

> restart

> Ecua := diff(y(x, t), x\$2) = c^2 · diff(y(x, t), t\$2)

$$Ecua := \frac{\partial^2}{\partial x^2} y(x, t) = c^2 \left( \frac{\partial^2}{\partial t^2} y(x, t) \right) \quad (1)$$

> EcuaDos := subs(c^2 = 1, Ecua)

$$EcuaDos := \frac{\partial^2}{\partial x^2} y(x, t) = \frac{\partial^2}{\partial t^2} y(x, t) \quad (2)$$

> EcuaSeparable := eval(subs(y(x, t) = F(x) · G(t), EcuaDos))

$$EcuaSeparable := \left( \frac{d^2}{dx^2} F(x) \right) G(t) = F(x) \left( \frac{d^2}{dt^2} G(t) \right) \quad (3)$$

> EcuaSeparada := lhs(EcuaSeparable) / (F(x) · G(t)) = rhs(EcuaSeparable) / (F(x) · G(t))

$$EcuaSeparada := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \frac{\frac{d^2}{dt^2} G(t)}{G(t)} \quad (4)$$

> EcuaX := lhs(EcuaSeparada) = alpha

$$EcuaX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \alpha \quad (5)$$

> EcuaT := rhs(EcuaSeparada) = alpha

$$EcuaT := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \alpha \quad (6)$$

Para alpha igual a cero

> EcuaXCero := subs(alpha = 0, EcuaX)

$$EcuaXCero := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = 0 \quad (7)$$

> SolXCero := dsolve(EcuaXCero)

$$SolXCero := F(x) = c_1 x + c_2 \quad (8)$$

> SistemaCero := subs(x = 0, rhs(SolXCero) = 0), subs(x = 1, rhs(SolXCero) = 0) :  
SistemaCero[1]; SistemaCero[2];

$$c_2 = 0$$

$$c_1 + c_2 = 0$$

(9)

> ParametroCero := solve({SistemaCero}, {c1, c2})

$$ParametroCero := \{c_1 = 0, c_2 = 0\} \quad (10)$$

Para alpha positiva

> EcuaPosX := subs(alpha = β^2, EcuaX)

$$EcuaPosX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \beta^2 \quad (11)$$

> SolPosX := dsolve(EcuaPosX)

$$SolPosX := F(x) = c_1 e^{-\beta x} + c_2 e^{\beta x} \quad (12)$$

> SistemaPos := eval(subs(x=0, rhs(SolPosX)=0), subs(x=1, rhs(SolPosX)=0) :  
SistemaPos[1]; SistemaPos[2]

$$c_1 + c_2 = 0$$

$$c_1 e^{-\beta} + c_2 e^{\beta} = 0 \quad (13)$$

> ParametroPos := solve({SistemaPos}, {c1, c2})

$$ParametroPos := \{c_1 = 0, c_2 = 0\} \quad (14)$$

Para alpha negativa

> EcuaNegX := subs(alpha=-β², EcuaX)

$$EcuaNegX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = -\beta^2 \quad (15)$$

> SolNegX := dsolve(EcuaNegX)

$$SolNegX := F(x) = c_1 \sin(\beta x) + c_2 \cos(\beta x) \quad (16)$$

> SistemaNegX := subs(x=0, rhs(SolNegX)=0), subs(x=1, rhs(SolNegX)=0) :  
SistemaNegX[1]; SistemaNegX[2]

$$c_2 = 0$$

$$c_1 \sin(\beta) + c_2 \cos(\beta) = 0 \quad (17)$$

> ParametroNegX := solve({SistemaNegX[1], subs(beta=n·Pi, sin(n·Pi)=0,  
SistemaNegX[2])}, {c1, c2})

$$ParametroNegX := \{c_1 = c_1, c_2 = 0\} \quad (18)$$

> SolPartX := subs(beta=n·Pi, sin(n·Pi)=0, c2=0, SolNegX)

$$SolPartX := F(x) = c_1 \sin(n \pi x) \quad (19)$$

> EcuaT

$$\frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \alpha \quad (20)$$

> SolPartT := dsolve(subs(alpha=-β², β=n·Pi, EcuaT))

$$SolPartT := G(t) = c_1 \sin(n \pi t) + c_2 \cos(n \pi t) \quad (21)$$

> SolGralCompleta := y(x, t) = subs(c1=1, rhs(SolPartX)) · rhs(SolPartT)

$$SolGralCompleta := y(x, t) = \sin(n \pi x) (c_1 \sin(n \pi t) + c_2 \cos(n \pi t)) \quad (22)$$

>  $SolGralTotal := y(x, t) = Sum(subs(c_1 = 1, rhs(SolPartX)) \cdot subs(c_1 = a[n], c_2 = b[n], rhs(SolPartT)), n = 1 .. infinity)$

$$SolGralTotal := y(x, t) = \sum_{n=1}^{\infty} \sin(n \pi x) (a_n \sin(n \pi t) + b_n \cos(n \pi t)) \quad (23)$$

>  $eval(subs(t=0, SolGralTotal))$

$$y(x, 0) = \sum_{n=1}^{\infty} \sin(n \pi x) b_n \quad (24)$$

>  $b[n] := subs\left(\sin(n \cdot \text{Pi}) = 0, \cos(n \cdot \text{Pi}) = (-1) \cdot n, simplify\left(\left(\frac{1}{\left(\frac{5}{10}\right)}\right) \cdot int\left(\left(\frac{\frac{5}{1000}}{\frac{5}{10}} \cdot x\right) \cdot \sin(n \cdot \text{Pi} \cdot x), x = 0 .. \frac{5}{10}\right)\right) + \left(\frac{1}{\left(\frac{5}{10}\right)}\right) \cdot int\left(\left(-\frac{\left(\frac{5}{1000}\right)}{\frac{5}{10}} \cdot x + \frac{1}{100}\right) \cdot \sin(n \cdot \text{Pi} \cdot x), x = \frac{5}{10} .. 1\right)\right)\right)$

$$b_n := \frac{\sin\left(\frac{n \pi}{2}\right)}{25 n^2 \pi^2} \quad (25)$$

>  $eval(rhs(subs(t=0, diff(SolGralTotal, t))) = 0)$

$$\sum_{n=1}^{\infty} \sin(n \pi x) a_n n \pi = 0 \quad (26)$$

>  $a[n] := 0$

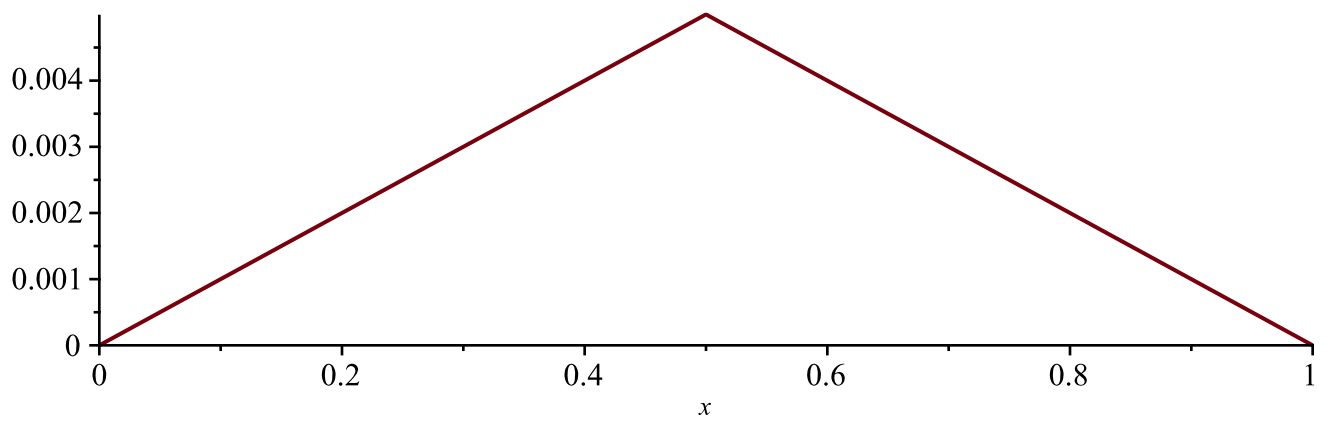
$$a_n := 0 \quad (27)$$

>  $SolGralTotal$

$$y(x, t) = \sum_{n=1}^{\infty} \frac{\sin(n \pi x) \sin\left(\frac{n \pi}{2}\right) \cos(n \pi t)}{25 n^2 \pi^2} \quad (28)$$

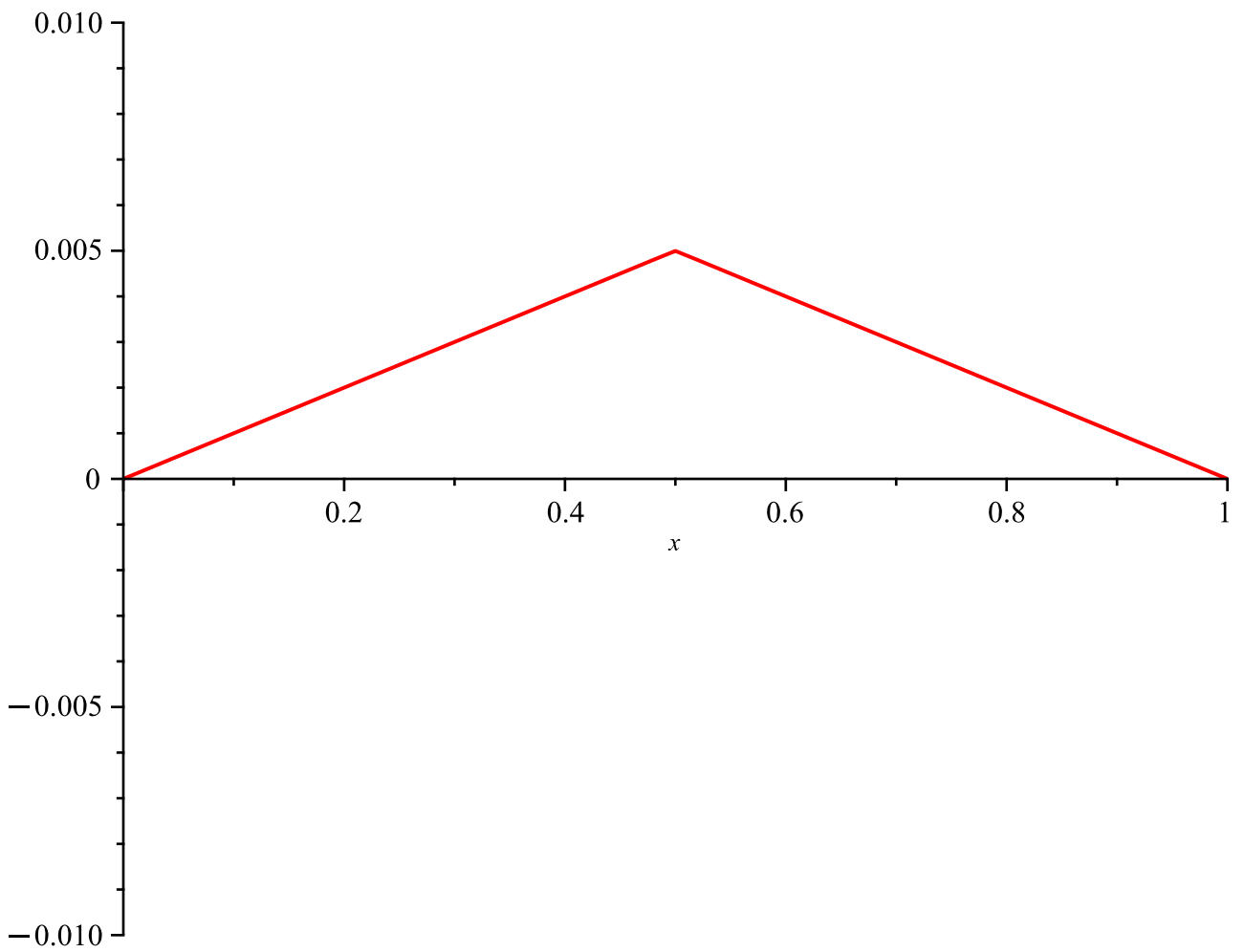
>  $SolPart500 := y(x, t) = sum\left(\frac{\sin(n \pi x) \sin\left(\frac{n \pi}{2}\right) \cos(n \pi t)}{25 n^2 \pi^2}, n = 1 .. 500\right) :$

>  $plot(subs(t=0, rhs(SolPart500)), x = 0 .. 1)$



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> with(plots) :
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> animate(rhs(SolPart500), x = 0 .. 1, t = 0 .. 4, frames = 150, view = [0 .. 1, -0.01 .. 0.01])
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