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>
SERIE 2026-1-3 SOLUCIÓN
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> restart
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1) Resuelva
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> Sistema := diff(x(t), t) = x(t) + 2·y(t) + exp(t), diff(y(t), t) = 2·x(t) + y(t) + exp(3 t) :
Sistema[1]; Sistema[2]
```

$$\begin{aligned}\frac{d}{dt} x(t) &= x(t) + 2 y(t) + e^t \\ \frac{d}{dt} y(t) &= 2 x(t) + y(t) + e^{3t}\end{aligned}\quad (1)$$

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> CondIni := x(0) = 0, y(0) = 0
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$$CondIni := x(0) = 0, y(0) = 0 \quad (2)$$

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Respuesta 1)
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> AA := array([ [1, 2], [2, 1] ])
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$$AA := \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad (3)$$

```
> BB := array([ exp(t), exp(3 t) ])
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$$BB := \begin{bmatrix} e^t & e^{3t} \end{bmatrix} \quad (4)$$

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> Xcero := array([0, 0])
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$$Xcero := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (5)$$

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> with(linalg) :
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> MatExp := exponential(AA, t)
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$$MatExp := \begin{bmatrix} \frac{e^{-t}}{2} + \frac{e^{3t}}{2} & \frac{e^{3t}}{2} - \frac{e^{-t}}{2} \\ \frac{e^{3t}}{2} - \frac{e^{-t}}{2} & \frac{e^{-t}}{2} + \frac{e^{3t}}{2} \end{bmatrix} \quad (6)$$

```
> SolHom := evalm(MatExp &* Xcero)
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$$SolHom := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (7)$$

```
> MatExpTau := map(rcurry(eval, t='t - tau'), MatExp)
```

$$MatExpTau := \begin{bmatrix} \frac{e^{-t+\tau}}{2} + \frac{e^{3t-3\tau}}{2} & \frac{e^{3t-3\tau}}{2} - \frac{e^{-t+\tau}}{2} \\ \frac{e^{3t-3\tau}}{2} - \frac{e^{-t+\tau}}{2} & \frac{e^{-t+\tau}}{2} + \frac{e^{3t-3\tau}}{2} \end{bmatrix} \quad (8)$$

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> BBtau := map(rcurry(eval, t='tau'), BB)
```

$$BBtau := \begin{bmatrix} e^\tau & e^{3\tau} \end{bmatrix} \quad (9)$$

```
> ProdTau := evalm(MatExpTau &* BBtau)
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$$ProdTau := \left[\left(\frac{e^{-t+\tau}}{2} + \frac{e^{3t-3\tau}}{2} \right) e^{\tau} + \left(\frac{e^{3t-3\tau}}{2} - \frac{e^{-t+\tau}}{2} \right) e^{3\tau}, \left(\frac{e^{3t-3\tau}}{2} - \frac{e^{-t+\tau}}{2} \right) e^{\tau} + \left(\frac{e^{-t+\tau}}{2} + \frac{e^{3t-3\tau}}{2} \right) e^{3\tau} \right] \quad (10)$$

> SolNoHom := map(int, ProdTau, tau = 0 .. t)

$$SolNoHom := \left[-\frac{e^{-t}}{8} + \frac{e^{3t}}{8} + \frac{t e^{3t}}{2}, \frac{e^{-t}}{8} + \frac{3 e^{3t}}{8} + \frac{t e^{3t}}{2} - \frac{e^t}{2} \right] \quad (11)$$

> SolPart := x(t) = SolHom[1] + SolNoHom[1], y(t) = SolHom[2] + SolNoHom[2] :
SolPart[1]; SolPart[2]

$$x(t) = -\frac{e^{-t}}{8} + \frac{e^{3t}}{8} + \frac{t e^{3t}}{2}$$

$$y(t) = \frac{e^{-t}}{8} + \frac{3 e^{3t}}{8} + \frac{t e^{3t}}{2} - \frac{e^t}{2} \quad (12)$$

> ComprobarUno := subs(t = 0, SolPart[1])

$$ComprobarUno := x(0) = 0 \quad (13)$$

> ComprobarDos := subs(t = 0, SolPart[2])

$$ComprobarDos := y(0) = 0 \quad (14)$$

> ComprobarTres := simplify(eval(subs(x(t) = rhs(SolPart[1]), y(t) = rhs(SolPart[2]),
lhs(Sistema[1]) - rhs(Sistema[1]) = 0)))

$$ComprobarTres := 0 = 0 \quad (15)$$

> ComprobarTres := simplify(eval(subs(x(t) = rhs(SolPart[1]), y(t) = rhs(SolPart[2]),
lhs(Sistema[2]) - rhs(Sistema[2]) = 0)))

$$ComprobarTres := 0 = 0 \quad (16)$$

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FIN RESPUESTA 1)

> restart

2) Resuelva

> Ecua := diff(y(t), t\$2) + 6·diff(y(t), t) + 5·y(t) = exp(t)·Dirac(t - 1)

$$Ecua := \frac{d^2}{dt^2} y(t) + 6 \frac{d}{dt} y(t) + 5 y(t) = e^t \text{Dirac}(t - 1) \quad (17)$$

> CondIni := y(0) = 0, D(y)(0) = 4

$$CondIni := y(0) = 0, D(y)(0) = 4 \quad (18)$$

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Respuesta 2)

> with(inttrans) :

> EcuaTL := subs(CondIni, laplace(Ecua, t, s))

$$EcuaTL := s^2 \mathcal{L}(y(t), t, s) - 4 + 6 s \mathcal{L}(y(t), t, s) + 5 \mathcal{L}(y(t), t, s) = e^{1-s} \quad (19)$$

> SolTL := isolate(EcuaTL, laplace(y(t), t, s))

$$SolTL := \mathcal{L}(y(t), t, s) = \frac{e^{1-s} + 4}{s^2 + 6s + 5} \quad (20)$$

$$\begin{aligned} &> \text{SolPart} := \text{simplify}(\text{invlaplace}(\text{SolTL}, s, t)) \\ &\quad \text{SolPart} := y(t) = \frac{\text{Heaviside}(t-1) \sinh(2t-2) e^{4-3t}}{2} + 2 e^{-3t} \sinh(2t) \end{aligned} \quad (21)$$

$$\begin{aligned} &> \text{ComprobarUno} := \text{simplify}(\text{eval}(\text{subs}(t=0, \text{SolPart}))) \\ &\quad \text{ComprobarUno} := y(0) = 0 \end{aligned} \quad (22)$$

$$\begin{aligned} &> \text{ComprobarDos} := \text{D}(y)(0) = \text{simplify}(\text{eval}(\text{subs}(t=0, \text{rhs}(\text{diff}(\text{SolPart}, t)))))) \\ &\quad \text{ComprobarDos} := \text{D}(y)(0) = 4 \end{aligned} \quad (23)$$

$$\begin{aligned} &> \text{Ecua} \\ &\quad \frac{d^2}{dt^2} y(t) + 6 \frac{d}{dt} y(t) + 5 y(t) = e^t \text{Dirac}(t-1) \end{aligned} \quad (24)$$

$$\begin{aligned} &> \text{ComprobarTres} := \text{simplify}(\text{eval}(\text{subs}(y(t) = \text{rhs}(\text{SolPart}), \text{lhs}(\text{Ecua}) - \text{rhs}(\text{Ecua}) = 0))) \\ &\quad \text{ComprobarTres} := 0 = 0 \end{aligned} \quad (25)$$

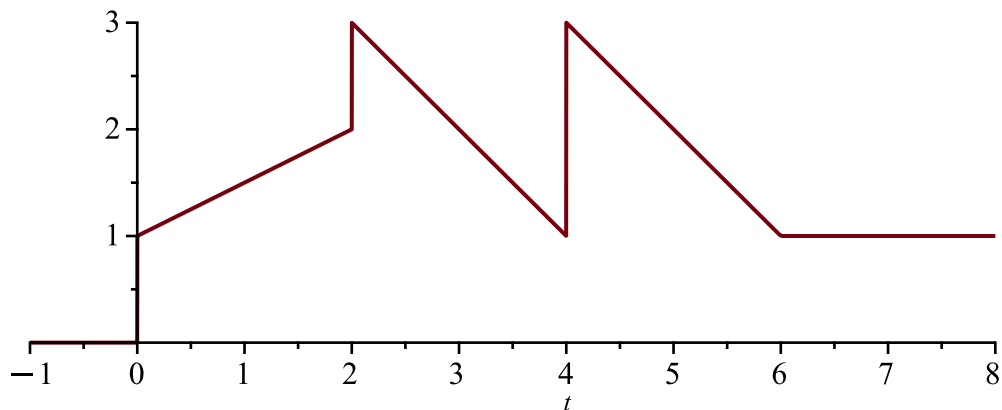
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FIN RESPUESTA 2)

> restart

3) Obtenga la TL

$$\begin{aligned} &> f := \text{Heaviside}(t) + \frac{(t \cdot \text{Heaviside}(t))}{2} - \frac{((t-2) \cdot \text{Heaviside}(t-2))}{2} + \text{Heaviside}(t-2) \\ &\quad - (t-2) \cdot \text{Heaviside}(t-2) + (t-4) \cdot \text{Heaviside}(t-4) + 2 \cdot \text{Heaviside}(t-4) - (t-4) \\ &\quad \cdot \text{Heaviside}(t-4) + (t-6) \cdot \text{Heaviside}(t-6); \text{plot}(f, t=-1..8, \text{scaling} \\ &\quad = \text{CONSTRAINED}) \\ &\quad f := \text{Heaviside}(t) + \frac{t \text{Heaviside}(t)}{2} - \frac{3(t-2) \text{Heaviside}(t-2)}{2} + \text{Heaviside}(t-2) \\ &\quad + 2 \text{Heaviside}(t-4) + (t-6) \text{Heaviside}(t-6) \end{aligned}$$



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> with(inttrans) :
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> F := laplace(f, t, s)
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$$F := \frac{e^{-2s} (2s - 3) + 2e^{-6s} + 1}{2s^2} + \frac{1 + 2e^{-4s}}{s} \quad (26)$$

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FIN RESPUESTA 3)
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> restart
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4) Resuelva
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> Ecua := diff(y(t), t$2) - 2*diff(y(t), t) + y(t) = Heaviside(t - 2)
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$$Ecua := \frac{d^2}{dt^2} y(t) - 2 \frac{d}{dt} y(t) + y(t) = \text{Heaviside}(t - 2) \quad (27)$$

```
> CondIni := y(0) = 0, D(y)(0) = 0
```

$$CondIni := y(0) = 0, D(y)(0) = 0 \quad (28)$$

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>
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Respuesta 4)
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```
> with(inttrans) :
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> EcuaTL := subs(CondIni, laplace(Ecua, t, s))
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(29)

$$EcuaTL := s^2 \mathcal{L}(y(t), t, s) - 2s \mathcal{L}(y(t), t, s) + \mathcal{L}(y(t), t, s) = \frac{e^{-2s}}{s} \quad (29)$$

> SolTL := isolate(EcuaTL, laplace(y(t), t, s))

$$SolTL := \mathcal{L}(y(t), t, s) = \frac{e^{-2s}}{s(s^2 - 2s + 1)} \quad (30)$$

> SolPart := invlaplace(SolTL, s, t)

$$SolPart := y(t) = \text{Heaviside}(t - 2) (1 + e^{t-2} (t - 3)) \quad (31)$$

> Comprobar := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))

$$Comprobar := 0 = 0 \quad (32)$$

>

FIN RESPUESTA 4)

> restart

5) Resolver

> Ecua := diff(y(t), t) - y(t) = (t - 1) * Heaviside(t - 3)

$$Ecua := \frac{d}{dt} y(t) - y(t) = (t - 1) \text{Heaviside}(t - 3) \quad (33)$$

> CondIni := y(0) = 0

$$CondIni := y(0) = 0 \quad (34)$$

> with(inttrans) :

> EcuaTL := subs(CondIni, laplace(Ecua, t, s))

$$EcuaTL := s \mathcal{L}(y(t), t, s) - \mathcal{L}(y(t), t, s) = \frac{e^{-3s} (2s + 1)}{s^2} \quad (35)$$

> SolTL := isolate(EcuaTL, laplace(y(t), t, s))

$$SolTL := \mathcal{L}(y(t), t, s) = \frac{e^{-3s} (2s + 1)}{s^2 (s - 1)} \quad (36)$$

> SolPart := invlaplace(SolTL, s, t)

$$SolPart := y(t) = -\text{Heaviside}(t - 3) (-3 e^{t-3} + t) \quad (37)$$

> ComprobarUno := simplify(subs(t = 0, SolPart))

$$ComprobarUno := y(0) = 0 \quad (38)$$

> ComprobarDos := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))

$$ComprobarDos := 0 = 0 \quad (39)$$

>

FIN RESPUESTA 5)

> restart

6) Obtener

$$F := \frac{s}{(2 \cdot s^2 - 4 \cdot s + 20)}$$

$$F := \frac{s}{2s^2 - 4s + 20} \quad (40)$$

$$\begin{aligned}
 &> \text{with(inttrans)} : \\
 &> f := \text{invlaplace}(F, s, t) \\
 &f := \frac{e^t (3 \cos(3 t) + \sin(3 t))}{6} \quad (41)
 \end{aligned}$$

$$\begin{aligned}
 &> G := \frac{4}{\exp(2 \cdot s) \cdot (s - 3)} \\
 &G := \frac{4}{e^{2s} (s - 3)} \quad (42)
 \end{aligned}$$

$$\begin{aligned}
 &> g := \text{invlaplace}(G, s, t) \\
 &g := 4 \text{Heaviside}(t - 2) e^{3t-6} \quad (43)
 \end{aligned}$$

>
FIN RESPUESTA 6)

$$\begin{aligned}
 &> \text{restart} \\
 &7) \text{ Resolver} \\
 &> Ecua := h(t) = t \cdot \exp(t) + \text{int}(v \cdot h(t - v), v = 0 .. t) \\
 &Ecua := h(t) = t e^t + \int_0^t v h(t - v) dv \quad (44)
 \end{aligned}$$

$$\begin{aligned}
 &> \text{with(inttrans)} : \\
 &> EcuaTL := \text{laplace}(Ecua, t, s) \\
 &EcuaTL := \mathcal{L}(h(t), t, s) = \frac{1}{(s - 1)^2} + \frac{\mathcal{L}(h(t), t, s)}{s^2} \quad (45)
 \end{aligned}$$

$$\begin{aligned}
 &> SolTL := \text{simplify}(\text{isolate}(EcuaTL, \text{laplace}(h(t), t, s))) \\
 &SolTL := \mathcal{L}(h(t), t, s) = \frac{s^2}{(s - 1)^3 (s + 1)} \quad (46)
 \end{aligned}$$

$$\begin{aligned}
 &> SolPart := \text{invlaplace}(SolTL, s, t) \\
 &SolPart := h(t) = -\frac{e^{-t}}{8} + \frac{e^t (2 t^2 + 6 t + 1)}{8} \quad (47)
 \end{aligned}$$

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FIN RESPUESTA 7)

$$\begin{aligned}
 &> \text{restart} \\
 &8) \text{ Obtenga} \\
 &> \text{with(inttrans)} : \\
 &> f := t \cdot \text{Dirac}(t - 1) \cdot \text{Heaviside}(t - 3) \\
 &f := t \text{Dirac}(t - 1) \text{Heaviside}(t - 3) \quad (48)
 \end{aligned}$$

$$\begin{aligned}
 &> F := \text{laplace}(f, t, s) \\
 &F := 0 \quad (49)
 \end{aligned}$$

$$\begin{aligned}
 &> H := \frac{s \cdot \exp(-3 \cdot s)}{s^2 + 4 \cdot s + 5} \\
 &H := \frac{s e^{-3s}}{s^2 + 4s + 5} \quad (50)
 \end{aligned}$$

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> h := invlaplace(H, s, t)
h := Heaviside(t - 3) e-2 t + 6 (cos(t - 3) - 2 sin(t - 3))

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(51)

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FIN RESPUESTA 8)

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> restart
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9)
> f := cos(2·t)·Heaviside(t - Pi)
f := cos(2 t) Heaviside(t - π)

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(52)

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=
> with(inttrans) :
> F := laplace(f, t, s)
F :=  $\frac{e^{-s \pi} s}{s^2 + 4}$ 

```

(53)