

```

>
> restart
> Ecua := y' = (x - 4*y - 1)^2
                                Ecua :=  $\frac{d}{dx} y(x) = (x - 4 y(x) - 1)^2$  (1)
>
> CondIni := y(0) = 0
                                CondIni :=  $y(0) = 0$  (2)
> EcuaEcua := eval(subs(isolate(z(x) = x - 4*y(x) - 1, y(x)), Ecua))
                                EcuaEcua :=  $-\frac{\frac{d}{dx} z(x)}{4} + \frac{1}{4} = z(x)^2$  (3)
>
> EcuaDos := isolate(EcuaEcua, diff(z(x), x))
                                EcuaDos :=  $\frac{d}{dx} z(x) = -4 z(x)^2 + 1$  (4)
>
> with(DEtools):
> odeadvisor(EcuaDos)
                                [_quadrature] (5)
>
> intfactor(EcuaDos)
                                 $\frac{1}{(2 z(x) - 1) (2 z(x) + 1)}$  (6)
>
> FI := simplify( $\frac{1}{(2 z - 1) (2 z + 1)}$ )
                                FI :=  $\frac{1}{4 z^2 - 1}$  (7)
>
> M := 4 z^2 - 1
                                M :=  $4 z^2 - 1$  (8)
>
> N := 1
                                N := 1 (9)
>
> MM := M*FI
                                MM := 1 (10)
>
> NN := N*FI
                                NN :=  $\frac{1}{4 z^2 - 1}$  (11)
>
> P := 1; Q := 1; R := 1; S :=  $\frac{1}{4 z^2 - 1}$ 
                                P := 1
                                Q := 1
                                R := 1
                                S :=  $\frac{1}{4 z^2 - 1}$  (12)

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$$\begin{aligned} &> \text{SolGral} := \text{simplify}\left(\text{int}\left(\frac{P}{R}, x\right) + \text{int}\left(\frac{S}{Q}, z\right)\right) = _C1 \\ &\quad \text{SolGral} := x - \frac{\ln(2z+1)}{4} + \frac{\ln(2z-1)}{4} = _C1 \end{aligned} \quad (13)$$

$$\begin{aligned} &> \text{subs}(x=0, y(0)=0, z(x)=x-4\cdot y(x)-1) \\ &\quad z(0) = -1 \end{aligned} \quad (14)$$

$$\begin{aligned} &> \text{Para} := \text{simplify}(\text{subs}(x=0, z=-1, \text{SolGral})) \\ &\quad \text{Para} := \frac{\ln(3)}{4} = _C1 \end{aligned} \quad (15)$$

$$\begin{aligned} &> \text{SolPartDos} := \text{subs}(_C1 = \text{lhs}(\text{Para}), \text{SolGral}) \\ &\quad \text{SolPartDos} := x - \frac{\ln(2z+1)}{4} + \frac{\ln(2z-1)}{4} = \frac{\ln(3)}{4} \end{aligned} \quad (16)$$

$$\begin{aligned} &> \text{SolPartFinal} := \text{simplify}(\text{isolate}(\text{simplify}(\text{subs}(z=x-4\cdot y(x)-1, \text{SolPartDos})), y(x))) \\ &\quad \text{SolPartFinal} := y(x) = \frac{(2x-3)e^{4x} - 6x + 3}{8e^{4x} - 24} \end{aligned} \quad (17)$$

$$\begin{aligned} &> \text{Ecua} \\ &\quad \frac{d}{dx} y(x) = (x - 4y(x) - 1)^2 \end{aligned} \quad (18)$$

$$\begin{aligned} &> \text{SolValeria} := \text{dsolve}(\{\text{CondIni}, \text{Ecua}\}) \\ &\quad \text{SolValeria} := y(x) = \frac{(2x-3)e^{4x} - 6x + 3}{8e^{4x} - 24} \end{aligned} \quad (19)$$

$$\begin{aligned} &> \text{ComprobarDos} := \text{simplify}(\text{eval}(\text{subs}(y(x) = \text{rhs}(\text{SolPartFinal}), \text{lhs}(\text{Ecua}) - \text{rhs}(\text{Ecua}) \\ &\quad = 0))) \\ &\quad \text{ComprobarDos} := 0 = 0 \end{aligned} \quad (20)$$

$$\begin{aligned} &> \text{restart} \\ &> \text{SolGral} := y(x) = _C1 \cdot x \\ &\quad \text{SolGral} := y(x) = _C1 x \end{aligned} \quad (21)$$

$$\begin{aligned} &> \text{DerSolGral} := \text{diff}(\text{SolGral}, x) \\ &\quad \text{DerSolGral} := \frac{d}{dx} y(x) = _C1 \end{aligned} \quad (22)$$

$$\begin{aligned} &> \text{ParaSolGral} := \text{isolate}(\text{SolGral}, _C1) \\ &\quad \text{ParaSolGral} := _C1 = \frac{y(x)}{x} \end{aligned} \quad (23)$$

$$\begin{aligned} &> \text{Ecua} := \text{lhs}(\text{DerSolGral}) = \text{rhs}(\text{ParaSolGral}) \\ &\quad \text{Ecua} := \frac{d}{dx} y(x) = \frac{y(x)}{x} \end{aligned} \quad (24)$$

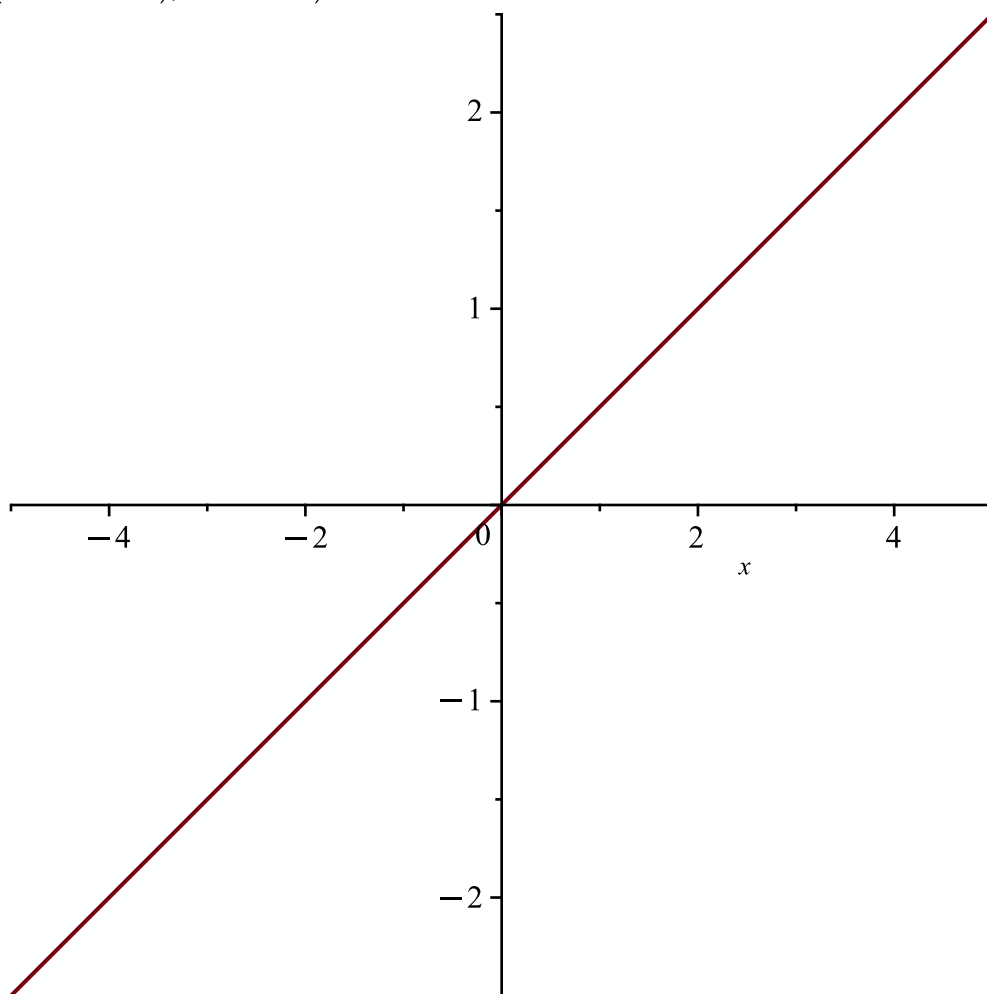
$$\begin{aligned} &> \\ &\text{a)} \\ &> \text{ParaDos} := \text{isolate}(\text{subs}(x=2, y(2)=1, \text{SolGral}), _C1) \\ &\quad \text{ParaDos} := _C1 = \frac{1}{2} \end{aligned} \quad (25)$$

> *SolPartDos* := subs(*ParaDos*, *SolGral*)

$$\textcolor{blue}{SolPartDos} := y(x) = \frac{x}{2}$$

(26)

> plot(rhs(*SolPartDos*), x = -5 .. 5)



>

b)

> *ParaTres* := isolate(subs(x = -2, y(-2) = 1, *SolGral*), _C1)

$$\textcolor{blue}{ParaTres} := _C1 = -\frac{1}{2}$$

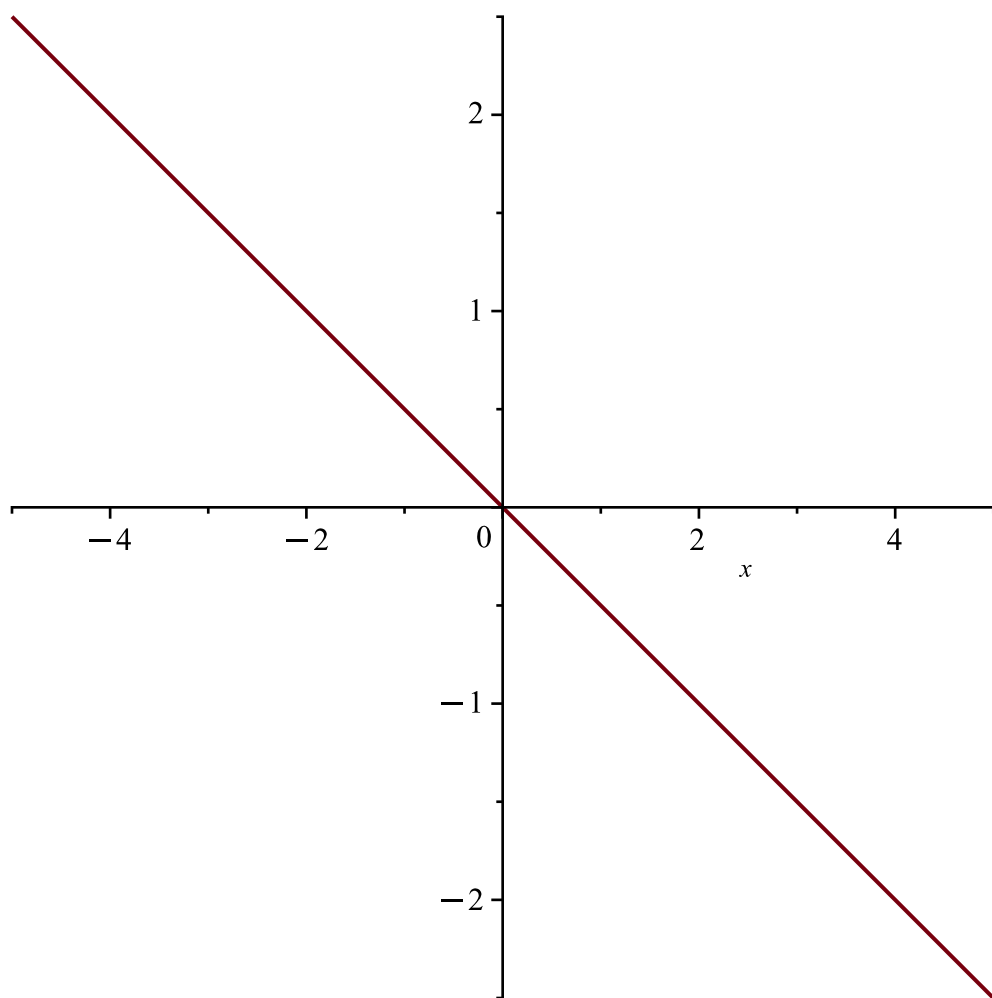
(27)

> *SolPartTres* := subs(*ParaTres*, *SolGral*)

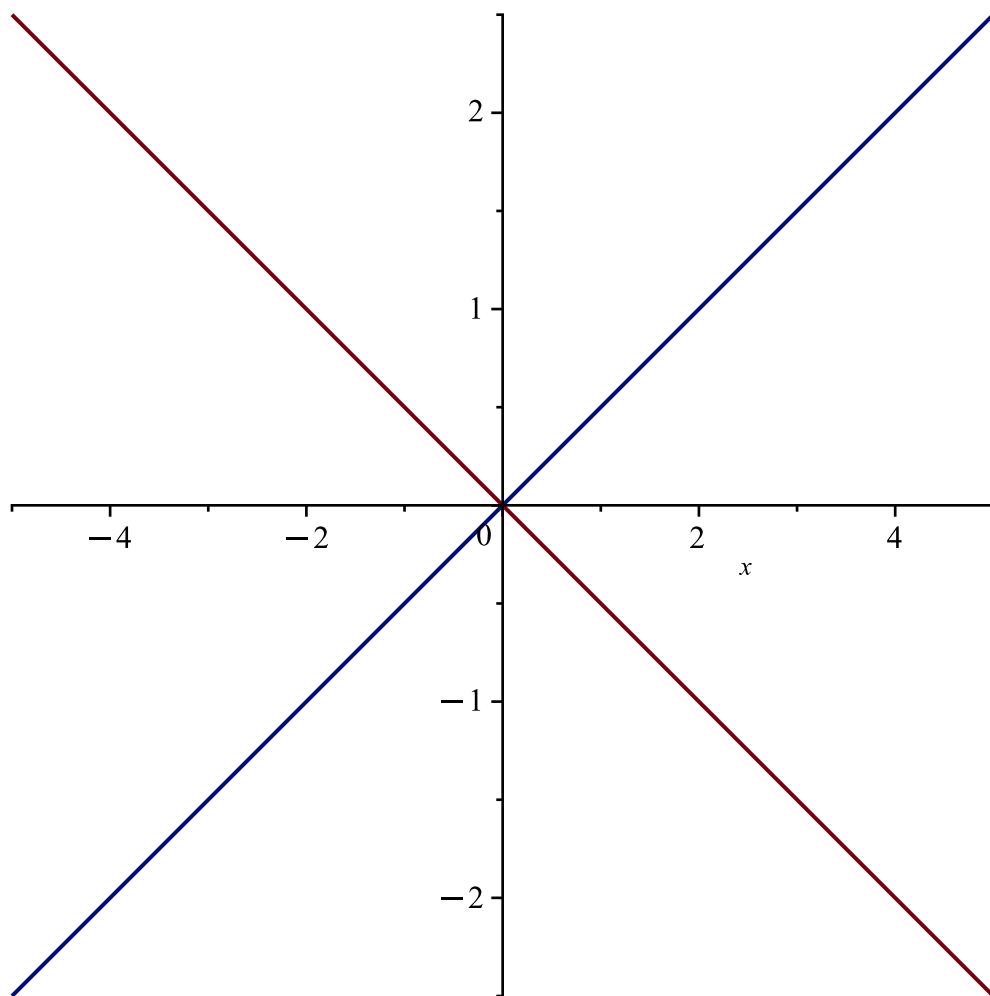
$$\textcolor{blue}{SolPartTres} := y(x) = -\frac{x}{2}$$

(28)

> plot(rhs(*SolPartTres*), x = -5 .. 5)



```
> plot( {rhs(SolPartDos), rhs(SolPartTres) }, x=-5 ..5)
```



```
> restart
```

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6)
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> Ecua := -y + (x - 2·x2·y2)·y'=0
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$$Ecua := -y(x) + (x - 2x^2 y(x)^2) \left(\frac{d}{dx} y(x) \right) = 0 \quad (29)$$

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> CondIni := y(-1) = 2
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$$CondIni := y(-1) = 2 \quad (30)$$

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> with(DEtools) :
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> odeadvisor(Ecua)
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[[_homogeneous, class G], _rational] (31)
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> FI := intfactor(Ecua)
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$$FI := \frac{1}{x^2} \quad (32)$$

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> M := -y
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$$M := -y \quad (33)$$

```
> N := (x - 2x2y2)
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$$N := -2x^2 y^2 + x \quad (34)$$

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> MM := M·FI
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$$MM := -\frac{y}{x^2} \quad (35)$$

> $NN := \text{expand}(N \cdot FI)$

$$NN := -2y^2 + \frac{1}{x} \quad (36)$$

> $\text{Comprobar} := \text{diff}(MM, y) = \text{diff}(NN, x)$

$$\text{Comprobar} := -\frac{1}{x^2} = -\frac{1}{x^2} \quad (37)$$

ES EXACTA

> $\text{IntMMx} := \text{int}(MM, x)$

$$\text{IntMMx} := \frac{y}{x} \quad (38)$$

> $\text{SolGral} := \text{IntMMx} + \text{int}((NN - \text{diff}(\text{IntMMx}, y)), y) = _C1$

$$\text{SolGral} := \frac{y}{x} - \frac{2y^3}{3} = _C1 \quad (39)$$

> $\text{Parametro} := \text{isolate}(\text{subs}(x=-1, y=2, \text{SolGral}), _C1)$

$$\text{Parametro} := _C1 = -\frac{22}{3} \quad (40)$$

> $\text{SolPart} := \text{subs}(_C1 = \text{rhs}(\text{Parametro}), \text{SolGral})$

$$\text{SolPart} := \frac{y}{x} - \frac{2y^3}{3} = -\frac{22}{3} \quad (41)$$

> $\text{SolPartDos} := \text{subs}(y=y(x), \text{lhs}(\text{SolPart}) \cdot 3 = \text{rhs}(\text{SolPart}) \cdot 3)$

$$\text{SolPartDos} := \frac{3y(x)}{x} - 2y(x)^3 = -22 \quad (42)$$

> $\text{DerEcua} := \text{isolate}(\text{Ecua}, \text{diff}(y(x), x))$

$$\text{DerEcua} := \frac{d}{dx} y(x) = \frac{y(x)}{x - 2x^2 y(x)^2} \quad (43)$$

> $\text{DerSolPartDos} := \text{simplify}(\text{isolate}(\text{diff}(\text{SolPartDos}, x), \text{diff}(y(x), x)))$

$$\text{DerSolPartDos} := \frac{d}{dx} y(x) = -\frac{y(x)}{2x^2 y(x)^2 - x} \quad (44)$$

> $\text{ComprobarDos} := \text{simplify}(\text{rhs}(\text{DerEcua}) - \text{rhs}(\text{DerSolPartDos})) = 0$

$$\text{ComprobarDos} := 0 = 0 \quad (45)$$

> $\text{ComprobarTres} := \text{subs}(x=-1, y(-1)=2, \text{lhs}(\text{SolPartDos}) - \text{rhs}(\text{SolPartDos})) = 0$

$$\text{ComprobarTres} := 0 = 0 \quad (46)$$

> Ecua

$$-y(x) + (x - 2x^2 y(x)^2) \left(\frac{d}{dx} y(x) \right) = 0 \quad (47)$$

> SolPartDos

$$\frac{3y(x)}{x} - 2y(x)^3 = -22 \quad (48)$$

> *ComprobarAilin* := *simplify*(*dsolve*({*CondIni*, *Ecua* }))

$$\text{ComprobarAilin} := y(x) = - \frac{1}{4x \left(\frac{\sqrt{2} \sqrt{\frac{242x^3 - 1}{x}} + 22x}{x} \right)^{1/3}} \left(2^{1/3} \left(-I\sqrt{3} \left(\frac{\sqrt{2} \sqrt{\frac{242x^3 - 1}{x}} + 22x}{x} \right)^{2/3} x + I 2^{1/3} \sqrt{3} + \left(\frac{\sqrt{2} \sqrt{\frac{242x^3 - 1}{x}} + 22x}{x} \right)^{2/3} x + 2^{1/3} \right) \right) \quad (49)$$

> *restart*

8)

> *Ecua* := $y^2 \cdot \sin(x) + (1 + 5 \cdot y \cdot \cos(x)) \cdot y' = 0$

$$\text{Ecua} := y(x)^2 \sin(x) + (1 + 5 y(x) \cos(x)) \left(\frac{d}{dx} y(x) \right) = 0 \quad (50)$$

> *with*(*DEtools*) :

> *odeadvisor*(*Ecua*)

$$[[_1st_order, _with_symmetry_ [F(x)*G(y),0]], [_Abel, 2nd\ type, class\ B]] \quad (51)$$

> *intfactor*(*Ecua*)

$$\frac{1}{y(x)^7} \quad (52)$$

> *FI* := $\frac{1}{y^7}$

$$FI := \frac{1}{y^7} \quad (53)$$

> *M* := $y^2 \sin(x)$

$$M := y^2 \sin(x) \quad (54)$$

> *N* := $1 + 5 y \cos(x)$

$$N := 1 + 5 y \cos(x) \quad (55)$$

> *MM* := *M*·*FI*

$$MM := \frac{\sin(x)}{y^5} \quad (56)$$

> *NN* := *expand*(*N*·*FI*)

$$NN := \frac{1}{y^7} + \frac{5 \cos(x)}{y^6} \quad (57)$$

> *Comprobar* := *diff*(*MM*, *y*) = *diff*(*NN*, *x*)

$$\text{Comprobar} := -\frac{5 \sin(x)}{y^6} = -\frac{5 \sin(x)}{y^6} \quad (58)$$

ES EXACTA

$$> \text{IntNNy} := \text{int}(\text{NN}, y)$$

$$\text{IntNNy} := -\frac{1}{6 y^6} - \frac{\cos(x)}{y^5} \quad (59)$$

$$> \text{SolGral} := \text{IntNNy} + \text{int}((\text{MM} - \text{diff}(\text{IntNNy}, x)), x) = _CI$$

$$\text{SolGral} := -\frac{1}{6 y^6} - \frac{\cos(x)}{y^5} = c_I \quad (60)$$

$$> \text{SolGralDos} := -\frac{1}{6 y(x)^6} - \frac{\cos(x)}{y(x)^5} = c_I$$

$$\text{SolGralDos} := -\frac{1}{6 y(x)^6} - \frac{\cos(x)}{y(x)^5} = c_I \quad (61)$$

$$> \text{Int}(\text{NN}, y) = \text{int}(\text{NN}, y)$$

$$\int \left(\frac{1}{y^7} + \frac{5 \cos(x)}{y^6} \right) dy = -\frac{1}{6 y^6} - \frac{\cos(x)}{y^5} \quad (62)$$

>

$$> \text{DerEcua} := \text{isolate}(\text{Ecua}, \text{diff}(y(x), x))$$

$$\text{DerEcua} := \frac{d}{dx} y(x) = -\frac{y(x)^2 \sin(x)}{1 + 5 y(x) \cos(x)} \quad (63)$$

$$> \text{DerSolDos} := \text{simplify}(\text{isolate}(\text{diff}(\text{SolGralDos}, x), \text{diff}(y(x), x)))$$

$$\text{DerSolDos} := \frac{d}{dx} y(x) = -\frac{y(x)^2 \sin(x)}{1 + 5 y(x) \cos(x)} \quad (64)$$

$$> \text{ComprobarDos} := \text{simplify}(\text{rhs}(\text{DerEcua}) - \text{rhs}(\text{DerSolDos})) = 0$$

$$\text{ComprobarDos} := 0 = 0 \quad (65)$$

> restart

$$> \text{Ecua} := x \cdot y' = \sqrt{x^2 - y^2} + y$$

$$\text{Ecua} := x \left(\frac{d}{dx} y(x) \right) = \sqrt{x^2 - y(x)^2} + y(x) \quad (66)$$

> with(DEtools):

> odeadvisor(Ecua)

$$[[_{\text{homogeneous}}, \text{class } A], _rational, _dAlembert] \quad (67)$$

$$> M := -(\sqrt{x^2 - y^2} + y)$$

$$M := -\sqrt{x^2 - y^2} - y \quad (68)$$

$$> N := x$$

$$N := x \quad (69)$$

$$> \text{EcuaDos} := \text{simplify}(\text{isolate}(\text{eval}(\text{subs}(y(x) = u(x) \cdot x, \text{Ecua})), \text{diff}(u(x), x)))$$

$$\text{EcuaDos} := \frac{d}{dx} u(x) = \frac{\sqrt{x^2 (1 - u(x)^2)}}{x^2} \quad (70)$$

$$\begin{aligned} &> MM := - \frac{(\sqrt{(1-u^2)})}{x} \\ &MM := - \frac{\sqrt{-u^2+1}}{x} \end{aligned} \quad (71)$$

$$\begin{aligned} &> NN := 1 \\ &NN := 1 \end{aligned} \quad (72)$$

$$\begin{aligned} &> P := -\frac{1}{x}; Q := \sqrt{-u^2+1}; R := 1; S := 1 \\ &P := -\frac{1}{x} \\ &Q := \sqrt{-u^2+1} \\ &R := 1 \\ &S := 1 \end{aligned} \quad (73)$$

$$\begin{aligned} &> SolGral := \text{int}\left(\frac{P}{R}, x\right) + \text{int}\left(\frac{S}{Q}, u\right) = _CI \\ &SolGral := -\ln(x) + \arcsin(u) = _CI \end{aligned} \quad (74)$$

$$\begin{aligned} &> SolGralDos := \text{subs}\left(u = \frac{y(x)}{x}, SolGral\right) \\ &SolGralDos := -\ln(x) + \arcsin\left(\frac{y(x)}{x}\right) = _CI \end{aligned} \quad (75)$$

$$\begin{aligned} &> DerSolGralDos := \text{expand}(\text{isolate}(\text{simplify}(\text{diff}(SolGralDos, x)), \text{diff}(y(x), x))) \\ &DerSolGralDos := \frac{d}{dx} y(x) = \sqrt{-\frac{y(x)^2}{x^2} + 1} + \frac{y(x)}{x} \end{aligned} \quad (76)$$

$$\begin{aligned} &> DerSolFinal := \frac{d}{dx} y(x) = \frac{\sqrt{-y(x)^2 + x^2}}{x} + \frac{y(x)}{x} \\ &DerSolFinal := \frac{d}{dx} y(x) = \frac{\sqrt{x^2 - y(x)^2}}{x} + \frac{y(x)}{x} \end{aligned} \quad (77)$$

$$\begin{aligned} &> Ecua \\ &x \left(\frac{d}{dx} y(x) \right) = \sqrt{x^2 - y(x)^2} + y(x) \end{aligned} \quad (78)$$

$$\begin{aligned} &> DerEcua := \text{isolate}(Ecua, \text{diff}(y(x), x)) \\ &DerEcua := \frac{d}{dx} y(x) = \frac{\sqrt{x^2 - y(x)^2} + y(x)}{x} \end{aligned} \quad (79)$$

$$\begin{aligned} &> ComprobarDos := \text{simplify}(\text{rhs}(DerEcua) - \text{rhs}(DerSolFinal)) = 0 \\ &ComprobarDos := 0 = 0 \end{aligned} \quad (80)$$

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