

Ex 10 (1) NL

$$M(x, y) + N(x, y) \frac{dy}{dx} = 0$$

MVS.

$$P(x) Q(y) + R(x) S(y) \cdot \frac{dy}{dx} = 0$$

$$P(x) Q(y) dx + R(x) S(y) dy = 0$$

$$\frac{1}{R(x) Q(y)}$$

$$\frac{P(x)}{R(x)} dx + \frac{S(y)}{Q(y)} dy = 0$$

$$\int \frac{P(x)}{R(x)} dx + \int \frac{S(y)}{Q(y)} dy = C, \quad \text{Sf}$$

Coefficientes homogéneos

$$M(x, y) + N(x, y) \frac{dy}{dx} = 0$$

MC H.

$$\text{such } x \Rightarrow \lambda x \quad y \Rightarrow \lambda y$$

$$\left. \begin{aligned} M(\lambda x, \lambda y) &= \lambda^m (M(x, y)) \\ N(\lambda x, \lambda y) &= \lambda^n (N(x, y)) \end{aligned} \right\} m=n$$

$$2x(x^2 + y^2) \frac{dy}{dx} = y(y^2 + 2x^2)$$

$$\underbrace{-y(y^2 + 2x^2)}_{M(x, y)} + \underbrace{2x(x^2 + y^2)}_{N(x, y)} \frac{dy}{dx} = 0$$

$$\begin{aligned} M(\lambda x, \lambda y) &= -(\lambda y)((\lambda y)^2 + 2(\lambda x)^2) \\ &= -\lambda y(\lambda^2 y^2 + 2\lambda^2 x^2) \\ &= -\lambda y(\lambda^2)(y^2 + 2x^2) \\ &= \lambda^3 (-y(y^2 + 2x^2)) \quad m=3 \end{aligned}$$

$$\begin{aligned} N(\lambda x, \lambda y) &= 2(\lambda x)((\lambda x)^2 + (\lambda y)^2) \\ &= 2\lambda x(\lambda^2 x^2 + \lambda^2 y^2) \\ &= 2\lambda x(\lambda^2)(x^2 + y^2) \\ &= \lambda^3 (2x(x^2 + y^2)) \quad n=3 \end{aligned}$$

$$\therefore m=n$$

$$-y(y^2 + 2x^2) + 2x(x^2 + y^2) \frac{dy}{dx} = 0$$

$$u = \frac{y}{x} \rightarrow y = x \cdot u \quad y(x) = x \cdot u(x)$$

$$-(x \cdot u)(x^2 u^2 + 2x^2) + \frac{dy}{dx} = x \cdot \frac{du}{dx} + u$$

$$-x^3 u^3 - 2x^3 u + (2x^3 + 2u^2 x^3) \left( x \frac{du}{dx} + u \right) = 0$$

$$-x^3 u^3 - 2x^3 u + (2x^3 + 2u^2 x^3) \left( x \frac{du}{dx} + u \right) = 0$$

$$(-x^3 u^3 - 2x^3 u + 2x^3 + 2u^2 x^3) \left( x \frac{du}{dx} + u \right) = 0$$

$$u^3 x^3 + x^4 (2 + 2u^2) \frac{du}{dx} = 0$$

$$P(x) = x^3$$

$$Q(u) = u^3$$

$$R(x) = x^4$$

$$S(u) = 2 + 2u^2$$

Variables  
Separables

$$\int \frac{x^3}{x^4} dx + \int \frac{2(1+u^2)}{u^3} du = C_1$$

$$\int \frac{dx}{x} + 2 \int \frac{1+u^2}{u^3} du = C_1$$

$$\ln x + 2 \int \frac{du}{u^3} + 2 \int \frac{du}{u} = C_1$$

$$\ln x + \frac{2}{-2} u^{-2} + 2 \ln(u) = C_1$$

$$-\frac{1}{u^2} + \ln x + \ln(u^2) = C_1$$

$$-\frac{1}{u^2} + \ln x + \ln(u^2) = C_1$$

$$\textcircled{SG} - \frac{1}{\left(\frac{y}{x}\right)^2} + \ln x + \ln\left(\left(\frac{y}{x}\right)^2\right) = C_1$$

$$M(x, y) + N(x, y) \frac{dy}{dx} = 0$$

$$F(x, y) = c,$$

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = - \frac{M(x, y)}{N(x, y)}$$

$$\frac{dy}{dx} = - \frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}$$

$$x \frac{dy}{dx} = \sqrt{x^2 - y^2} + y$$

$$M(\lambda x, \lambda y) = -\sqrt{(\lambda x)^2 - (\lambda y)^2} - \lambda y$$

$$\boxed{\begin{aligned} y(x) &= x \cdot u(x) \\ \frac{dy}{dx} &= x \frac{du}{dx} + u \end{aligned}} \quad \begin{aligned} &= -\sqrt{x^2 x^2 - x^2 y^2} - \lambda y \\ &= \sqrt{x^2} (-\sqrt{x^2 - y^2}) - \lambda y \\ &= \lambda \left[ -\sqrt{x^2 - y^2} - y \right] \quad m=1 \end{aligned}$$

$$N(\lambda x, \lambda y) = (\lambda x)^n \quad m=n$$

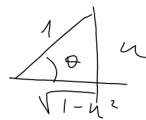
$$= \lambda(x) \quad n=1 \quad \text{Ch.}$$

$$x \left( x \frac{du}{dx} + u \right) = \sqrt{x^2 - (xu)^2} + xu$$

$$x^2 \frac{du}{dx} + x \cancel{u} = \sqrt{x^2 - x^2 u^2} + \cancel{xu}$$

$$x^2 \frac{du}{dx} + \cancel{xu} = \sqrt{x^2} (\sqrt{1-u^2}) + \cancel{xu}$$

$$x^2 \frac{du}{dx} + \cancel{xu} = x \sqrt{1-u^2} + \cancel{xu}$$



$$\sin \theta = \frac{u}{1}$$

$$u = \sin \theta$$

$$du = \cos(\theta) d\theta \quad \frac{\sqrt{1-u^2}}{1} = \cos(\theta)$$

$$\int \frac{\cancel{\cos(\theta)} d\theta}{\cancel{\cos(\theta)}} = \int d\theta \Rightarrow \theta$$

$$\theta = \arcsin(u)$$

$$\arcsin(u) = \ln x + C$$

$$\textcircled{56} \quad \boxed{\arcsin\left(\frac{y}{x}\right) = \ln x + C}$$