

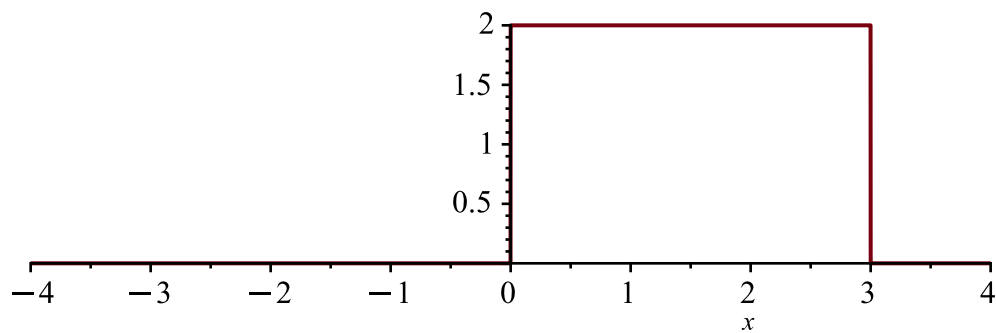
```

> restart
> SolGral := u(x,y) = f(x)·exp(x·y) + g(x)·exp(-x·y) +  $\frac{\exp(y)}{1-x^2}$ 
      SolGral := u(x,y) = f(x) exy + g(x) e-xy +  $\frac{e^y}{-x^2+1}$  (1)
=
> SolHom := u(x,y) = f(x)·exp(x·y) + g(x)·exp(-x·y)
      SolHom := u(x,y) = f(x) exy + g(x) e-xy (2)
=
> SolQ := u(x,y) =  $\frac{\exp(y)}{1-x^2}$ 
      SolQ := u(x,y) =  $\frac{e^y}{-x^2+1}$  (3)
=
> DerSolHomX := diff(SolHom, x)
      DerSolHomX :=  $\frac{\partial}{\partial x} u(x,y) = \left(\frac{d}{dx} f(x)\right) e^{xy} + f(x) y e^{xy} + \left(\frac{d}{dx} g(x)\right) e^{-xy} - g(x) y e^{-xy}$  (4)
=
> DerSolHomY := diff(SolHom, y)
      DerSolHomY :=  $\frac{\partial}{\partial y} u(x,y) = f(x) x e^{xy} - g(x) x e^{-xy}$  (5)
=
> DerDerSolHomY := diff(DerSolHomY, y)
      DerDerSolHomY :=  $\frac{\partial^2}{\partial y^2} u(x,y) = f(x) x^2 e^{xy} + g(x) x^2 e^{-xy}$  (6)
=
> EcuaHom := diff(u(x,y), y$2) - x^2·u(x,y) = 0
      EcuaHom :=  $\frac{\partial^2}{\partial y^2} u(x,y) - x^2 u(x,y) = 0$  (7)
=
> SolHom
      u(x,y) = f(x) exy + g(x) e-xy (8)
=
> ComprobarUno := simplify(eval(subs(u(x,y) = rhs(SolHom), EcuaHom)))
      ComprobarUno := 0 = 0 (9)
=
> SolQ
      u(x,y) =  $\frac{e^y}{-x^2+1}$  (10)
=
> Q := simplify(eval(subs(u(x,y) = rhs(SolQ), lhs(EcuaHom))))
      Q := ey (11)
=
> EcuaFinal := lhs(EcuaHom) = Q
      EcuaFinal :=  $\frac{\partial^2}{\partial y^2} u(x,y) - x^2 u(x,y) = e^y$  (12)
=
> with(PDEtools):
> SolHomFinal := pdsolve(EcuaFinal)
      SolHomFinal := u(x,y) = exy f2(x) + e-xy f1(x) -  $\frac{e^y}{x^2-1}$  (13)

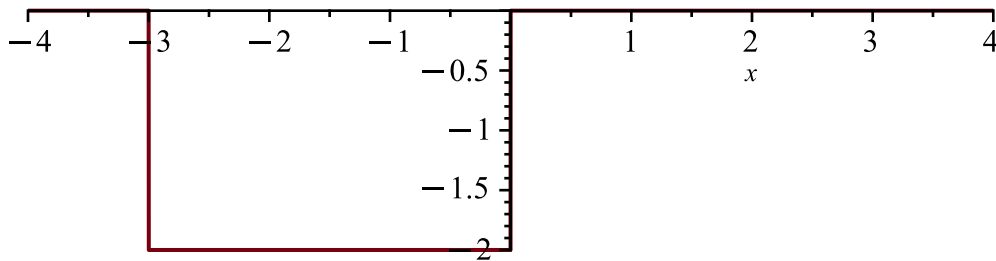
```

> restart

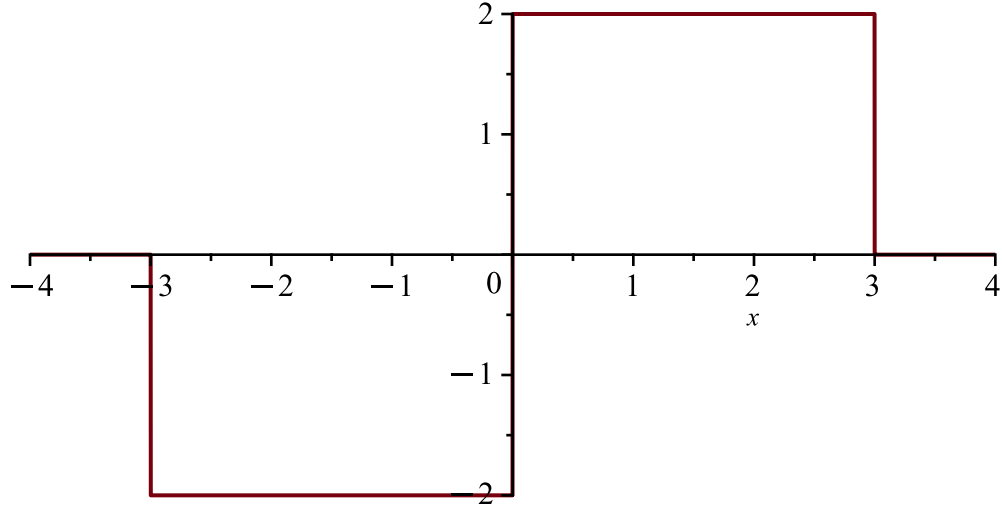
> $f := 2 \cdot \text{Heaviside}(x) - 2 \cdot \text{Heaviside}(x - 3) : \text{plot}(f, x = -4 .. 4, \text{scaling} = \text{CONSTRAINED})$



> $g := -2 \cdot \text{Heaviside}(x + 3) + 2 \cdot \text{Heaviside}(x) : \text{plot}(g, x = -4 .. 4, \text{scaling} = \text{CONSTRAINED})$



> $h := f + g : \text{plot}(h, x = -4 .. 4, \text{scaling} = \text{CONSTRAINED})$



$$\begin{aligned} &> L := 4 \\ &L := 4 \end{aligned} \tag{14}$$

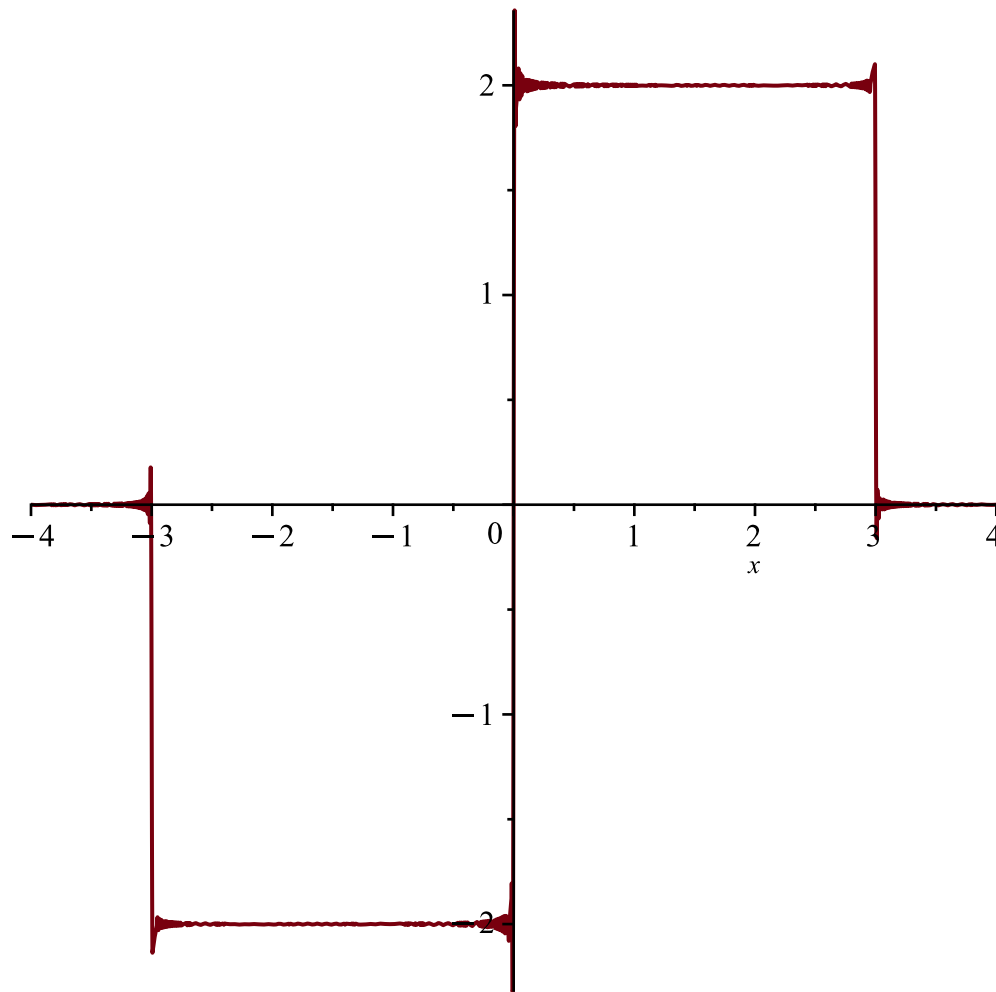
$$\begin{aligned} &> a[0] := \frac{1}{L} \cdot \text{int}(h, x = -L..L) \\ &a_0 := 0 \end{aligned} \tag{15}$$

$$\begin{aligned} &> a[n] := \frac{1}{L} \cdot \text{int}\left(h \cdot \cos\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), x = -L..L\right) \\ &a_n := 0 \end{aligned} \tag{16}$$

$$\begin{aligned} &> b[n] := \text{simplify}\left(\frac{1}{L} \cdot \text{int}\left(h \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), x = -L..L\right)\right) \\ &b_n := \frac{4 - 4 \cos\left(\frac{3 n \pi}{4}\right)}{n \pi} \end{aligned} \tag{17}$$

$$\begin{aligned} &> STFseno := \text{Sum}\left(b[n] \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), n = 1..infinity\right) \\ &STFseno := \sum_{n=1}^{\infty} \frac{\left(4 - 4 \cos\left(\frac{3 n \pi}{4}\right)\right) \sin\left(\frac{n \pi x}{4}\right)}{n \pi} \end{aligned} \tag{18}$$

```
> STF500 := sum(b[n]·sin( (n·Pi/L) ·x), n=1..500) :
> plot(STF500, x=-L..L)
```



```
> restart
> Sistema := diff(x[1](t), t) = 2·x[1](t) - 2·x[2](t) + 6·exp(3 t), diff(x[2](t), t) = -x[1](t)
+ x[2](t) - sin(5·t) : Sistema[1]; Sistema[2]
```

$$\frac{d}{dt} x_1(t) = 2x_1(t) - 2x_2(t) + 6e^{3t}$$

$$\frac{d}{dt} x_2(t) = -x_1(t) + x_2(t) - \sin(5t) \quad (19)$$

```
> AA := array([ [2, -2], [-1, 1] ])
```

$$AA := \begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix} \quad (20)$$

```
> BB := array([ 6·exp(3 t), -sin(5 t) ])
```

$$BB := \begin{bmatrix} 6e^{3t} & -\sin(5t) \end{bmatrix} \quad (21)$$

```
> Xcero := array([ c[1], c[2] ])
```

$$Xcero := \begin{bmatrix} c_1 & c_2 \end{bmatrix} \quad (22)$$

> with(linalg) :

> MatExp := exponential(AA, t)

$$MatExp := \begin{bmatrix} \frac{1}{3} + \frac{2e^{3t}}{3} & -\frac{2e^{3t}}{3} + \frac{2}{3} \\ -\frac{e^{3t}}{3} + \frac{1}{3} & \frac{2}{3} + \frac{e^{3t}}{3} \end{bmatrix} \quad (23)$$

> SolHom := evalm(MatExp &* Xcero) : x[1](t) = SolHom[1]; x[2](t) = SolHom[2]

$$x_1(t) = \left(\frac{1}{3} + \frac{2e^{3t}}{3} \right) c_1 + \left(-\frac{2e^{3t}}{3} + \frac{2}{3} \right) c_2$$

$$x_2(t) = \left(-\frac{e^{3t}}{3} + \frac{1}{3} \right) c_1 + \left(\frac{2}{3} + \frac{e^{3t}}{3} \right) c_2 \quad (24)$$

> MatExpTau := map(rcurry(eval, t='t - tau'), MatExp)

$$MatExpTau := \begin{bmatrix} \frac{1}{3} + \frac{2e^{3t-3\tau}}{3} & -\frac{2e^{3t-3\tau}}{3} + \frac{2}{3} \\ -\frac{e^{3t-3\tau}}{3} + \frac{1}{3} & \frac{2}{3} + \frac{e^{3t-3\tau}}{3} \end{bmatrix} \quad (25)$$

> BBtau := map(rcurry(eval, t='tau'), BB)

$$BBtau := \begin{bmatrix} 6e^{3\tau} & -\sin(5\tau) \end{bmatrix} \quad (26)$$

> ProdTau := simplify(evalm(MatExpTau &* BBtau)) : ProdTau[1]; ProdTau[2]

$$\frac{2\sin(5\tau)e^{3t-3\tau}}{3} + 4e^{3t} - \frac{2\sin(5\tau)}{3} + 2e^{3\tau}$$

$$-2e^{3t} - \frac{\sin(5\tau)e^{3t-3\tau}}{3} + 2e^{3\tau} - \frac{2\sin(5\tau)}{3} \quad (27)$$

> SolNoHom := map(int, ProdTau, tau=0..t) : x[1](t) = SolNoHom[1]; x[2](t) = SolNoHom[2]

$$x_1(t) = -\frac{4}{5} + \frac{13e^{3t}}{17} + \frac{3\cos(5t)}{85} - \frac{\sin(5t)}{17} + 4e^{3t}t$$

$$x_2(t) = -\frac{4}{5} + \frac{21e^{3t}}{34} + \frac{31\cos(5t)}{170} + \frac{\sin(5t)}{34} - 2e^{3t}t \quad (28)$$

> ComprobarDos := x[1](0) = simplify(subs(t=0, SolNoHom[1]))

$$ComprobarDos := x_1(0) = 0 \quad (29)$$

> ComprobarTres := x[2](0) = simplify(subs(t=0, SolNoHom[2]))

$$ComprobarTres := x_2(0) = 0 \quad (30)$$

> ComprobarCuatro := x[1](0) = simplify(subs(t=0, SolHom[1]))

$$ComprobarCuatro := x_1(0) = c_1 \quad (31)$$

> ComprobarCinco := x[2](0) = simplify(subs(t=0, SolHom[2]))

$$ComprobarCinco := x_2(0) = c_2 \quad (32)$$

> SolFinal := evalm(SolHom + SolNoHom) : x[1](t) = SolFinal[1], x[2](t) = SolFinal[2]

$$\begin{aligned}
x_1(t) &= \left(\frac{1}{3} + \frac{2e^{3t}}{3} \right) c_1 + \left(-\frac{2e^{3t}}{3} + \frac{2}{3} \right) c_2 - \frac{4}{5} + \frac{13e^{3t}}{17} + \frac{3\cos(5t)}{85} - \frac{\sin(5t)}{17} \\
&+ 4e^{3t}t, x_2(t) = \left(-\frac{e^{3t}}{3} + \frac{1}{3} \right) c_1 + \left(\frac{2}{3} + \frac{e^{3t}}{3} \right) c_2 - \frac{4}{5} + \frac{21e^{3t}}{34} + \frac{31\cos(5t)}{170} \\
&+ \frac{\sin(5t)}{34} - 2e^{3t}t
\end{aligned} \tag{33}$$

> Sistema[1]; Sistema[2]

$$\begin{aligned}
\frac{d}{dt} x_1(t) &= 2x_1(t) - 2x_2(t) + 6e^{3t} \\
\frac{d}{dt} x_2(t) &= -x_1(t) + x_2(t) - \sin(5t)
\end{aligned} \tag{34}$$

> ComprobarSeis := simplify(eval(subs(x[1](t) = SolFinal[1], x[2](t) = SolFinal[2],
lhs(Sistema[1]) - rhs(Sistema[1]) = 0)))

$$\text{ComprobarSeis} := 0 = 0 \tag{35}$$

> ComprobarSiete := simplify(eval(subs(x[1](t) = SolFinal[1], x[2](t) = SolFinal[2],
lhs(Sistema[2]) - rhs(Sistema[2]) = 0)))

$$\text{ComprobarSiete} := 0 = 0 \tag{36}$$

> restart

> Ecua := diff(y(t), t\$2) - 5*diff(y(t), t) + 6*y(t) = (t - 3)*Heaviside(t - 3) + Dirac(t - 1)

$$Ecua := \frac{d^2}{dt^2} y(t) - 5 \frac{d}{dt} y(t) + 6y(t) = (t - 3) \text{Heaviside}(t - 3) + \text{Dirac}(t - 1) \tag{37}$$

> CondIni := y(0) = 5, D(y)(0) = -4

$$\text{CondIni} := y(0) = 5, D(y)(0) = -4 \tag{38}$$

> with(inttrans)

[addtable, fourier, fouriercos, fouriersin, hankel, hilbert, invfourier, invhilbert, invlaplace, invmellin, laplace, mellin, savetable, setup] \tag{39}

> EcuaTL := subs(CondIni, laplace(Ecua, t, s))

$$EcuaTL := s^2 \mathcal{L}(y(t), t, s) + 29 - 5s - 5s \mathcal{L}(y(t), t, s) + 6 \mathcal{L}(y(t), t, s) = \frac{e^{-3s}}{s^2} + e^{-s} \tag{40}$$

> SolTL := isolate(EcuaTL, laplace(y(t), t, s))

$$SolTL := \mathcal{L}(y(t), t, s) = \frac{\frac{e^{-3s}}{s^2} + e^{-s} + 5s - 29}{s^2 - 5s + 6} \tag{41}$$

> SolPart := simplify(invlaplace(SolTL, s, t))

$$\begin{aligned}
SolPart := y(t) &= -\frac{e^{2t-6} \text{Heaviside}(t-3)}{4} - e^{2t-2} \text{Heaviside}(t-1) \\
&+ \frac{e^{3t-9} \text{Heaviside}(t-3)}{9} + e^{3t-3} \text{Heaviside}(t-1) + \frac{\text{Heaviside}(t-3) (6t-13)}{36} \\
&+ 19e^{2t} - 14e^{3t}
\end{aligned} \tag{42}$$

$$\begin{aligned} & \text{ComprobarUno} := \text{simplify}(\text{subs}(t=0, \text{SolPart})) \\ & \text{ComprobarUno} := y(0) = 5 \end{aligned} \tag{43}$$

$$\begin{aligned} & \text{ComprobarDos} := \text{D}(y)(0) = \text{simplify}(\text{subs}(t=0, \text{rhs}(\text{diff}(\text{SolPart}, t)))) \\ & \text{ComprobarDos} := \text{D}(y)(0) = -4 \end{aligned} \tag{44}$$

$$\begin{aligned} & \text{Ecua} \\ & \frac{d^2}{dt^2} y(t) - 5 \frac{d}{dt} y(t) + 6 y(t) = (t - 3) \text{Heaviside}(t - 3) + \text{Dirac}(t - 1) \end{aligned} \tag{45}$$

$$\begin{aligned} & \text{ComprobarTres} := \text{simplify}(\text{eval}(\text{subs}(y(t) = \text{rhs}(\text{SolPart}), \text{lhs}(\text{Ecua}) - \text{rhs}(\text{Ecua}) = 0))) \\ & \text{ComprobarTres} := 0 = 0 \end{aligned} \tag{46}$$

>