

FACULTAD DE INGENIERIA
DIVISION DE CIENCIAS BASICAS
ECUACIONES DIFERENCIALES
GRUPO 10 SEMESTRE 2025-2
SERIE 3

> restart

1) por Laplace

> Sistema := diff(x[1](t), t) = 2·x[1](t) - 3·x[2](t), diff(x[2](t), t) = -2·x[1](t) + x[2](t) : Sistema[1]; Sistema[2]

$$\frac{d}{dt} x_1(t) = 2 x_1(t) - 3 x_2(t)$$

$$\frac{d}{dt} x_2(t) = -2 x_1(t) + x_2(t) \quad (1)$$

> CondIni := x[1](0) = 8, x[2](0) = 3

$$CondIni := x_1(0) = 8, x_2(0) = 3 \quad (2)$$

RESPUESTA

> with(inttrans) :

> SisTransLapl := subs(CondIni, laplace(Sistema[1], t, s)), subs(CondIni, laplace(Sistema[2], t, s)) : SisTransLapl[1]; SisTransLapl[2]

$$s \mathcal{L}(x_1(t), t, s) - 8 = 2 \mathcal{L}(x_1(t), t, s) - 3 \mathcal{L}(x_2(t), t, s)$$

$$s \mathcal{L}(x_2(t), t, s) - 3 = -2 \mathcal{L}(x_1(t), t, s) + \mathcal{L}(x_2(t), t, s) \quad (3)$$

> VarTrans := solve({SisTransLapl}, {laplace(x_1(t), t, s), laplace(x_2(t), t, s)})

$$VarTrans := \left\{ \mathcal{L}(x_1(t), t, s) = \frac{8s - 17}{s^2 - 3s - 4}, \mathcal{L}(x_2(t), t, s) = \frac{3s - 22}{s^2 - 3s - 4} \right\} \quad (4)$$

> Solucion := invlaplace(VarTrans[1], s, t), invlaplace(VarTrans[2], s, t) : Solucion[1]; Solucion[2]

$$x_1(t) = 5 e^{-t} + 3 e^{4t}$$

$$x_2(t) = 5 e^{-t} - 2 e^{4t} \quad (5)$$

> ComprobarUno := eval(subs(Solucion[1], Solucion[2], lhs(Sistema[1]) - rhs(Sistema[1]) = 0))

$$ComprobarUno := 0 = 0 \quad (6)$$

> ComprobarDos := eval(subs(Solucion[1], Solucion[2], lhs(Sistema[2]) - rhs(Sistema[2]) = 0))

$$ComprobarDos := 0 = 0 \quad (7)$$

> restart

2)

> Ecua := diff(y(t), t\$2) + 4·y(t) = Dirac(t - Pi)

$$Ecua := \frac{d^2}{dt^2} y(t) + 4 y(t) = \text{Dirac}(t - \pi) \quad (8)$$

$$\begin{aligned} &> \text{CondIni} := y(0) = 0, D(y)(0) = 0 \\ &\text{CondIni} := y(0) = 0, D(y)(0) = 0 \end{aligned} \quad (9)$$

RESPUESTA

$$\begin{aligned} &> \text{with(inttrans)} : \\ &> \text{EcuaTransLap} := \text{subs}(\text{CondIni}, \text{laplace}(\text{Ecua}, t, s)) \\ &\text{EcuaTransLap} := s^2 \mathcal{L}(y(t), t, s) + 4 \mathcal{L}(y(t), t, s) = e^{-s\pi} \end{aligned} \quad (10)$$

$$\begin{aligned} &> \text{SolTransLap} := \text{isolate}(\text{EcuaTransLap}, \text{laplace}(y(t), t, s)) \\ &\text{SolTransLap} := \mathcal{L}(y(t), t, s) = \frac{e^{-s\pi}}{s^2 + 4} \end{aligned} \quad (11)$$

$$\begin{aligned} &> \text{SolPart} := \text{invlaplace}(\text{SolTransLap}, s, t) \\ &\text{SolPart} := y(t) = \frac{\text{Heaviside}(t - \pi) \sin(2t)}{2} \end{aligned} \quad (12)$$

$$\begin{aligned} &> \text{Comprobar} := \text{simplify}(\text{eval}(\text{subs}(y(t) = \text{rhs}(\text{SolPart}), \text{lhs}(\text{Ecua}) - \text{rhs}(\text{Ecua}) = 0))) \\ &\text{Comprobar} := 0 = 0 \end{aligned} \quad (13)$$

> restart

3)

$$\begin{aligned} &> \text{Ecua} := \text{diff}(y(t), t) + 3 \cdot y(t) = 4 \cdot \text{Heaviside}(t - 1) \\ &\text{Ecua} := \frac{d}{dt} y(t) + 3 y(t) = 4 \text{Heaviside}(t - 1) \end{aligned} \quad (14)$$

$$\begin{aligned} &> \text{CondIni} := y(0) = 1 \\ &\text{CondIni} := y(0) = 1 \end{aligned} \quad (15)$$

RESPUESTA

$$\begin{aligned} &> \text{with(inttrans)} : \\ &> \text{EcuaTransLap} := \text{subs}(\text{CondIni}, \text{laplace}(\text{Ecua}, t, s)) \\ &\text{EcuaTransLap} := s \mathcal{L}(y(t), t, s) - 1 + 3 \mathcal{L}(y(t), t, s) = \frac{4 e^{-s}}{s} \end{aligned} \quad (16)$$

$$\begin{aligned} &> \text{SolTransLap} := \text{simplify}(\text{isolate}(\text{EcuaTransLap}, \text{laplace}(y(t), t, s))) \\ &\text{SolTransLap} := \mathcal{L}(y(t), t, s) = \frac{4 e^{-s} + s}{s(s + 3)} \end{aligned} \quad (17)$$

$$\begin{aligned} &> \text{SolPart} := \text{invlaplace}(\text{SolTransLap}, s, t) \\ &\text{SolPart} := y(t) = \frac{4 \text{Heaviside}(t - 1) (1 - e^{-3t+3})}{3} + e^{-3t} \end{aligned} \quad (18)$$

$$\begin{aligned} &> \text{Comprobar} := \text{simplify}(\text{eval}(\text{subs}(y(t) = \text{rhs}(\text{SolPart}), \text{lhs}(\text{Ecua}) - \text{rhs}(\text{Ecua}) = 0))) \\ &\text{Comprobar} := 0 = 0 \end{aligned} \quad (19)$$

> restart

4)

$$\begin{aligned} &> \text{Ecua} := \text{diff}(y(t), t^2) + 6 \cdot \text{diff}(y(t), t) + 5 \cdot y(t) = \exp(t) \cdot \text{Dirac}(t - 1) \\ &\text{Ecua} := \frac{d^2}{dt^2} y(t) + 6 \frac{d}{dt} y(t) + 5 y(t) = e^t \text{Dirac}(t - 1) \end{aligned} \quad (20)$$

$$> \text{CondIni} := y(0) = 0, D(y)(0) = 4$$

$$CondIni := y(0) = 0, D(y)(0) = 4 \quad (21)$$

RESPUESTA

> with(inttrans) :

> EcuaTransLap := subs(CondIni, laplace(Ecua, t, s))

$$EcuaTransLap := s^2 \mathcal{L}(y(t), t, s) - 4 + 6s \mathcal{L}(y(t), t, s) + 5 \mathcal{L}(y(t), t, s) = e^{1-s} \quad (22)$$

> SolTransLap := isolate(EcuaTransLap, laplace(y(t), t, s))

$$SolTransLap := \mathcal{L}(y(t), t, s) = \frac{e^{1-s} + 4}{s^2 + 6s + 5} \quad (23)$$

> SolPart := invlaplace(SolTransLap, s, t)

$$SolPart := y(t) = \frac{\text{Heaviside}(t-1) \sinh(2t-2) e^{4-3t}}{2} + 2 e^{-3t} \sinh(2t) \quad (24)$$

> Comprobar := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))

$$Comprobar := 0 = 0 \quad (25)$$

> restart

5)

> Ecua := f(t) = 4 · t² - exp(-t) - int(f(tau) · exp(t - tau), tau = 0 .. t)

$$Ecua := f(t) = 4 t^2 - e^{-t} - \left(\int_0^t f(\tau) e^{t-\tau} d\tau \right) \quad (26)$$

RESPUESTA

> with(inttrans) :

> EcuaTransLap := laplace(Ecua, t, s)

$$EcuaTransLap := \mathcal{L}(f(t), t, s) = \frac{8}{s^3} - \frac{1}{1+s} - \frac{\mathcal{L}(f(t), t, s)}{s-1} \quad (27)$$

> SolTransLap := simplify(isolate(EcuaTransLap, laplace(f(t), t, s)))

$$SolTransLap := \mathcal{L}(f(t), t, s) = \frac{-s^4 + s^3 + 8s^2 - 8}{s^4(1+s)} \quad (28)$$

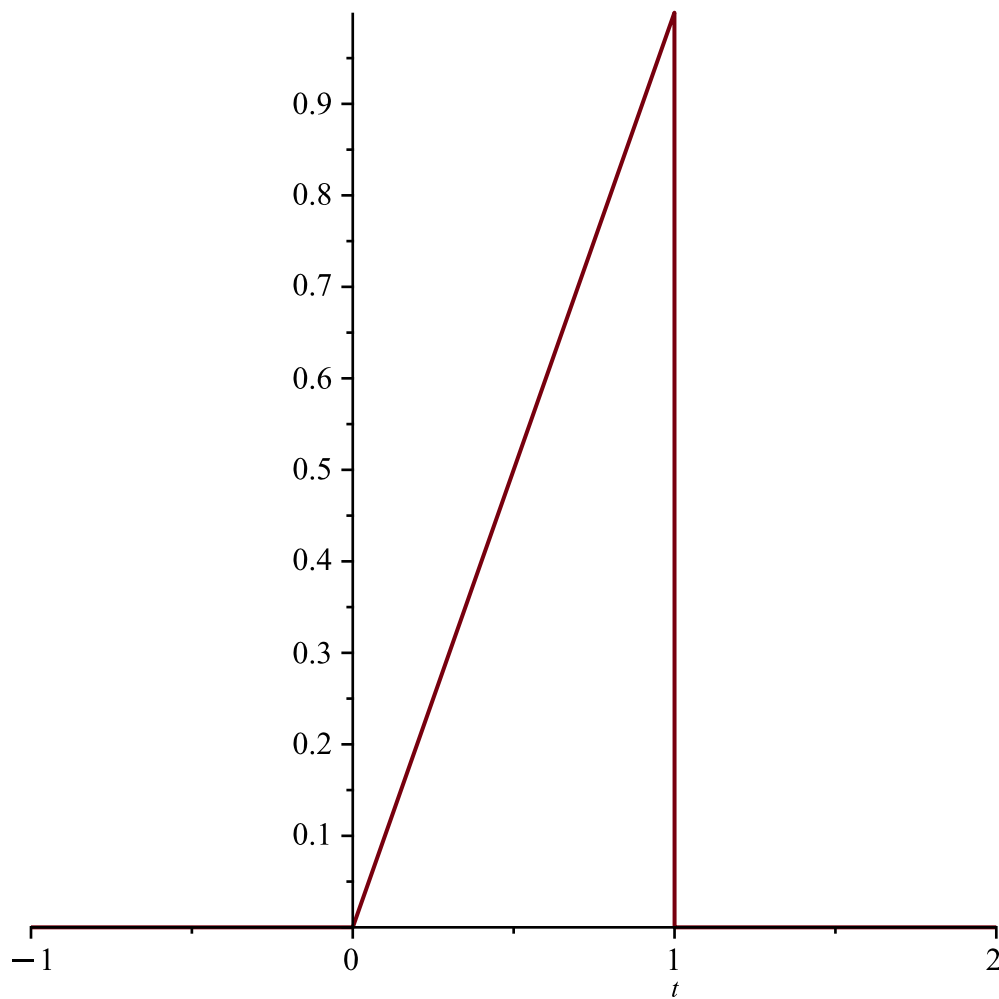
> SolPart := invlaplace(SolTransLap, s, t)

$$SolPart := f(t) = -2 e^{-t} - \frac{4 t^3}{3} + 4 t^2 + 1 \quad (29)$$

> restart

6)

> f(t) := t · Heaviside(t) - (t - 1) · Heaviside(t - 1) - Heaviside(t - 1) : plot(f(t), t = -1 .. 2)



> $Ecua := \text{diff}(y(t), t) + 2 \cdot y(t) = f(t)$

$$Ecua := \frac{d}{dt} y(t) + 2 y(t) = t \text{Heaviside}(t) - (t - 1) \text{Heaviside}(t - 1) - \text{Heaviside}(t - 1) \quad (30)$$

> $CondIni := y(0) = 0$

$$CondIni := y(0) = 0 \quad (31)$$

RESPUESTA

> $\text{with}(\text{inttrans}) :$

> $EcuaTransLap := \text{subs}(CondIni, \text{laplace}(Ecua, t, s))$

$$EcuaTransLap := s \mathcal{L}(y(t), t, s) + 2 \mathcal{L}(y(t), t, s) = \frac{-(s + 1) e^{-s} + 1}{s^2} \quad (32)$$

> $SolTransLap := \text{isolate}(EcuaTransLap, \text{laplace}(y(t), t, s))$

$$SolTransLap := \mathcal{L}(y(t), t, s) = \frac{-(s + 1) e^{-s} + 1}{s^2 (s + 2)} \quad (33)$$

> $SolPart := \text{invlaplace}(SolTransLap, s, t)$

$$SolPart := y(t) = -\frac{1}{4} + \frac{t \text{Heaviside}(-t + 1)}{2} + \frac{e^{-2t}}{4} + \frac{\text{Heaviside}(t - 1) (e^{-t+1} \sinh(t - 1) + e^{-2t+2})}{2} \quad (34)$$

> restart

7)

> Ecua := diff(y(t), t\$2) + 4·diff(y(t), t) = 3

$$Ecua := \frac{d^2}{dt^2} y(t) + 4 \frac{d}{dt} y(t) = 3 \quad (35)$$

> CondIni := y(0) = -1, D(y)(0) = $\frac{19}{4}$

$$CondIni := y(0) = -1, D(y)(0) = \frac{19}{4} \quad (36)$$

RESPUESTA

> with(inttrans) :

> EcuaTransLap := subs(CondIni, laplace(Ecua, t, s))

$$EcuaTransLap := s^2 \mathcal{L}(y(t), t, s) - \frac{3}{4} + s + 4 s \mathcal{L}(y(t), t, s) = \frac{3}{s} \quad (37)$$

> SolTransLap := simplify(isolate(EcuaTransLap, laplace(y(t), t, s)))

$$SolTransLap := \mathcal{L}(y(t), t, s) = \frac{-4 s^2 + 3 s + 12}{4 s^2 (s + 4)} \quad (38)$$

> SolPart := invlaplace(SolTransLap, s, t)

$$SolPart := y(t) = \frac{3 t}{4} - e^{-4 t} \quad (39)$$

> Comprobar := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))

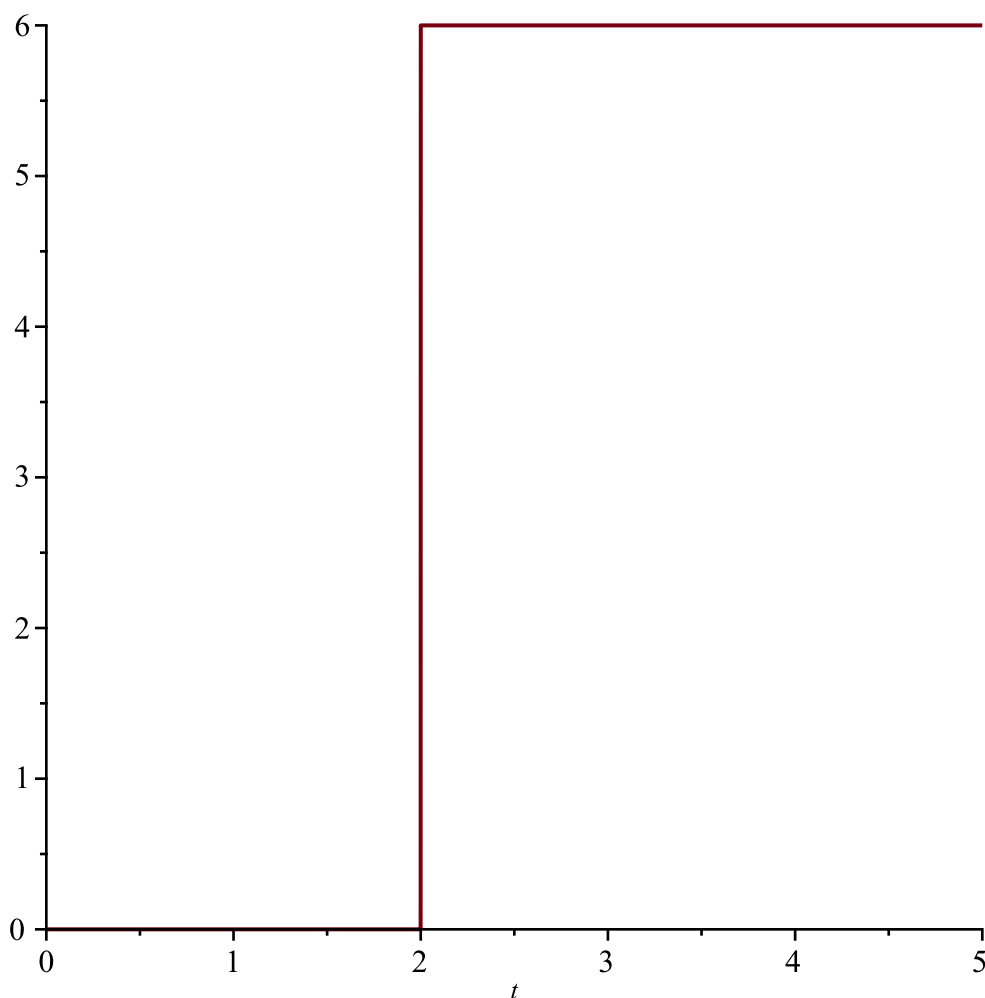
$$Comprobar := 0 = 0 \quad (40)$$

> restart

8)

> f(t) := 6·Heaviside(t - 2); plot(f(t), t = 0 .. 5)

$$f := t \mapsto 6 \cdot \text{Heaviside}(t - 2)$$



$$\begin{aligned} &> \text{Ecua} := \text{diff}(y(t), t) - 7 \cdot y(t) = f(t) \\ &\quad \text{Ecua} := \frac{d}{dt} y(t) - 7 y(t) = 6 \text{Heaviside}(t - 2) \end{aligned} \quad (41)$$

$$\begin{aligned} &> \text{CondIni} := y(0) = 1 \\ &\quad \text{CondIni} := y(0) = 1 \end{aligned} \quad (42)$$

RESPUESTA

$$\begin{aligned} &> \text{with(inttrans)} : \\ &> \text{EcuaTransLap} := \text{subs}(\text{CondIni}, \text{laplace}(\text{Ecua}, t, s)) \\ &\quad \text{EcuaTransLap} := s \mathcal{L}(y(t), t, s) - 1 - 7 \mathcal{L}(y(t), t, s) = \frac{6 e^{-2s}}{s} \end{aligned} \quad (43)$$

$$\begin{aligned} &> \text{SolTransLap} := \text{simplify}(\text{isolate}(\text{EcuaTransLap}, \text{laplace}(y(t), t, s))) \\ &\quad \text{SolTransLap} := \mathcal{L}(y(t), t, s) = \frac{6 e^{-2s} + s}{s(s - 7)} \end{aligned} \quad (44)$$

$$\begin{aligned} &> \text{SolPart} := \text{invlaplace}(\text{SolTransLap}, s, t) \\ &\quad \text{SolPart} := y(t) = -\frac{6 \text{Heaviside}(t - 2)}{7} + e^{7t} + \frac{6(1 - \text{Heaviside}(2 - t)) e^{7t - 14}}{7} \end{aligned} \quad (45)$$

$$\begin{aligned} &> \text{Comprobar} := \text{simplify}(\text{eval}(\text{subs}(y(t) = \text{rhs}(\text{SolPart}), \text{lhs}(\text{Ecua}) - \text{rhs}(\text{Ecua}) = 0))) \\ &\quad \text{Comprobar} := 0 = 0 \end{aligned} \quad (46)$$

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|> restart
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|9)
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|> F := 
$$\frac{s}{(2 \cdot s^2 - 4 \cdot s + 20)}$$

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$$F := \frac{s}{2 s^2 - 4 s + 20} \quad (47)$$

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|> G := 
$$\frac{4}{\exp(2 \cdot s) \cdot (s - 3)}$$

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$$G := \frac{4}{e^{2 s} (s - 3)} \quad (48)$$

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|> with(inttrans) :
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|> f := invlaplace(F, s, t)
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$$f := \frac{e^t (3 \cos(3 t) + \sin(3 t))}{6} \quad (49)$$

```
|> g := invlaplace(G, s, t)
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$$g := 4 \operatorname{Heaviside}(t - 2) e^{3 t - 6} \quad (50)$$

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|> restart
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|FIN SERIE 3
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