t-\tau)}}{11} \right) \tau^2 \right]$$

> *SolNoHom* := map(int, ProdTau, tau=0..t)

$$\begin{aligned} \text{SolNoHom} := & \left[ \frac{3}{242} - \frac{52 \sqrt{22} e^{-\sqrt{22} t}}{231} + \frac{52 \sqrt{22} e^{\sqrt{22} t}}{231} + \frac{9617 e^{-\sqrt{22} t}}{10164} + \frac{9617 e^{\sqrt{22} t}}{10164} \right. \\ & + \frac{3 t^2}{22} - \frac{40 e^t}{21}, -\frac{2}{121} - \frac{155 \sqrt{22} e^{-\sqrt{22} t}}{10164} + \frac{155 \sqrt{22} e^{\sqrt{22} t}}{10164} + \frac{989 e^{-\sqrt{22} t}}{2541} \\ & \left. + \frac{989 e^{\sqrt{22} t}}{2541} - \frac{2 t^2}{11} - \frac{16 e^t}{21} + \frac{t}{11} \right] \end{aligned} \quad (20)$$

> *ComprobarDos* := map(rcurry(eval, t=0'), SolNoHom)

$$\text{ComprobarDos} := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (21)$$

> *ComprobarTres* := map(rcurry(eval, t=0'), SolHom)

$$\text{ComprobarTres} := \begin{bmatrix} \_C1 & \_C2 \end{bmatrix} \quad (22)$$

> *SolCompleta* := evalm(SolHom + SolNoHom):x[1](t)  
= expand(evalf(simplify(SolCompleta[1]), 3)); x[2](t)  
= expand(evalf(simplify(SolCompleta[2]), 3));

$$\begin{aligned} x_1(t) = & 0.073800 e^{-4.69 t} \_C1 - 0.319800 e^{-4.69 t} \_C2 - 0.106272 e^{-4.69 t} + 0.925944 e^{4.69 t} \_C1 \\ & + 0.319800 e^{4.69 t} \_C2 + 1.99752 e^{4.69 t} + 0.136 t^2 - 1.90 e^t + 0.0124 \\ x_2(t) = & -0.213528 e^{-4.69 t} \_C1 + 0.925944 e^{-4.69 t} \_C2 + 0.317832 e^{-4.69 t} + 0.213528 e^{4.69 t} \_C1 \\ & + 0.073800 e^{4.69 t} \_C2 + 0.461496 e^{4.69 t} - 0.182 t^2 + 0.0909 t - 0.762 e^t - 0.0165 \end{aligned} \quad (23)$$

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