

# EXÁMENES FINALES COLEGIADOS.

1ª Vuelta: VIERNES 8 DIC. 13:00 a 16:00  
SALONES J205A & J204

2ª Vuelta: VIERNES 15 DIC. 11:30 a 14:30  
SALÓN J205A

SE PODRÁ RESOLVER EN MAPLE Y/O MANO  
DEBERÁ RESPONDERSE COMPLETO.

# TEMA 4. Ecs. EN DERIVADAS PARCIALES.

	TESORERO	REAL
EDO.	80%	20%
EDP -	20%	80%

1<sup>er</sup> Método: EDP / CON TÉRMINOS  
DEL MISMO ORDEN.

$$\frac{\partial^2 z}{\partial x^2} - 6 \frac{\partial^2 z}{\partial x \partial y} + 8 \frac{\partial^2 z}{\partial y^2} = 0$$

$$z(x, y) = f(y + mx)$$

$$\frac{\partial z}{\partial x} = m f' \quad \frac{\partial^2 z}{\partial x^2} = m^2 f''$$

$$\frac{\partial^2 z}{\partial x \partial y} = m f'' \quad \frac{\partial^2 z}{\partial y^2} = f''$$

$$[m^2 f''] - 6 [m f''] + 8 f'' = 0$$

$$(m^2 - 6m + 8) f'' = 0 \quad f'' = 0 \quad f' = c_1 \quad f = c_1(y + mx) + c_2$$

$$m^2 - 6m + 8 = 0 \quad (m-2)(m-4) = 0 \quad m_1 = 2$$

$$m_2 = 4$$

$$z(x, y) = f_1(y + 2x) + f_2(y + 4x)$$

## 2º Método: Variables Separables

$$\frac{\partial^2 z}{\partial y^2} - 6 \frac{\partial^2 z}{\partial x \partial y} + 8 \frac{\partial z}{\partial x} = z$$

$$H_0 = z(x, y) = F(x) \cdot G(y)$$

$$\frac{\partial z}{\partial y} = F \cdot G' \quad \frac{\partial^2 z}{\partial y^2} = F \cdot G''$$

$$\frac{\partial^2 z}{\partial x \partial y} = F' \cdot G' \quad \frac{\partial z}{\partial x} = F' \cdot G$$

$$F \cdot G'' - 6 F' \cdot G' + 8 F' \cdot G = F G$$

$$F \cdot G'' - F G = 6 F' G' + 8 F' G$$

$$F(G'' - G) = F'(6G' + 8G)$$

$$\frac{G'' - G}{6G' + 8G} = \frac{F'}{F}$$

$$H_1 = F(x) + G(y)$$

$$\frac{F'}{F} = \alpha \quad \frac{G'' - G}{6G' + 8G} = \alpha$$

$$\alpha = 0 \quad F' = 0 \quad F(x) = C_1$$

$$G'' - G = 0 \quad G(y) = k_1 e^{-y} + k_2 e^y$$

$$m^2 - 1 = 0$$

$$m = \pm 1$$

$$\text{para } \alpha = 0 \quad z(x, y) = G(y) = k_1 e^{-y} + k_2 e^y$$

$$\boxed{z(x, y) = k_1 e^{-y} + k_2 e^y}$$

para  $\alpha > 0$   $\alpha = \beta^2 \quad \forall \beta \neq 0 \in \mathbb{R}$

$$\frac{F'}{F} = \beta^2 \quad F' - \beta^2 F = 0$$

$$m - \beta^2 = 0 \quad m = \beta^2$$

$$F(x) = G e^{\beta^2 x}$$

$$\frac{G'' - G}{6G' - 8G} = \beta^2$$

$$G'' - G = \beta^2 (6G' - 8G)$$

$$G'' - 6\beta^2 G' + (8\beta^2 - 1)G = 0$$

$$m^2 - 6\beta^2 m + (8\beta^2 - 1) = 0$$

$$m_{1,2} = \frac{6\beta^2 \pm \sqrt{36\beta^4 - 4(8\beta^2 - 1)}}{2}$$

$$m_{1,2} = \frac{6\beta^2 \pm \sqrt{36\beta^4 - 32\beta^2 + 4}}{2}$$

$$m_1 = 3\beta^2 + \sqrt{9\beta^2 - 8}$$

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