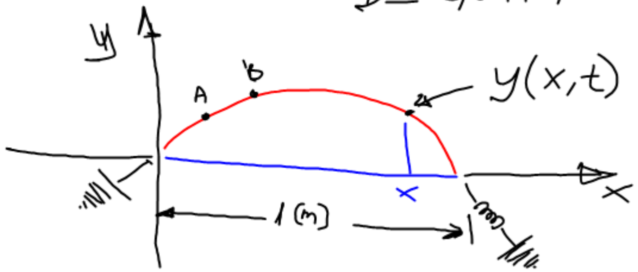
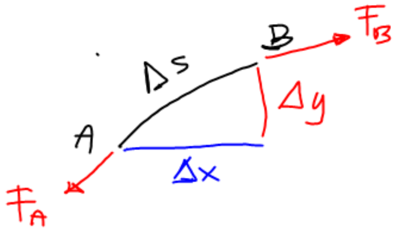
PROBLEMA DE APLICACIÓN: CUERDA DE GUITARRA.





$$F = ma$$

$$F_{H_A} = F_{H_B} \qquad F = T_{V_B} - T_{V_A}$$

$$a = \frac{\partial^2 y(x,t)}{\partial t^2} \qquad m = \rho \Delta s$$

$$\boxed{T_{V_B} - T_{V_A} = \rho\, \Delta s \cdot \frac{\partial^2 y}{\partial t^2}}$$

$$\operatorname{sen}\alpha \doteq \tan\alpha = \frac{\Delta y}{\Delta x}$$

$$\alpha < 4$$

$$T_{V_A} = T\,\frac{\Delta y}{\Delta x} \qquad \Delta x \to 0 \qquad T_{V_A} = T\,\frac{\partial y}{\partial x}$$

$$T_{V_B} = T\,\frac{\partial y}{\partial x} + \frac{\partial}{\partial x}\left(T\,\frac{\partial y}{\partial x}\cdot \Delta s\right)$$

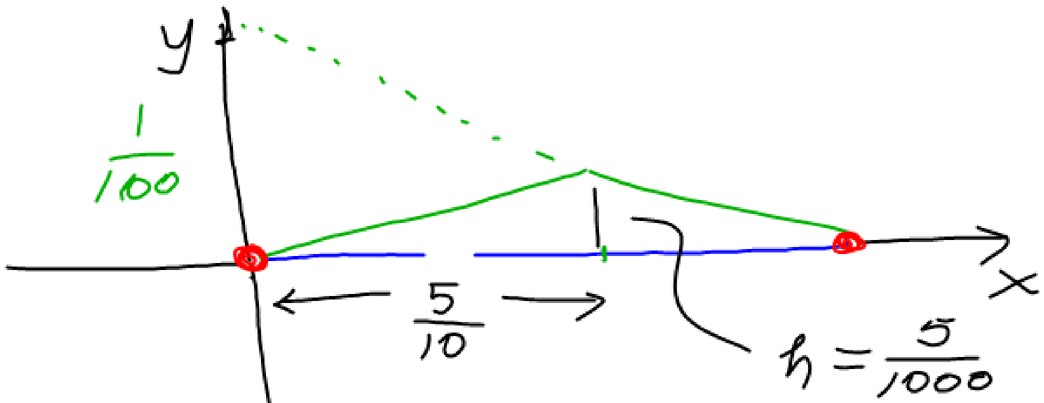
$$T_{V_B} = T\,\frac{\partial y}{\partial x} + T\,\frac{\partial^2 y}{\partial x^2}$$

$$T\,\frac{\partial^2 y}{\partial x^2} = \rho\,\frac{\partial^2 y}{\partial t^2}$$

$$T > 0 \qquad \rho > 0 \qquad \frac{\rho}{T} = c^2$$

$$\boxed{\frac{\partial^2 y}{\partial x^2} = c^2\,\frac{\partial^2 y}{\partial t^2}}$$

EDenDP(2)



Condiciones frontera

$$y(0,t) = 0$$

$$y(1,t) = 0$$

condiciones iniciales

$$y(x,0) \Rightarrow g(x) = \begin{cases} \dfrac{\frac{5}{1000}}{\frac{5}{10}} x \;; & 0 \leq x \leq \frac{5}{10} \\[2ex] \frac{1}{100} - \dfrac{\frac{5}{1000}}{\frac{5}{10}} x \;; & \frac{5}{10} < x < 1 \end{cases}$$

condiciones iniciales.

$$g(x) = \begin{cases} \frac{1}{100} x \;; & 0 \leq x \leq \frac{5}{10} \\[2ex] \frac{1}{100} - \frac{1}{100} x \;; & \frac{5}{10} < x < 1 \end{cases}$$

$$\frac{\partial y(x,0)}{dt} = 0$$

$$\boxed{c^2 = 1}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^3}$$

H_0

$$y(x,t) = F(x) \cdot G(t)$$

$$\frac{\partial y}{\partial x} = F'G \qquad \frac{\partial y}{\partial t} = FG'$$

$$\frac{\partial^2 y}{\partial x^2} = F''G \qquad \frac{\partial^2 y}{\partial t^2} = FG''$$

$$F''G = FG''$$

$$\frac{F''}{F} = \frac{G''}{G}$$

Metodo Variables Separables

$$\frac{F''}{F} = \alpha \qquad \frac{G''}{G} = \alpha$$

para $\alpha = 0$

$$\frac{F''}{F} = 0 \qquad \frac{G''}{G} = 0$$

$F'' = 0 \quad F \neq 0$

$F'(x) = C_1 \quad \boxed{F(x) = C_1 x + C_2}$

$F(0) = 0$

$F(1) = 0$

$F(0) \Rightarrow 0 = \cancel{C_1}(0) + C_2 \quad \boxed{C_2 = 0}$

$F(x) = C_1 x$

$F(1) \Rightarrow 0 = C_1(0) \quad \boxed{C_1 = 0}$

$F(x) = 0 \quad y(x,t) = (0) \cdot G(t) \Rightarrow 0.$

para $\alpha = \beta^2 \quad \forall \beta \neq 0.$

$$\frac{F''(x)}{F(x)} = \beta^2 \qquad F''(x) - \beta^2 F(x) = 0$$

EDO (2) L cc H.

$$\boxed{F(x) = C_1 e^{\beta x} + C_2 e^{-\beta x}} \quad m^2 - \beta^2 = 0 \qquad m = \pm\beta$$

$F(0) = 0$

$$F(0) \Rightarrow 0 = C_1 e^{\beta(0)} + C_2 e^{-\beta(0)}$$

$$C_1 + C_2 = 0 \qquad C_1 = -C_2$$

$$F(1) \Rightarrow 0 = C_1 e^{\beta} + C_2 e^{-\beta}$$

$$C_1 e^{\beta} + \frac{C_2}{e^{\beta}} = 0$$

$$C_1 e^{\beta} - \frac{C_1}{e^{\beta}} = 0$$

$$C_1\left(e^{\beta} - \frac{1}{e^{\beta}}\right) = 0 \quad \begin{cases} C_1 = 0 \\ e^{\beta} = \frac{1}{e^{\beta}} \end{cases}$$

$$e^{2\beta} = 1 \qquad 2\beta = 0 \qquad \beta = 0$$

para $\alpha < 0$ $\quad \alpha = -\beta^2$ $\quad \forall \beta \neq 0$

$$\frac{F''(x)}{F(x)} = -\beta^2 \qquad F''(x) = -\beta^2 F(x)$$

$$F''(x) + \beta^2 F(x) = 0 \qquad m^2 + \beta^2 = 0$$

$$m = \pm \beta i$$

$$F(x) = C_1 \cos(\beta x) + C_2 \operatorname{sen}(\beta x).$$

$$F(0) \Rightarrow 0 = C_1(1) + C_2(0) \Rightarrow \boxed{C_1 = 0}$$

$$F(1) \Rightarrow 0 = C_2 \operatorname{sen}(\beta(1)) \qquad \beta = n\pi \quad n = 1, 2, 3 \ldots$$

$$\boxed{C_2 \neq 0} \qquad \frac{G''}{G} = -\beta^2$$

$$F(x) = C_2 \operatorname{sen}(n\pi x) \qquad G'' + \beta^2 G = 0$$

$$m^2 + \beta^2 = 0$$

$$G(t) = C_1 \cos(n\pi t) + C_2 \operatorname{sen}(n\pi t)$$

$$\boxed{Z(x,t) = \operatorname{sen}(n\pi)\left(C_{10} \cos(n\pi t) + C_{20} \operatorname{sen}(n\pi t)\right)}$$