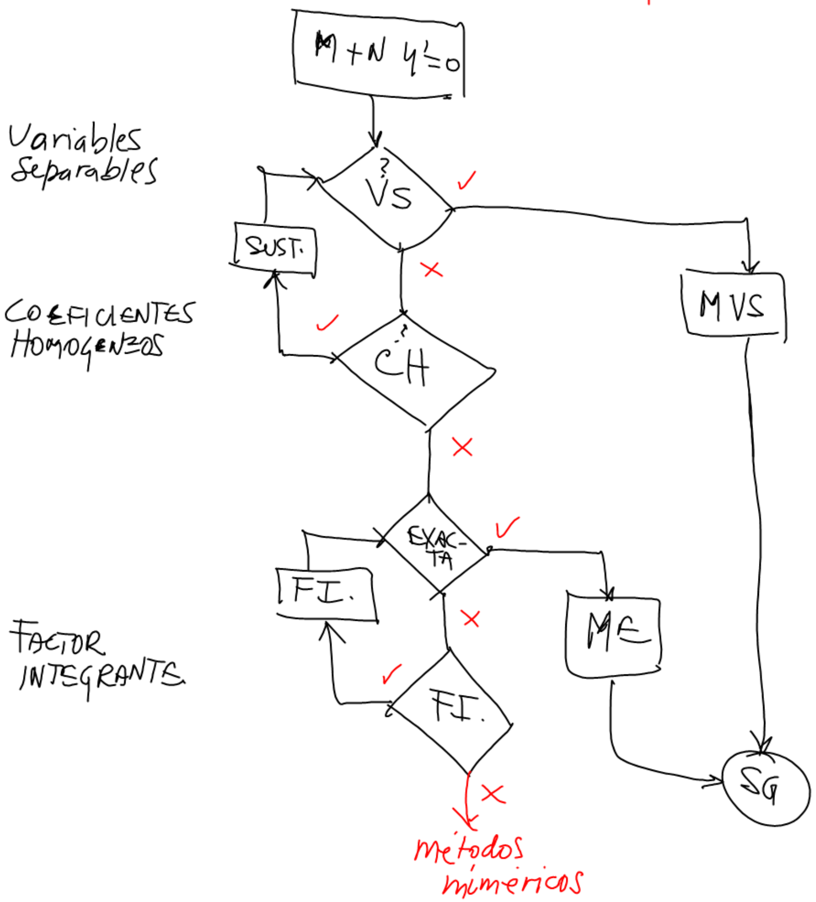
Ecuación Diferencial Primer Orden no Lineal

$$\frac{dy}{dx} = F(x,y)$$

$$\frac{dy}{dx} = -\frac{M(x,y)}{N(x,y)}$$

$$N(x,y)\frac{dy}{dx} = -M(x,y)$$

$$\boxed{M(x,y) + N(x,y)\frac{dy}{dx} = 0} \quad \text{NL}$$



MÉTODO VARIABLES SEPARABLES

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

$$P(x)\cdot Q(y) + R(x)\cdot S(y)\cdot\frac{dy}{dx} = 0$$

$$\frac{1}{Q(y)R(x)}\left(P(x)\cdot Q(y) + R(x)\cdot S(y)\cdot\frac{dy}{dx}\right) = 0$$

$$\frac{P(x)}{R(x)} + \frac{S(y)}{Q(y)}\cdot\frac{dy}{dx} = 0$$

$$\frac{P(x)}{R(x)}dx + \frac{S(y)}{Q(y)}dy = 0$$

SG

$$\boxed{\int\frac{P(x)}{R(x)}dx + \int\frac{S(y)}{Q(y)}dy = C}$$

$$F(x,y) = C$$

$$(1+e^{x})\,y\,\frac{dy}{dx} = e^{x}$$

$$\underbrace{-e^{x}}_{M} + \underbrace{(1+e^{x})\,y}_{N}\,\frac{dy}{dx} = 0$$

$$P(x) = e^{x} \qquad Q(y) = -1$$

$$R(x) = 1+e^{x} \qquad S(y) = y$$

SOL GRAL

$$\int \frac{e^{x}}{1+e^{x}}\,dx + \int \frac{y}{-1}\,dy = C$$

$$u = 1+e^{x}$$

$$du = e^{x}dx$$

$$\int \frac{du}{u} - \int y\,dy = C,$$

$$\ln u - \frac{y^2}{2} = C$$

SG

$$\boxed{\ln(1+e^{x}) - \frac{y^2}{2} = C}$$

$$(1+e^{x})\,y\,\frac{dy}{dx} = e^{x}$$

MÉTODO DE COEFICIENTES HOMOGENEOS

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

$$\left.\begin{aligned} M(\lambda x, \lambda y) &= \lambda^m M(x,y) \\ N(\lambda x, \lambda y) &= \lambda^n N(x,y) \end{aligned}\right\} m = n$$

Sust. $y(x) = x \cdot u(x)$ $\quad \frac{dy}{dx} = x\frac{du}{dx} + u$

$$x \cdot \frac{dy}{dx} = \sqrt{x^2-y^2} + y$$

$M = -\sqrt{x^2-y^2} - y$

$N = x$

$$M(\lambda x, \lambda y) = -\sqrt{(\lambda x)^2 - \lambda(y)^2} - (\lambda y)$$

$$= -\sqrt{\lambda^2 x^2 - \lambda^2 y^2} + \lambda(-y)$$

$$= -\sqrt{\lambda^2}\sqrt{x^2-y^2} - \lambda y$$

$$= -\lambda\sqrt{x^2-y^2} - \lambda y$$

$$= \lambda\left(-\sqrt{x^2-y^2} - y\right) \qquad m=1$$

$N = x$

$$N(\lambda x, \lambda y) = \lambda x \qquad n=1 \qquad m=n$$

$$y(x) = x \cdot u(x) \qquad \frac{dy}{dx} = x\frac{du}{dx} + u$$

$$x\left(x\frac{du}{dx} + u\right) = \sqrt{x^2 - x^2u^2} + xu$$

$$x^2\frac{du}{dx} + \cancel{x}u = \sqrt{x^2(1-u^2)} + \cancel{x}u$$

$$x^2\frac{du}{dx} = x\sqrt{1-u^2}$$

$$-x\sqrt{1-u^2} + x^2\frac{du}{dx} = 0$$

$P = -x \qquad Q = \sqrt{1-u^2}$

$R = x^2 \qquad S = 1$

$$\boxed{-\int \frac{1}{x}\,dx + \int \frac{du}{\sqrt{1-u^2}} = C_1}$$