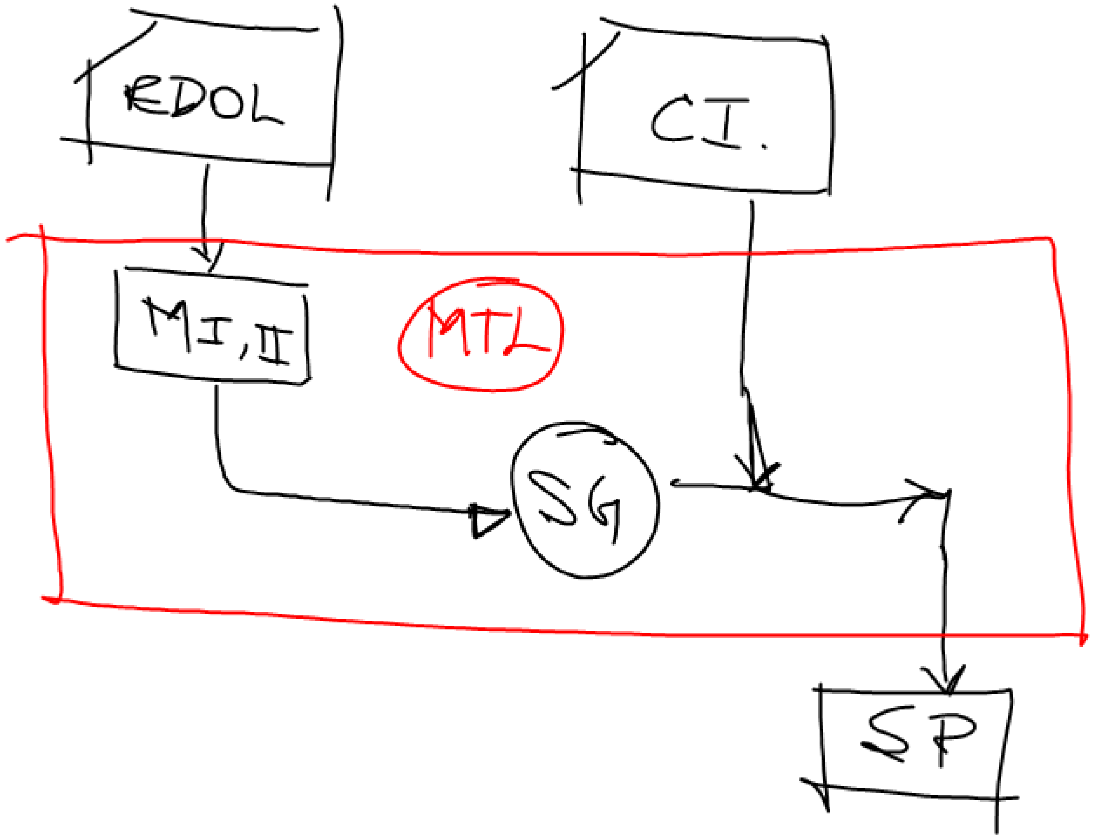
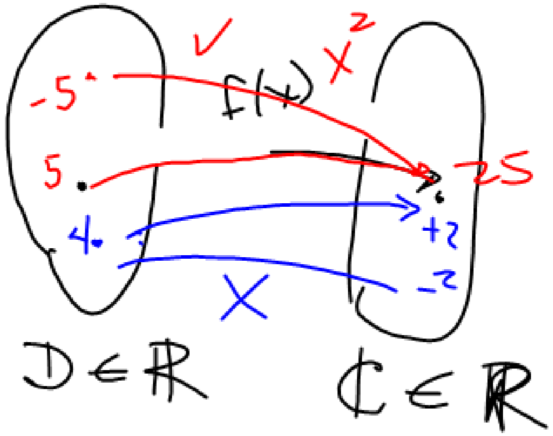
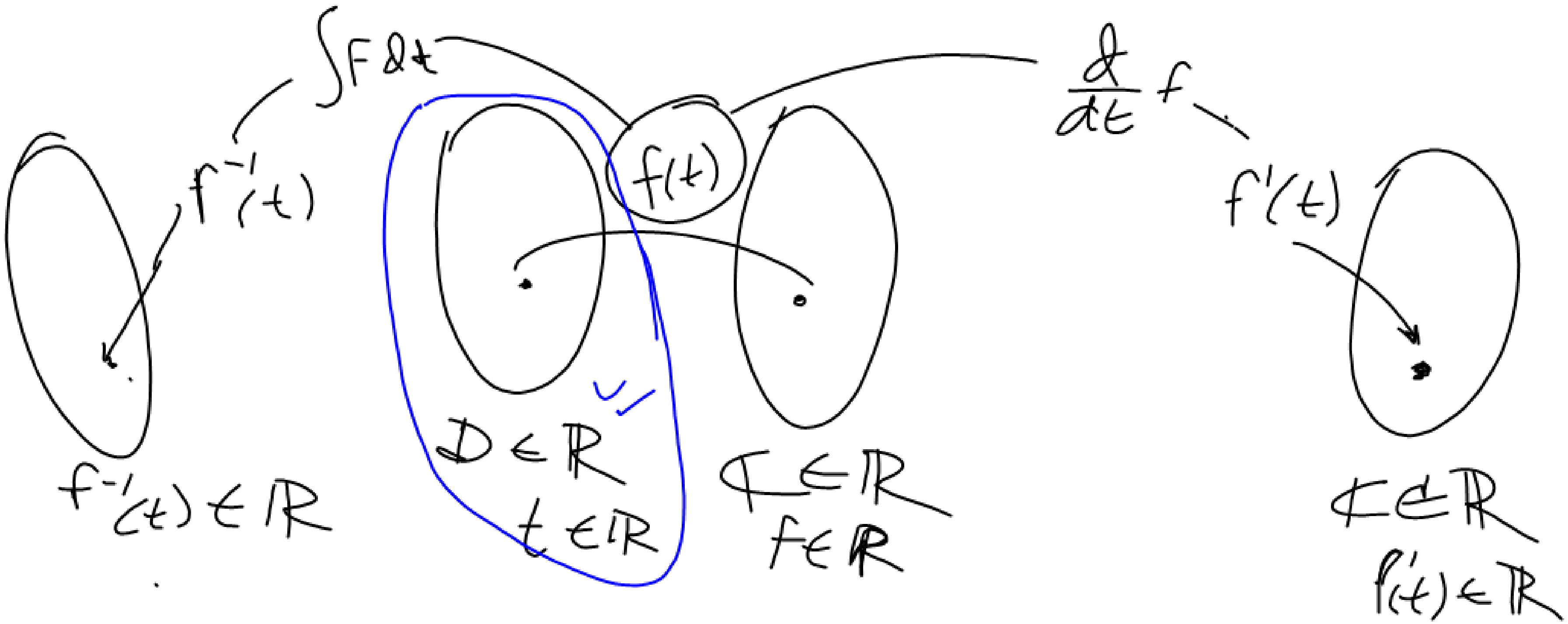
TEMA 3.- TRANSFORMADA DE LAPLACE & SISTEMAS EDOL SIMULTANEAS.





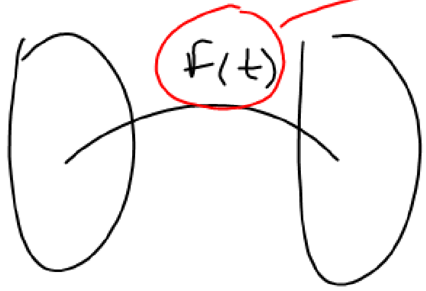
$y = x^2$

$y = \pm\sqrt{x}$



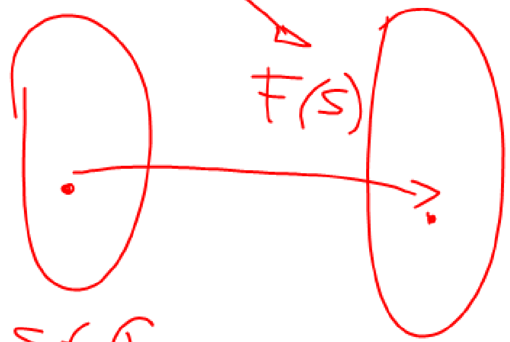
$\mathcal{L}\{f\} = F$

$f(t)$



$t \in \mathbb{R}$ $f \in \mathbb{R}$

$F(s)$



$s \in \mathbb{C}$ $F \in \mathbb{R}$

$a, b \in \mathbb{R}$ $\quad a f(t) + b g(t)$ $\quad aF(s) + bG(s)$

$f'(t)$ $\quad sF(s) - f(0)$

$\int f \, dt$ $\quad \dfrac{F(s)}{s}$

$$T\{f(t)\} = \int_{-\infty}^{\infty} N(t,s) f(t)\,dt = F(s)$$

núcleo   argumento.   resultado

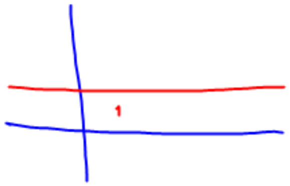
$t \in \mathbb{R}$ $s \in \mathbb{C}$

Transformada de Laplace

$$N(t,s) = \begin{cases} 0 \,;\ t<0 \\ e^{-st} \,;\ t \geq 0 \end{cases}$$

$$\boxed{\mathcal{L}\{f(t)\} = \int_{0}^{\infty} e^{-st} f(t)\,dt = F(s)}$$

$$\mathcal{L}\{1\} = \int_0^{\infty} e^{-st}(1)\,dt = \left[\int e^{-st}\,dt\right]_0^{\infty}$$



$$= \left[-\frac{1}{s}\int(-s)e^{-st}\,dt\right]_0^{\infty}$$

$$= -\frac{1}{s}\left[e^{-st}\right]_0^{\infty}$$

$$= -\frac{1}{s}\left(\lim_{t\to\infty} e^{-st} - 1\right)$$

$$\lim_{t\to\infty} e^{-st} = \lim_{t\to\infty}\frac{1}{e^{st}} \Rightarrow \lim_{a\to\infty}\frac{1}{a} \Rightarrow 0$$

$$\boxed{\mathcal{L}\{1\} = -\frac{1}{s}(-1) \Rightarrow \frac{1}{s}}$$

$$\mathcal{L}\{\pi\} = \pi\,\mathcal{L}\{1\} \Rightarrow \frac{\pi}{s}$$

$$\mathcal{L}\{e^{5t}\} = \int_0^\infty e^{-st} e^{5t}\,dt$$

$$= \left[\int e^{-(s-5)t}\,dt \right]_0^\infty$$

$$= \left[-\frac{1}{s-5} \int -(s-5) e^{-(s-5)t}\,dt \right]_0^\infty$$

$$= -\frac{1}{s-5} \left[\int -(s-5) e^{-(s-5)t}\,dt \right]_0^\infty$$

$$= -\frac{1}{s-5} \left[e^{-(s-5)t} \right]_0^\infty$$

$$= -\frac{1}{s-5} (-1)$$

$$\boxed{\mathcal{L}\{e^{5t}\} = \frac{1}{s-5}}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a} \quad a \in \mathbb{R}$$

$$\mathcal{L}\{e^{-6t}\} = \frac{1}{s+6}$$

$$\mathcal{L}\{t\} = \int_0^\infty e^{-st}(t)\,dt$$

$$= \left[\int t e^{-st}\,dt\right]_0^\infty$$

$$\int u\,dV = uV - \int V\,du.$$

$u = t \quad du = 1$

$dV = e^{-st}dt \quad V = -\frac{1}{s}e^{-st}$

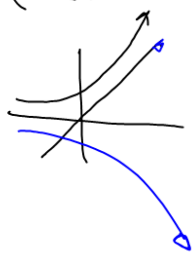
$$\int te^{-st}dt = -\frac{t}{s}e^{-st} - \frac{1}{s}\left(-\frac{1}{s}\int -se^{-st}dt - 1\right)$$

$$= \left[-\frac{t}{s}e^{-st} + \frac{1}{s^2}e^{-st}\right]_0^\infty$$

$$= \frac{-1}{s}\left[\lim_{t\to\infty} te^{-st} - 0\right] - \frac{1}{s^2}\left[\lim_{t\to\infty} e^{-st} - 1\right]$$

$$\lim_{t\to\infty} te^{-st} = \lim_{t\to\infty} t \cdot \lim_{t\to\infty} \frac{1}{e^{-st}})$$

$\infty \quad = 0 \quad 0$



$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}$$

$f(t)$	$F(s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
te^{at}	$\frac{1}{(s-a)^2}$
$\cos(bt)$	$\frac{s}{s^2+b^2}$
$\text{sen}(bt)$	$\frac{b}{s^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$e^{at}\text{sen}(bt)$	$\frac{b}{(s-a)^2+b^2}$

$$F(s) = \mathcal{L}\{f(t)\} \Rightarrow \int_0^\infty e^{-st} f(t)\,dt.$$

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} e^{st} F(s)\,ds.$$