

$$\frac{\partial^2 z}{\partial x \partial y} + 6 \frac{\partial^2 z}{\partial y^2} - 2 \frac{\partial z}{\partial x} = 0$$

$$Z(x, y) = F(x) \cdot G(y)$$

$$\frac{\partial z}{\partial x} = F' G \quad \frac{\partial z}{\partial y} = F G'$$

$$\frac{\partial^2 z}{\partial x \partial y} = F' G' \quad \frac{\partial^2 z}{\partial y^2} = F G''$$

$$F' G' + 6 F G'' - 2 F' G = 0$$

$$F' (G' - 2 G) = -6 F G''$$

$$\frac{F'}{-6F} = \frac{G''}{G' - 2G} \quad \alpha = 3$$

$$\frac{F'}{-6F} = 3 \quad \frac{G''}{G' - 2G} = 3$$

$$\begin{aligned} F' &= -18F \\ F' + 18F &= 0 \\ m + 18 &= 0 \\ m &= -18 \end{aligned}$$

$$F(x) = c_1 e^{-18x}$$

$$G'' = 3(G' - 2G)$$

$$G'' - 3G' + 6G = 0$$

$$m^2 - 3m + 6 = 0$$

$$m = \frac{3 \pm \sqrt{9 - 24}}{2}$$

$$m = \frac{3}{2} \pm \frac{\sqrt{15}}{2} i$$

$$G(y) = c_1 e^{\frac{3}{2}y} \cos\left(\frac{\sqrt{15}}{2}y\right) + c_2 e^{\frac{3}{2}y} \sin\left(\frac{\sqrt{15}}{2}y\right)$$

$$Z(x, y) = e^{-18x} \left(c_1 e^{\frac{3}{2}y} \cos\left(\frac{\sqrt{15}}{2}y\right) + c_2 e^{\frac{3}{2}y} \sin\left(\frac{\sqrt{15}}{2}y\right) \right)$$