

TEMA 3b) SISTEMAS DE ECUACIONES DIFERENCIALES.

$$S(2 \times 2) \begin{cases} \frac{dx_1(t)}{dt} = 2x_1(t) + 3x_2(t) \\ \frac{dx_2(t)}{dt} = x_1(t) + 4x_2(t) \end{cases} \quad \begin{matrix} SP \\ \begin{cases} x_1(0) = -3 \\ x_2(0) = 2 \end{cases} \end{matrix}$$

$$\bar{X} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad \dot{\bar{X}} = \frac{d}{dt} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad \text{SG} \quad \boxed{\begin{matrix} x_1(0) = c_1 \\ x_2(0) = c_2 \end{matrix}}$$

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad \bar{X}(0) = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

$$\frac{d}{dt} \bar{X} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \bar{X}$$

$$\bar{X} = \left[e^{At} \right] \bar{X}(0)$$

$$\frac{dx_1}{dt} = 2x_1 + 3x_2 \quad \text{--- ①} \quad x_1(0) = -3$$

$$\frac{dx_2}{dt} = x_1 + 4x_2 \quad \text{--- ②} \quad x_2(0) = 2$$

$$\boxed{x_1 = \frac{dx_2}{dt} - 4x_2}$$

$$\frac{d}{dt} \left(\frac{dx_2}{dt} - 4x_2 \right) = \frac{d^2 x_2}{dt^2} - 4 \frac{dx_2}{dt}$$

$$\textcircled{1} \quad \frac{dx_1}{dt} = 2x_1 + 3x_2$$

$$\left[\frac{d^2 x_2}{dt^2} - 4 \frac{dx_2}{dt} \right] = 2 \left[\frac{dx_2}{dt} - 4x_2 \right] + 3x_2$$

$$\boxed{\frac{d^2 x_2}{dt^2} - 6 \frac{dx_2}{dt} + 5x_2 = 0}$$

$$S(z) \in \mathcal{DO}(1)_{\text{LCC}} = \mathcal{EDO}(z)_{\text{LCC}}$$

$$(D^2 - 6D + 5)x_2 = 0$$

$$(D-1)(D-5)x_2 = 0$$

$$\boxed{x_2(t) = c_1 e^t + c_2 e^{5t}}$$

$$\frac{dx_2}{dt} = c_1 e^t + 5c_2 e^{5t}$$

$$x_1(t) = (c_1 e^t + 5c_2 e^{5t}) - 4(c_1 e^t + c_2 e^{5t})$$

$$\text{SG} \quad \boxed{x_1(t) = -3c_1 e^t + c_2 e^{5t}}$$

$$\boxed{x_2(t) = c_1 e^t + c_2 e^{5t}}$$

$$x_1(0) \Rightarrow -3c_1(1) + c_2(1) = -3$$

$$x_2(0) \Rightarrow c_1(1) + c_2(1) = 2$$

$$c_2 = 2 - c_1$$

$$-3c_1 + (2 - c_1) = -3$$

$$-3c_1 - c_1 + 2 = -3$$

$$-4c_1 = -5$$

$$\boxed{c_1 = \frac{5}{4}}$$

SP

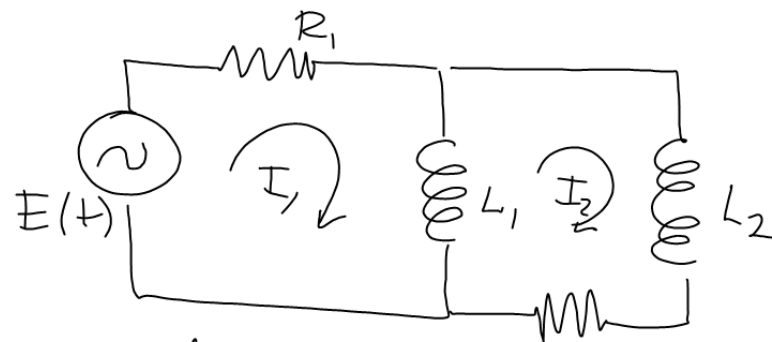
$$\boxed{x_1(t) = -\frac{15}{4}e^t + \frac{3}{4}e^{5t}}$$

$$\boxed{x_2(t) = \frac{5}{4}e^t + \frac{3}{4}e^{5t}}$$

$$\boxed{c_2 = 2 - \frac{5}{4} \Rightarrow \frac{3}{4}}$$

$$x_1(0) = -\frac{15}{4}(1) + \frac{3}{4}(1) \Rightarrow -\frac{12}{4} \Rightarrow -3$$

$$x_2(0) = \frac{5}{4}(1) + \frac{3}{4}(1) \Rightarrow \frac{8}{4} \Rightarrow 2$$



$$L_1 \frac{d}{dt}(I_2 - I_1) + R_1 I_1 = E(t)$$

$$-L_1 \frac{d}{dt}(I_1 - I_2) + L_2 \frac{dI_2}{dt} + R_2 I_2 = 0$$

$$I_1(t) \quad I_2(t)$$

$$L_1 \frac{dI_2}{dt} - L_1 \frac{dI_1}{dt} + R_1 I_1 = E(t)$$

$$-L_1 \frac{dI_1}{dt} + L_1 \frac{dI_2}{dt} + L_2 \frac{dI_2}{dt} + R_2 I_2 = 0$$

$$-L_1 \frac{dI_1}{dt} + (L_1 + L_2) \frac{dI_2}{dt} + R_2 I_2 = 0$$

$$+L_1 \frac{dI_1}{dt} - L_1 \frac{dI_2}{dt} - R_1 I_1 = -E(t)$$

$$(0) \quad L_2 \frac{dI_2}{dt} = -R_2 I_2 + R_1 I_1 - E(t)$$

$$\frac{dI_2}{dt} = \frac{R_1}{L_2} I_1 - \frac{R_2}{L_2} I_2 - \frac{E(t)}{L_2}$$

$$\frac{dy_3}{dt} - 4y_3 + 5y_2 - y_1 = 0$$

EDO(3) L'ord. $\frac{d^3 y}{dt^3} - 4 \frac{d^2 y}{dt^2} + 5 \frac{dy}{dt} - y = 0$

$$y = y_1$$

$$\frac{dy}{dt} = \frac{dy_1}{dt} = y_2$$

$$\frac{d^2 y}{dt^2} = \frac{dy_2}{dt} = y_3$$

$$\frac{d^3 y}{dt^3} = \frac{dy_3}{dt}$$

$$\frac{dy_1}{dt} = y_2$$

$$\frac{dy_2}{dt} = y_3$$

$$\frac{dy_3}{dt} = y_1 - 5y_2 + 4y_3$$

$$\begin{bmatrix} \frac{dy_1}{dt} \\ \frac{dy_2}{dt} \\ \frac{dy_3}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -5 & 4 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix}$$

$$\frac{d}{dt} \bar{X} = A \bar{X} \quad \bar{X} = [e^{At}] \bar{X}(0)$$

$$\frac{dx_1}{dt} = x_1 + x_2 \quad x_2 = \frac{dx_1}{dt} - x_1$$

$$\frac{dx_2}{dt} = -x_1 + x_2 \quad \frac{dx_2}{dt} = \frac{d^2x_1}{dt^2} - \frac{dx_1}{dt}$$

$$\left[\frac{d^2x_1}{dt^2} - \frac{dx_1}{dt} \right] = -x_1 + \left[\frac{dx_1}{dt} - x_1 \right]$$

$$\frac{d^2x_1}{dt^2} - 2\frac{dx_1}{dt} + 2x_1 = 0$$

$$(D^2 - 2D + 2)x_1 = 0$$

$$m^2 - 2m + 2 = 0 \quad \frac{-(-2) \pm \sqrt{4 - 4(2)}}{2}$$

$$m_{1,2} = 1 \pm i$$

$$x_1 = c_1 e^t \cos(t) + c_2 e^t \sin(t)$$

$$\frac{dx_1}{dt} = c_1 (e^t \cos(t) - e^t \sin(t)) + c_2 (e^t \sin(t) + e^t \cos(t))$$

$$x_1 = -c_1 e^t \cos(t) - c_2 e^t \sin(t)$$

$$x_2 = -c_1 e^t \sin(t) + c_2 e^t \cos(t)$$

$$x_1(t) = c_1 e^t \cos(t) + c_2 e^t \sin(t)$$

$$x_2(t) = -c_1 e^t \sin(t) + c_2 e^t \cos(t)$$