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**FACULTAD DE INGENIERIA  
 DIVISION DE CIENCIAS BASICAS  
 ECUACIONES DIFERENCIALES  
 GRUPO 10 SEMESTRE 2025-2  
 SERIE 1**

> *restart*

1) Sea la ecuación diferencial

> *Ecuacion* :=  $y' = (x - 4 \cdot y - 1)^2$

$$\text{Ecuacion} := \frac{d}{dx} y(x) = (x - 4y(x) - 1)^2 \quad (1)$$

> *CondIni* :=  $y(0) = 0$

$$\text{CondIni} := y(0) = 0 \quad (2)$$

>

**SOLUCIÓN**

> *Sug* :=  $z(x) = x - 4y(x) - 1$

$$\text{Sug} := z(x) = x - 4y(x) - 1 \quad (3)$$

> *Sub* := *isolate*(*Sug*,  $y(x)$ )

$$\text{Sub} := y(x) = -\frac{z(x)}{4} + \frac{x}{4} - \frac{1}{4} \quad (4)$$

> *EcuaDos* := *isolate*(*eval*(*subs*( $y(x) = \text{rhs}(\text{Sub})$ ), *Ecuacion*)), *diff*( $z(x)$ ,  $x$ )

$$\text{EcuaDos} := \frac{d}{dx} z(x) = -4z(x)^2 + 1 \quad (5)$$

> *EcuaTres* :=  $-\text{rhs}(\text{EcuaDos}) + \text{lhs}(\text{EcuaDos}) = 0$

$$\text{EcuaTres} := 4z(x)^2 - 1 + \frac{d}{dx} z(x) = 0 \quad (6)$$

>  $M := 4z^2 - 1; N := 1$

$$\begin{aligned} M &:= 4z^2 - 1 \\ N &:= 1 \end{aligned} \quad (7)$$

**VARIABLES SEPARABLES**

>  $P := 4; Q := z^2 - \frac{1}{4}; R := 1; S := 1$

$$P := 4$$

$$Q := z^2 - \frac{1}{4}$$

$$R := 1$$

$$S := 1$$

(8)

>  $\text{SolIntermedia} := \text{int}\left(\frac{P}{R}, x\right) + \text{int}\left(\frac{S}{Q}, z\right) = -C1$

$$\text{SolIntermedia} := 4x - \ln(2z + 1) + \ln(2z - 1) = -C1 \quad (9)$$

>  $\text{SolInter} := 4x - \ln(2z(x) + 1) + \ln(2z(x) - 1) = -C1$

$$SolInter := 4x - \ln(2z(x) + 1) + \ln(2z(x) - 1) = _C1 \quad (10)$$

> Sug

$$z(x) = x - 4y(x) - 1 \quad (11)$$

$$\begin{aligned} > SolGralFinal &:= expand(subs(Sug, SolInter)) \\ SolGralFinal &:= 4x - \ln(2x - 8y(x) - 1) + \ln(2x - 8y(x) - 3) = _C1 \end{aligned} \quad (12)$$

> DerSolGral := simplify(isolate(diff(SolGralFinal, x), diff(y(x), x)))

$$DerSolGral := \frac{d}{dx} y(x) = (x - 4y(x) - 1)^2 \quad (13)$$

> Ecuacion

$$\frac{d}{dx} y(x) = (x - 4y(x) - 1)^2 \quad (14)$$

$$\begin{aligned} > Comprobar &:= rhs(DerSolGral) - rhs(Ecuacion) = 0 \\ Comprobar &:= 0 = 0 \end{aligned} \quad (15)$$

> CondIni

$$y(0) = 0 \quad (16)$$

$$\begin{aligned} > Para &:= subs(x=0, y(0)=0, SolGralFinal) \\ Para &:= -\ln(-1) + \ln(-3) = _C1 \end{aligned} \quad (17)$$

$$\begin{aligned} > SolPart &:= subs(_C1 = lhs(Para), SolGralFinal) \\ SolPart &:= 4x - \ln(2x - 8y(x) - 1) + \ln(2x - 8y(x) - 3) = \ln(3) \end{aligned} \quad (18)$$

> DerSolPart := simplify(isolate(diff(SolPart, x), diff(y(x), x)))

$$DerSolPart := \frac{d}{dx} y(x) = (x - 4y(x) - 1)^2 \quad (19)$$

> Ecuacion

$$\frac{d}{dx} y(x) = (x - 4y(x) - 1)^2 \quad (20)$$

> restart

2) Resuelva

$$\begin{aligned} > Ecuacion &:= y' = \frac{\sin(y)}{(x \cdot \cos(y) - \sin(y)^2)} \\ Ecuacion &:= \frac{d}{dx} y(x) = \frac{\sin(y(x))}{x \cos(y(x)) - \sin(y(x))^2} \end{aligned} \quad (21)$$

$$> CondIni := y(0) = \frac{\pi}{2}$$

$$CondIni := y(0) = \frac{\pi}{2} \quad (22)$$

$$> Ecua := -(\sin(y(x))) + (x \cos(y(x)) - \sin(y(x))^2) \cdot diff(y(x), x) = 0$$

$$Ecua := -\sin(y(x)) + (x \cos(y(x)) - \sin(y(x))^2) \left( \frac{d}{dx} y(x) \right) = 0 \quad (23)$$

SOLUCIÓN

> with(DEtools) :

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> intfactor(Ecua)

$$\frac{1}{\sin(y(x))^2} \quad (24)$$


> IntFact :=  $\frac{1}{\sin(y)^2}$ 

$$IntFact := \frac{1}{\sin(y)^2} \quad (25)$$


> M := -sin(y); N := (x cos(y) - sin(y)^2)

$$M := -\sin(y)$$


$$N := x \cos(y) - \sin(y)^2 \quad (26)$$


> MM := simplify(IntFact·M); NN := simplify(IntFact·N)

$$MM := -\csc(y)$$


$$NN := x \cot(y) \csc(y) - 1 \quad (27)$$


> comprobar := diff(MM, y) - diff(NN, x) = 0

$$comprobar := 0 = 0 \quad (28)$$


> IntMMx := int(MM, x)

$$IntMMx := -\csc(y) x \quad (29)$$


> SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = _C1

$$SolGral := -\csc(y) x - y = _C1 \quad (30)$$


> SolGralFinal := -csc(y(x)) x - y(x) = _C1

$$SolGralFinal := -\csc(y(x)) x - y(x) = _C1 \quad (31)$$


> DerSolFinal := isolate(diff(SolGralFinal, x), diff(y(x), x))

$$DerSolFinal := \frac{d}{dx} y(x) = \frac{\csc(y(x))}{\csc(y(x)) \cot(y(x)) x - 1} \quad (32)$$


> Ecuacion

$$\frac{d}{dx} y(x) = \frac{\sin(y(x))}{x \cos(y(x)) - \sin(y(x))^2} \quad (33)$$


> Comprobar := simplify(rhs(DerSolFinal) - rhs(Ecuacion)) = 0

$$Comprobar := 0 = 0 \quad (34)$$


> Para := subs(x=0, y=Pi/2, SolGral)

$$Para := -\frac{\pi}{2} = _C1 \quad (35)$$


> SolPartFinal := subs(_C1 = lhs(Para), SolGralFinal)

$$SolPartFinal := -\csc(y(x)) x - y(x) = -\frac{\pi}{2} \quad (36)$$


> ComprobarDos := subs(x=0, SolPartFinal)

$$ComprobarDos := -y(0) = -\frac{\pi}{2} \quad (37)$$


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$$\begin{aligned} > \text{DerSolPart} &:= \text{isolate}(\text{diff}(\text{SolPartFinal}, x), \text{diff}(y(x), x)) \\ \text{DerSolPart} &:= \frac{d}{dx} y(x) = \frac{\csc(y(x))}{\csc(y(x)) \cot(y(x)) x - 1} \end{aligned} \quad (38)$$

$$\begin{aligned} > \text{ComprobarTres} &:= \text{simplify}(\text{rhs}(\text{DerSolPart}) - \text{rhs}(\text{Ecuacion})) = 0 \\ \text{ComprobarTres} &:= 0 = 0 \end{aligned} \quad (39)$$

> restart

3)

$$\begin{aligned} > \text{Ecua} &:= y' + \frac{y}{x} = \log(x) \\ \text{Ecua} &:= \frac{d}{dx} y(x) + \frac{y(x)}{x} = \ln(x) \end{aligned} \quad (40)$$

>

SOLUCIÓN

$$\begin{aligned} > P &:= \frac{1}{x}; Q := \log(x) \\ P &:= \frac{1}{x} \\ Q &:= \ln(x) \end{aligned} \quad (41)$$

$$\begin{aligned} > \text{ExpIntP} &:= \exp(\text{int}(P, x)) \\ \text{ExpIntP} &:= x \end{aligned} \quad (42)$$

$$\begin{aligned} > \text{ExpIntPNeg} &:= \exp(-\text{int}(P, x)) \\ \text{ExpIntPNeg} &:= \frac{1}{x} \end{aligned} \quad (43)$$

$$\begin{aligned} > \text{SolGral} &:= y(x) = _C1 \cdot \text{ExpIntPNeg} + \text{simplify}(\text{ExpIntPNeg} \cdot \text{int}(\text{ExpIntP} \cdot Q, x)) \\ \text{SolGral} &:= y(x) = \frac{C1}{x} + \frac{x \ln(x)}{2} - \frac{x}{4} \end{aligned} \quad (44)$$

> Ecua

$$\frac{d}{dx} y(x) + \frac{y(x)}{x} = \ln(x) \quad (45)$$

$$\begin{aligned} > \text{Comprobar} &:= \text{simplify}(\text{eval}(\text{subs}(y(x) = \text{rhs}(\text{SolGral}), \text{lhs}(\text{Ecua}) - \text{rhs}(\text{Ecua}) = 0))) \\ \text{Comprobar} &:= 0 = 0 \end{aligned} \quad (46)$$

> restart

4)

$$\begin{aligned} > \text{Ecua} &:= \exp(x) \cdot (y - 1) + 2 \cdot (\exp(x) + 4) \cdot y' = 0 \\ \text{Ecua} &:= e^x (y(x) - 1) + 2 (e^x + 4) \left( \frac{d}{dx} y(x) \right) = 0 \end{aligned} \quad (47)$$

$$\begin{aligned} > \text{CondIni} &:= y(0) = 2 \\ \text{CondIni} &:= y(0) = 2 \end{aligned} \quad (48)$$

>

SOLUCION

> with(DEtools) :

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> odeadvisor(Ecua)
[_separable] (49)

> M := ex (y - 1); N := 2 (ex + 4)
M := ex (y - 1)
N := 2 ex + 8 (50)

> P := exp(x); Q := y - 1; R := exp(x) + 4; S := 2
P := ex
Q := y - 1
R := ex + 4
S := 2 (51)

> SolGral := int(P/R, x) + int(S/Q, y) = _C1
SolGral := ln(ex + 4) + 2 ln(y - 1) = _C1 (52)

> SolGralDos := simplify(exp(lhs(SolGral))) = _C1
SolGralDos := (ex + 4) (y - 1)2 = _C1 (53)

> SolGralFinal := (ex + 4) (y(x) - 1)2 = _C1
SolGralFinal := (ex + 4) (y(x) - 1)2 = _C1 (54)

> DerSolFinal := isolate(diff(SolGralFinal, x), diff(y(x), x))
DerSolFinal :=  $\frac{d}{dx} y(x) = -\frac{e^x (y(x) - 1)}{2 (e^x + 4)}$  (55)

> Ecua
ex (y(x) - 1) + 2 (ex + 4)  $\left( \frac{d}{dx} y(x) \right) = 0$  (56)

> DerEcua := isolate(Ecua, diff(y(x), x))
DerEcua :=  $\frac{d}{dx} y(x) = -\frac{e^x (y(x) - 1)}{2 e^x + 8}$  (57)

> Comprobar := simplify(rhs(DerSolFinal) - rhs(DerEcua)) = 0
Comprobar := 0 = 0 (58)

> Para := simplify(subs(x=0, y=2, SolGralDos))
Para := 5 = _C1 (59)

> SolPartFinal := subs(_C1 = lhs(Para), SolGralFinal)
SolPartFinal := (ex + 4) (y(x) - 1)2 = 5 (60)

> DerSolPart := isolate(diff(SolPartFinal, x), diff(y(x), x))
DerSolPart :=  $\frac{d}{dx} y(x) = -\frac{e^x (y(x) - 1)}{2 (e^x + 4)}$  (61)

> ComprobarDos := simplify(rhs(DerSolPart) - rhs(DerEcua)) = 0
ComprobarDos := 0 = 0 (62)

> restart

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5)

>  $Ecua := -x \cdot y' - x = -y$

$$Ecua := -x \left( \frac{d}{dx} y(x) \right) - x = -y(x) \quad (63)$$

>  $CondIni := y(1) = -1$

$$CondIni := y(1) = -1 \quad (64)$$

SOLUCION

>  $EcuaDos := lhs(Ecua) + x + y(x) = rhs(Ecua) + x + y(x)$

$$EcuaDos := -x \left( \frac{d}{dx} y(x) \right) + y(x) = x \quad (65)$$

>  $EcuaTres := expand\left( \frac{lhs(EcuaDos)}{-x} \right) = \frac{rhs(EcuaDos)}{-x}$

$$EcuaTres := \frac{d}{dx} y(x) - \frac{y(x)}{x} = -1 \quad (66)$$

>  $p := -\frac{1}{x}$

$$p := -\frac{1}{x} \quad (67)$$

>  $q := -1$

$$q := -1 \quad (68)$$

>  $SolGral := y(x) = _C1 \cdot \exp(-int(p, x)) + \exp(-int(p, x)) \cdot int(\exp(int(p, x)) \cdot q, x)$

$$SolGral := y(x) = _C1 x - x \ln(x) \quad (69)$$

>  $ComprobarUno := simplify(eval(subs(y(x) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarUno := 0 = 0 \quad (70)$$

>  $CondIni$

$$y(1) = -1 \quad (71)$$

>  $Para := simplify(isolate(-1 = subs(x = 1, rhs(SolGral)), _C1))$

$$Para := _C1 = -1 \quad (72)$$

>  $SolPart := subs(_C1 = rhs(Para), SolGral)$

$$SolPart := y(x) = -x - x \ln(x) \quad (73)$$

>  $ComprobarDos := simplify(eval(subs(y(x) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarDos := 0 = 0 \quad (74)$$

>  $restart$ 

7)

>  $Ecua := (2 \cdot x^2 \cdot y - x) \cdot y' = -y$

$$Ecua := (2 x^2 y(x) - x) \left( \frac{d}{dx} y(x) \right) = -y(x) \quad (75)$$

>  $with(DEtools) :$

>  $odeadvisor(Ecua)$

$$[[\text{homogeneous, class G}], \text{rational}, [\text{Abel, 2nd type, class B}]] \quad (76)$$

>  $\text{FactInt} := \text{intfactor}(\text{Ecua})$

$$\text{FactInt} := \frac{1}{x^2} \quad (77)$$

>  $\text{EcuaDos} := \text{simplify}(\text{FactInt} \cdot \text{lhs}(\text{Ecua}) - \text{FactInt} \cdot \text{rhs}(\text{Ecua})) = 0$

$$\text{EcuaDos} := \frac{(2xy(x) - 1) \left( \frac{d}{dx} y(x) \right)}{x} + \frac{y(x)}{x^2} = 0 \quad (78)$$

>  $\text{odeadvisor}(\text{EcuaDos})$

$$[ [\text{homogeneous}, \text{class } G], \text{exact}, \text{rational}, [ \text{Abel}, \text{2nd type}, \text{class } B ] ] \quad (79)$$

>  $M := \frac{y}{x^2}$

$$M := \frac{y}{x^2} \quad (80)$$

>  $N := \text{expand} \left( \frac{(2y \cdot x - 1)}{x} \right)$

$$N := 2y - \frac{1}{x} \quad (81)$$

>  $\text{diff}(M, y) = \text{diff}(N, x)$

$$\frac{1}{x^2} = \frac{1}{x^2} \quad (82)$$

>  $\text{IntMx} := \text{int}(M, x)$

$$\text{IntMx} := -\frac{y}{x} \quad (83)$$

>  $\text{SolGral} := \text{IntMx} + \text{int}((N - \text{diff}(\text{IntMx}, y)), y) = \_C1$

$$\text{SolGral} := -\frac{y}{x} + y^2 = \_C1 \quad (84)$$

>  $\text{SolFinal} := -\frac{y(x)}{x} + y(x)^2 = \_C1$

$$\text{SolFinal} := -\frac{y(x)}{x} + y(x)^2 = \_C1 \quad (85)$$

>  $\text{DerSolFinal} := \text{isolate}(\text{diff}(\text{SolFinal}, x), \text{diff}(y(x), x))$

$$\text{DerSolFinal} := \frac{d}{dx} y(x) = -\frac{y(x)}{x^2 \left( -\frac{1}{x} + 2y(x) \right)} \quad (86)$$

>  $\text{Ecua}$

$$(2x^2 y(x) - x) \left( \frac{d}{dx} y(x) \right) = -y(x) \quad (87)$$

>  $\text{DerEcua} := \text{isolate}(\text{Ecua}, \text{diff}(y(x), x))$

$$\text{DerEcua} := \frac{d}{dx} y(x) = -\frac{y(x)}{2x^2 y(x) - x} \quad (88)$$

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> Comprobar := simplify(rhs(DerSolFinal) - rhs(DerEcua)) = 0
                                         Comprobar := 0 = 0
(89)

> restart
8)
> Ecua := (x + 3)^2 · y' = 4 - 9 · y - 3 · x · y
          Ecua := (x + 3)^2  $\left( \frac{dy}{dx} \right) = 4 - 9y(x) - 3xy(x)$ 
(90)

>
SOLUCIÓN
> with(DEtools):
> odeadvisor(Ecua)
                                         [_linear]
(91)

> IntFact := intfactor(Ecua)
                                         IntFact := x + 3
(92)

> M := -(4 - 9y - 3xy)
                                         M := 3xy + 9y - 4
(93)

> N := (x + 3)^2
                                         N := (x + 3)^2
(94)

> MM := IntFact · M
                                         MM := (x + 3)(3xy + 9y - 4)
(95)

> NN := IntFact · N
                                         NN := (x + 3)^3
(96)

> expand(diff(MM, y))
                                         3x^2 + 18x + 27
(97)

> expand(diff(NN, x))
                                         3x^2 + 18x + 27
(98)

> IntMMx := expand(int(MM, x))
                                         IntMMx := yx^3 + 9yx^2 - 2x^2 + 27xy - 12x
(99)

> SolGral := simplify(IntMMx + int((NN - diff(IntMMx, y)), y)) = _C1
                                         SolGral := yx^3 + (9y - 2)x^2 + (27y - 12)x + 27y = _C1
(100)

> SolFinal := isolate(y(x)x^3 + (9y(x) - 2)x^2 + (27y(x) - 12)x + 27y(x) = _C1, y(x))
                                         SolFinal := y(x) =  $\frac{2x^2 + _C1 + 12x}{x^3 + 9x^2 + 27x + 27}$ 
(101)

> Ecua
                                         (x + 3)^2  $\left( \frac{dy}{dx} \right) = 4 - 9y(x) - 3xy(x)$ 
(102)

> Comprobar := simplify(eval(subs(y(x) = rhs(SolFinal), lhs(Ecua) - rhs(Ecua) = 0)))
                                         Comprobar := 0 = 0
(103)

> restart

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9)

>  $Ecua := x \cdot y^2 \cdot y' - y^3 - 1 = 0$

$$Ecua := x y(x)^2 \left( \frac{d}{dx} y(x) \right) - y(x)^3 - 1 = 0 \quad (104)$$

>  $\text{with(DEtools)} :$ 

>  $\text{odeadvisor}(Ecua)$

$$[_{\text{separable}}] \quad (105)$$

>  $M := -y^3 - 1$

$$M := -y^3 - 1 \quad (106)$$

>  $N := x y^2$

$$N := x y^2 \quad (107)$$

>  $P := -1; Q := y^3 + 1; R := x; S := y^2$

$$\begin{aligned} P &:= -1 \\ Q &:= y^3 + 1 \\ R &:= x \\ S &:= y^2 \end{aligned} \quad (108)$$

>  $SolGral := \text{int}\left(\frac{P}{R}, x\right) + \text{int}\left(\frac{S}{Q}, y\right) = _C1$

$$SolGral := -\ln(x) + \frac{\ln(y^3 + 1)}{3} = _C1 \quad (109)$$

>  $SolFinal := \text{simplify}(\exp(\text{lhs}(SolGral))) = _C1$

$$SolFinal := \frac{(y^3 + 1)^{1/3}}{x} = _C1 \quad (110)$$

>  $SolFinalDos := \text{lhs}(SolFinal) \cdot x = \text{rhs}(SolFinal) \cdot x$

$$SolFinalDos := (y^3 + 1)^{1/3} = _C1 x \quad (111)$$

>  $SolFinalTres := \text{lhs}(SolFinalDos)^3 = _C1 \cdot x^3$

$$SolFinalTres := y^3 + 1 = _C1 x^3 \quad (112)$$

>  $SolFinalCuatro := \frac{\text{lhs}(SolFinalTres)}{x^3} = \frac{\text{rhs}(SolFinalTres)}{x^3}$

$$SolFinalCuatro := \frac{y^3 + 1}{x^3} = _C1 \quad (113)$$

>  $Ecua$ 

$$x y(x)^2 \left( \frac{d}{dx} y(x) \right) - y(x)^3 - 1 = 0 \quad (114)$$

>  $SolucionFinal := \frac{y(x)^3 + 1}{x^3} = _C1$

$$SolucionFinal := \frac{y(x)^3 + 1}{x^3} = _C1 \quad (115)$$

>  $\text{DerSolucionFinal} := \text{isolate}(\text{diff}(\text{SolucionFinal}, x), \text{diff}(y(x), x))$

$$\text{DerSolucionFinal} := \frac{d}{dx} y(x) = \frac{y(x)^3 + 1}{x y(x)^2} \quad (116)$$

>  $\text{DerEcua} := \text{isolate}(\text{Ecua}, \text{diff}(y(x), x))$

$$\text{DerEcua} := \frac{d}{dx} y(x) = \frac{y(x)^3 + 1}{x y(x)^2} \quad (117)$$

>  $\text{Comprobar} := \text{rhs}(\text{DerSolucionFinal}) - \text{rhs}(\text{DerEcua}) = 0$

$$\text{Comprobar} := 0 = 0 \quad (118)$$

> *restart*

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