TEMA 3.-

- TRANSFORMADA DE LAPLACE.
- SISTEMAS DE ECUACIONES DIFERENCIALES LINEALES (MATRIZ EXPONENCIAL).

$$\mathcal{L}\{y'(x)\} = s \cdot \mathcal{L}\{y\} - y(0)$$

$$\mathcal{L}\{y''(x)\} = s^2 \mathcal{L}\{y\} - s\, y(0) - y'(0).$$

$$\mathcal{L}\{y'''(x)\} = s^3 \mathcal{L}\{y\} - s^2 (y(0)) - s(y'(0)) - y''(0)$$

$$y'' - 5y' + 6y = 2e^{2x} \qquad y(0) = 4 \quad y'(0) = -3$$

$$L\{y'' - 5y' + 6y\} = L\{2e^{2x}\}$$

$$L\{y''\} - 5L\{y'\} + 6L\{y\} = 2L\{e^{2x}\}$$

$$\left[s^2 L\{y\} - s\cdot(4) - (-3)\right] - 5\left[sL\{y\} - (4)\right] + 6L\{y\} = \frac{2}{s-2}$$

$$(s^2 - 5s + 6)L\{y\} - 4s + 23 = \frac{2}{s-2}$$

$$(s^2 - 5s + 6)L\{y\} = \frac{2}{s-2} + 4s - 23$$

$$= \frac{2 + (4s-23)(s-2)}{(s-2)}$$

$$(s^2 - 5s + 6)L\{y\} = \frac{4s^2 - 31s + 46 + 2}{(s-2)}$$

$$\boxed{L\{y\} = \frac{4s^2 - 31s + 48}{(s-2)(s-2)(s-3)}}$$

$$\frac{4s^2 - 31s + 48}{(s-2)^2(s-3)} = \frac{A}{(s-2)^2} + \frac{B}{s-2} + \frac{C}{s-3}$$

$$4s^2 - 31s + 48 = A(s-3) + B(s^2 - 5s + 6) + C(s-2)^2$$

$$4s^2 - 31s + 48 = (B + C)s^2 + (A - 5B - 4C)s + (-3A + 6B + 4C)$$

$B + C = 4$

$A - 5B - 4C = -31$ $\qquad A = -31 + 5B + 4C$

$-3A + 6B + 4C = 48$ $\qquad A = -31 + 65 - 36$

$-3(-31 + 5B + 4C) + 6B + 4C = 48$ $\qquad A = -2$

$93 - 15B - 12C + 6B + 4C = 48$

$-9B - 8C = 48 - 93$

$+9B + 8C = 45$

$-8B - 8C = -32$

$B = 13$

$B + C = 4$

$C = 4 - B$

$C = -9$

$$L\{y(x)\} = \frac{-2}{(s-2)^2} + \frac{13}{(s-2)} - \frac{9}{s-3}$$

$$y(x) = -2xe^{2x} + 13e^{2x} - 9e^{3x}$$

$y(0) = 4$

$$y'(x) = -4xe^{2x} - 2e^{2x} + 26e^{2x} - 27e^{3x}$$

$$= -4xe^{2x} + 24e^{2x} - 27e^{3x}$$

$y'(0) = -3$

$$\frac{dx_1(t)}{dt} = 4x_1(t) + 3x_2(t) + 8e^t$$

$$\frac{d\cancel{x_2}(t)}{dt} = 2x_1(t) - 4x_2(t) - t^2$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 8e^t \\ -t^2 \end{bmatrix}$$

$$\frac{d}{dt}\bar{x} = A\bar{x} + \bar{b}(t)$$

$$A = \begin{bmatrix} 4 & 3 \\ 2 & -4 \end{bmatrix} \qquad \bar{x}(0) = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\bar{x} = \left[e^{At}\right]\bar{x}(0) + \int_0^t e^{A(t-\tau)}\bar{b}(\tau)\,d\tau.$$

$$e^{At} = B_0(t)I + B_1(t)A \qquad e^{\lambda t} = B_0(t) + \lambda B_1(t)$$

$$\begin{vmatrix} 4-\lambda & 3 \\ 2 & -4-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(-4-\lambda) - 6 = 0$$

$$\lambda^2 - 16 - 6 = 0$$

$$\lambda^2 - 22 = 0$$

$$\lambda = 22 \qquad \lambda_1 = \sqrt{22} \qquad \lambda_2 = -\sqrt{22}$$

$$e^{\sqrt{22}t} = B_0 + \sqrt{22}B_1$$

$$e^{-\sqrt{22}t} = B_0 - \sqrt{22}B_1$$

$$B_0 = \frac{e^{\sqrt{22}t} + e^{-\sqrt{22}t}}{2}$$

$$B_1 = e^{\sqrt{22}t} - \frac{e^{\sqrt{22}t} + e^{-\sqrt{22}t}}{}$$

$$B_1 = \frac{e^{\sqrt{22}t} - e^{-\sqrt{22}t}}{2}$$

$$e^{At} = B_0\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + B_1\begin{bmatrix} 4 & 3 \\ 2 & -4 \end{bmatrix}$$