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> restart
> Sistema := diff(x(t), t) + 2·x(t) + 4·y(t) = 1 + 4·t, diff(y(t), t) + x(t) - y(t) =  $\frac{3}{2} \cdot t^2$  :
Sistema[1]; Sistema[2]

$$\frac{d}{dt} x(t) + 2 x(t) + 4 y(t) = 1 + 4 t$$


$$\frac{d}{dt} y(t) + x(t) - y(t) = \frac{3 t^2}{2}$$


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(1)

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> Xcero := array([_C1, _C2])
Xcero := [ _C1 _C2 ]

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(2)

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>
RESPUESTA

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> SistemaDos := diff(x(t), t) = -2·x(t) - 4·y(t) + 1 + 4·t, diff(y(t), t) = -x(t) + y(t) +  $\frac{3}{2} \cdot t^2$  :
SistemaDos[1]; SistemaDos[2]

$$\frac{d}{dt} x(t) = -2 x(t) - 4 y(t) + 1 + 4 t$$


$$\frac{d}{dt} y(t) = -x(t) + y(t) + \frac{3 t^2}{2}$$


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(3)

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> AA := array([[-2, -4], [-1, 1]])
AA := [ -2 -4 ]
      [-1  1 ]

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(4)

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> BB := array([rhs(Sistema[1]), rhs(Sistema[2])])
BB := [ 1 + 4 t   $\frac{3 t^2}{2}$  ]

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(5)

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>
> with(linalg) :
> MatExp := exponential(AA, t)
MatExp := [  $\frac{4 e^{-3 t}}{5} + \frac{e^{2 t}}{5}$    $-\frac{4 e^{2 t}}{5} + \frac{4 e^{-3 t}}{5}$  ]
           [  $-\frac{e^{2 t}}{5} + \frac{e^{-3 t}}{5}$    $\frac{e^{-3 t}}{5} + \frac{4 e^{2 t}}{5}$  ]

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(6)

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> Identidad := map(rcurry(eval, t='0'), MatExp)
Identidad := [ 1  0 ]
              [ 0  1 ]

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(7)

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CALCULAR LA SOLUCION HOMOGENEA

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> SolHom := evalm(MatExp &* Xcero) : x(t) = SolHom[1]; y(t) = SolHom[2]

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$$\begin{aligned}
 x(t) &= \left(\frac{4 e^{-3t}}{5} + \frac{e^{2t}}{5} \right) _{-C1} + \left(-\frac{4 e^{2t}}{5} + \frac{4 e^{-3t}}{5} \right) _{-C2} \\
 y(t) &= \left(-\frac{e^{2t}}{5} + \frac{e^{-3t}}{5} \right) _{-C1} + \left(\frac{e^{-3t}}{5} + \frac{4 e^{2t}}{5} \right) _{-C2}
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 > \text{ComprobarUno} := x(0) = \text{simplify}(\text{eval}(\text{subs}(t=0, \text{SolHom}[1]))) \\
 &\quad \text{ComprobarUno} := x(0) = _C1
 \end{aligned} \tag{9}$$

$$\begin{aligned}
 > \text{ComprobarDos} := y(0) = \text{simplify}(\text{eval}(\text{subs}(t=0, \text{SolHom}[2]))) \\
 &\quad \text{ComprobarDos} := y(0) = _C2
 \end{aligned} \tag{10}$$

$$\begin{aligned}
 > x(0) = \text{evalm}(\text{Xcero}[1]) \\
 &\quad x(0) = _C1
 \end{aligned} \tag{11}$$

$$\begin{aligned}
 > y(0) = \text{evalm}(\text{Xcero}[2]) \\
 &\quad y(0) = _C2
 \end{aligned} \tag{12}$$

CALCULAR LA SOLUCION NO HOMOGENEA

$$\begin{aligned}
 > \text{evalm}(\text{MatExp}) \\
 &\quad \begin{bmatrix} \frac{4 e^{-3t}}{5} + \frac{e^{2t}}{5} & -\frac{4 e^{2t}}{5} + \frac{4 e^{-3t}}{5} \\ -\frac{e^{2t}}{5} + \frac{e^{-3t}}{5} & \frac{e^{-3t}}{5} + \frac{4 e^{2t}}{5} \end{bmatrix}
 \end{aligned} \tag{13}$$

$$\begin{aligned}
 > \text{MatExpTau} := \text{map}(\text{rcurry}(\text{eval}, t='t - \text{tau}'), \text{MatExp}) \\
 &\quad \text{MatExpTau} := \begin{bmatrix} \frac{4 e^{-3t+3t}}{5} + \frac{e^{2t-2t}}{5} & -\frac{4 e^{2t-2t}}{5} + \frac{4 e^{-3t+3t}}{5} \\ -\frac{e^{2t-2t}}{5} + \frac{e^{-3t+3t}}{5} & \frac{e^{-3t+3t}}{5} + \frac{4 e^{2t-2t}}{5} \end{bmatrix}
 \end{aligned} \tag{14}$$

$$\begin{aligned}
 > \text{BBtau} := \text{map}(\text{rcurry}(\text{eval}, t='tau'), \text{BB}) \\
 &\quad \text{BBtau} := \begin{bmatrix} 1 + 4t & \frac{3t^2}{2} \end{bmatrix}
 \end{aligned} \tag{15}$$

$$\begin{aligned}
 > \text{ProdTau} := \text{evalm}(\text{MatExpTau} \& \text{BBtau}) : \text{ProdTau}[1]; \text{ProdTau}[2] \\
 &\quad \left(\frac{4 e^{-3t+3t}}{5} + \frac{e^{2t-2t}}{5} \right) (1 + 4t) + \frac{3 \left(-\frac{4 e^{2t-2t}}{5} + \frac{4 e^{-3t+3t}}{5} \right) t^2}{2} \\
 &\quad \left(-\frac{e^{2t-2t}}{5} + \frac{e^{-3t+3t}}{5} \right) (1 + 4t) + \frac{3 \left(\frac{e^{-3t+3t}}{5} + \frac{4 e^{2t-2t}}{5} \right) t^2}{2}
 \end{aligned} \tag{16}$$

$$\begin{aligned}
 > \text{SolNoHom} := \text{simplify}(\text{map}(\text{int}, \text{ProdTau}, \text{tau}=0..t)) : x(t) = \text{SolNoHom}[1]; y(t) \\
 &\quad = \text{SolNoHom}[2] \\
 &\quad x(t) = t^2 + t \\
 &\quad y(t) = -\frac{t^2}{2}
 \end{aligned} \tag{17}$$

$$\begin{aligned} &> \text{ComprobarTres} := x(0) = \text{simplify}(\text{eval}(\text{subs}(t=0, \text{SolNoHom}[1]))) \\ &\quad \text{ComprobarTres} := x(0) = 0 \end{aligned} \quad (18)$$

$$\begin{aligned} &> \text{ComprobarCuatro} := y(0) = \text{simplify}(\text{eval}(\text{subs}(t=0, \text{SolNoHom}[2]))) \\ &\quad \text{ComprobarCuatro} := y(0) = 0 \end{aligned} \quad (19)$$

$$\begin{aligned} &> \text{SolFinal} := \text{evalm}(\text{SolHom} + \text{SolNoHom}) : x(t) = \text{SolFinal}[1]; y(t) = \text{SolFinal}[2] \\ &\quad x(t) = \left(\frac{4e^{-3t}}{5} + \frac{e^{2t}}{5} \right) - C1 + \left(-\frac{4e^{2t}}{5} + \frac{4e^{-3t}}{5} \right) - C2 + t^2 + t \\ &\quad y(t) = \left(-\frac{e^{2t}}{5} + \frac{e^{-3t}}{5} \right) - C1 + \left(\frac{e^{-3t}}{5} + \frac{4e^{2t}}{5} \right) - C2 - \frac{t^2}{2} \end{aligned} \quad (20)$$

$$\begin{aligned} &> \text{ComprobarCinco} := x(0) = \text{simplify}(\text{eval}(\text{subs}(t=0, \text{SolFinal}[1]))) \\ &\quad \text{ComprobarCinco} := x(0) = _C1 \end{aligned} \quad (21)$$

$$\begin{aligned} &> \text{ComprobarSeis} := y(0) = \text{simplify}(\text{eval}(\text{subs}(t=0, \text{SolFinal}[2]))) \\ &\quad \text{ComprobarSeis} := y(0) = _C2 \end{aligned} \quad (22)$$

$$\begin{aligned} &> \text{evalm}(\text{Sistema}[1]); \text{evalm}(\text{Sistema}[2]) \\ &\quad \frac{d}{dt} x(t) + 2x(t) + 4y(t) = 1 + 4t \\ &\quad \frac{d}{dt} y(t) + x(t) - y(t) = \frac{3t^2}{2} \end{aligned} \quad (23)$$

$$\begin{aligned} &> \text{ComprobarSiete} := \text{simplify}(\text{eval}(\text{subs}(x(t) = \text{SolFinal}[1], y(t) = \text{SolFinal}[2], \\ &\quad \text{lhs}(\text{SistemaDos}[1]) - \text{rhs}(\text{SistemaDos}[1]) = 0))) \\ &\quad \text{ComprobarSiete} := 0 = 0 \end{aligned} \quad (24)$$

$$\begin{aligned} &> \text{ComprobarOcho} := \text{simplify}(\text{eval}(\text{subs}(x(t) = \text{SolFinal}[1], y(t) = \text{SolFinal}[2], \\ &\quad \text{lhs}(\text{SistemaDos}[2]) - \text{rhs}(\text{SistemaDos}[2]) = 0))) \\ &\quad \text{ComprobarOcho} := 0 = 0 \end{aligned} \quad (25)$$

$$\begin{aligned} &> \text{ComprobarNueve} := \text{simplify}(\text{eval}(\text{subs}(x(t) = \text{SolFinal}[1], y(t) = \text{SolFinal}[2], \\ &\quad \text{lhs}(\text{Sistema}[1]) - \text{rhs}(\text{Sistema}[1]) = 0))) \\ &\quad \text{ComprobarNueve} := 0 = 0 \end{aligned} \quad (26)$$

$$\begin{aligned} &> \text{ComprobarDiez} := \text{simplify}(\text{eval}(\text{subs}(x(t) = \text{SolFinal}[1], y(t) = \text{SolFinal}[2], \text{lhs}(\text{Sistema}[2]) \\ &\quad - \text{rhs}(\text{Sistema}[2]) = 0))) \\ &\quad \text{ComprobarDiez} := 0 = 0 \end{aligned} \quad (27)$$

> restart

$$\begin{aligned} &> \text{Sistema} := \text{diff}(x(t), t) = 3 \cdot x(t) - \frac{1}{2} \cdot y(t) - 3 \cdot t^2 - \frac{1}{2} \cdot t + \frac{3}{2}, \text{diff}(y(t), t) = 2 \cdot y(t) - 2 \cdot t \\ &\quad - 1 : \text{Sistema}[1]; \text{Sistema}[2] \\ &\quad \frac{d}{dt} x(t) = 3x(t) - \frac{y(t)}{2} - 3t^2 - \frac{t}{2} + \frac{3}{2} \\ &\quad \frac{d}{dt} y(t) = 2y(t) - 2t - 1 \end{aligned} \quad (28)$$

$$\begin{aligned} &> \text{CondIni} := x(0) = 1, y(0) = 1 \\ &\quad \text{CondIni} := x(0) = 1, y(0) = 1 \end{aligned} \quad (29)$$

>

RESULTADO

> $AA := \text{array}\left(\left[\left[3, -\frac{1}{2}\right], [0, 2]\right]\right)$

$$AA := \begin{bmatrix} 3 & -\frac{1}{2} \\ 0 & 2 \end{bmatrix} \quad (30)$$

> $Xcero := \text{array}([1, 1])$

$$Xcero := \begin{bmatrix} 1 & 1 \end{bmatrix} \quad (31)$$

> $BB := \text{array}\left(\left[-3t^2 - \frac{t}{2} + \frac{3}{2}, -2t - 1\right]\right) : BB[1]; BB[2];$

$$\begin{aligned} & -3t^2 - \frac{1}{2}t + \frac{3}{2} \\ & -2t - 1 \end{aligned} \quad (32)$$

> $\text{with}(\text{linalg}) :$

> $\text{MatExp} := \text{exponential}(AA, t)$

$$\text{MatExp} := \begin{bmatrix} e^{3t} & -\frac{e^{3t}}{2} + \frac{e^{2t}}{2} \\ 0 & e^{2t} \end{bmatrix} \quad (33)$$

> $\text{SolHom} := \text{evalm}(\text{MatExp} \&* Xcero) : x(t) = \text{SolHom}[1]; y(t) = \text{SolHom}[2]$

$$\begin{aligned} x(t) &= \frac{e^{3t}}{2} + \frac{e^{2t}}{2} \\ y(t) &= e^{2t} \end{aligned} \quad (34)$$

> $\text{MatExpTau} := \text{map}(\text{rcurry}(\text{eval}, t = t - x'), \text{MatExp})$

$$\text{MatExpTau} := \begin{bmatrix} e^{3t-3x} & -\frac{e^{3t-3x}}{2} + \frac{e^{2t-2x}}{2} \\ 0 & e^{2t-2x} \end{bmatrix} \quad (35)$$

> $\text{BBtau} := \text{map}(\text{rcurry}(\text{eval}, t = t - x'), BB)$

$$\text{BBtau} := \begin{bmatrix} -3x^2 - \frac{1}{2}x + \frac{3}{2} & -2x - 1 \end{bmatrix} \quad (36)$$

> $\text{ProdTau} := \text{evalm}(\text{MatExpTau} \&* \text{BBtau}) : \text{ProdTau}[1]; \text{ProdTau}[2]$

$$\begin{aligned} & e^{3t-3x} \left(-3x^2 - \frac{1}{2}x + \frac{3}{2} \right) + \left(-\frac{e^{3t-3x}}{2} + \frac{e^{2t-2x}}{2} \right) (-2x - 1) \\ & e^{2t-2x} (-2x - 1) \end{aligned} \quad (37)$$

> $\text{SolNoHom} := \text{map}(\text{int}, \text{ProdTau}, x = 0 .. t) : x(t) = \text{SolNoHom}[1]; y(t) = \text{SolNoHom}[2]$

$$\begin{aligned} x(t) &= \frac{e^{3t}}{2} - \frac{e^{2t}}{2} + t^2 + t \\ y(t) &= 1 - e^{2t} + t \end{aligned} \quad (38)$$

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> SolFinal := evalm(SolHom + SolNoHom) : x(t) = SolFinal[1]; y(t) = SolFinal[2]
      x(t) = e3t + t2 + t
      y(t) = 1 + t
(39)

> Sistema[1]
      d
      dt x(t) = 3 x(t) -  $\frac{y(t)}{2}$  - 3 t2 -  $\frac{t}{2}$  +  $\frac{3}{2}$ 
(40)

> ComprobarUno := simplify(eval(subs(x(t) = SolFinal[1], y(t) = SolFinal[2], lhs(Sistema[1])
      - rhs(Sistema[1]) = 0)))
      ComprobarUno := 0 = 0
(41)

> Sistema[2]
      d
      dt y(t) = 2 y(t) - 2 t - 1
(42)

> ComprobarDos := simplify(eval(subs(x(t) = SolFinal[1], y(t) = SolFinal[2], lhs(Sistema[2])
      - rhs(Sistema[2]) = 0)))
      ComprobarDos := 0 = 0
(43)

> ComprobarTres := x(0) = simplify(eval(subs(t = 0, SolFinal[1])))
      ComprobarTres := x(0) = 1
(44)

> ComprobarCuatro := y(0) = simplify(eval(subs(t = 0, SolFinal[2])))
      ComprobarCuatro := y(0) = 1
(45)

> CondIni[1]
      x(0) = 1
(46)

> CondIni[2]
      y(0) = 1
(47)

>

```