

FACULTAD DE INGENIERIA
DIVISION DE CIENCIAS BASICAS
ECUACIONES DIFERENCIALES
GRUPO 11
SEMESTRE 2027-1
SERIE 2
SOLUCIÓN

> restart

> restart

1) Resuelva el problema de valor inicial

>

$$\frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) = e^{-x}$$

$$y(0) = 1$$

$$D(y)(0) = -2$$

(1)

RESPUESTA

> $Ecua := \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) = e^{-x}$

$$Ecua := \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) = e^{-x}$$

(2)

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) = 0$$

(3)

> $Q := rhs(Ecua)$

$$Q := e^{-x}$$

(4)

> $EcuaCarac := m^2 + m = 0$

$$EcuaCarac := m^2 + m = 0$$

(5)

> $Raiz := solve(EcuaCarac)$

$$Raiz := -1, 0$$

(6)

> $yy[1] := \exp(Raiz[1] \cdot x)$

$$yy_1 := e^{-x}$$

(7)

> $yy[2] := \exp(Raiz[2] \cdot x)$

$$yy_2 := 1$$

(8)

> with(linalg) :

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^{-x} & 1 \\ -e^{-x} & 0 \end{bmatrix}$$

(9)

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & e^{-x} \end{bmatrix} \quad (10)$$

> Para := linsolve(WW, BB)

$$Para := \begin{bmatrix} -1 & e^{-x} \end{bmatrix} \quad (11)$$

> Aprima := Para[1]

$$Aprima := -1 \quad (12)$$

> Bprima := Para[2]

$$Bprima := e^{-x} \quad (13)$$

> SolGral := y(x) = simplify((int(Aprima, x) + _C1)·yy[1] + (int(Bprima, x) + _C2)·yy[2])

$$SolGral := y(x) = (-x + _C1 - 1) e^{-x} + _C2 \quad (14)$$

> SolFinal := subs(_C1 = _C10 + 1, _C2 = _C20, SolGral)

$$SolFinal := y(x) = (-x + _C10) e^{-x} + _C20 \quad (15)$$

> Comprobar := simplify(eval(subs(y(x) = rhs(SolFinal), Ecua)))

$$Comprobar := e^{-x} = e^{-x} \quad (16)$$

> CondIni := y(0) = 1, D(y)(0) = -2

$$CondIni := y(0) = 1, D(y)(0) = -2 \quad (17)$$

> EcuaUno := simplify(subs(x = 0, rhs(SolFinal)) = 1)

$$EcuaUno := _C10 + _C20 = 1 \quad (18)$$

> EcuaDos := simplify(subs(x = 0, rhs(diff(SolFinal, x))) = -2)

$$EcuaDos := -1 - _C10 = -2 \quad (19)$$

> Para := solve({EcuaUno, EcuaDos})

$$Para := \{ _C10 = 1, _C20 = 0 \} \quad (20)$$

> SolPart := subs(_C10 = rhs(Para[1]), _C20 = rhs(Para[2]), SolFinal)

$$SolPart := y(x) = (1 - x) e^{-x} \quad (21)$$

> restart

2) Resuelva la Ecuación Diferencial Ordinaria Lineal

>

$$\frac{d^2}{dx^2} y(x) + y(x) = \sec(x) \csc(x) \quad (22)$$

RESPUESTA

> Ecua := $\frac{d^2}{dx^2} y(x) + y(x) = \sec(x) \csc(x)$

$$Ecua := \frac{d^2}{dx^2} y(x) + y(x) = \sec(x) \csc(x) \quad (23)$$

> EcuaHom := lhs(Ecua) = 0

$$EcuaHom := \frac{d^2}{dx^2} y(x) + y(x) = 0 \quad (24)$$

> Q := rhs(Ecua)

$$Q := \sec(x) \csc(x) \quad (25)$$

$$\begin{aligned} &> \text{EcuaCarac} := m^2 + 1 = 0 \\ &\text{EcuaCarac} := m^2 + 1 = 0 \end{aligned} \quad (26)$$

$$\begin{aligned} &> \text{Raiz} := \text{solve}(\text{EcuaCarac}) \\ &\text{Raiz} := I, -I \end{aligned} \quad (27)$$

$$\begin{aligned} &> \text{yy}[1] := \cos(\text{Im}(\text{Raiz}[1]) \cdot x) \\ &\text{yy}_1 := \cos(x) \end{aligned} \quad (28)$$

$$\begin{aligned} &> \text{yy}[2] := \sin(\text{Im}(\text{Raiz}[1]) \cdot x) \\ &\text{yy}_2 := \sin(x) \end{aligned} \quad (29)$$

$$\begin{aligned} &> \text{with}(\text{linalg}) : \\ &> \text{WW} := \text{wronskian}([\text{yy}[1], \text{yy}[2]], x) \\ &\text{WW} := \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix} \end{aligned} \quad (30)$$

$$\begin{aligned} &> \text{BB} := \text{array}([0, Q,]) \\ &\text{BB} := \begin{bmatrix} 0 & \sec(x) & \csc(x) \end{bmatrix} \end{aligned} \quad (31)$$

$$\begin{aligned} &> \text{Para} := \text{simplify}(\text{linsolve}(\text{WW}, \text{BB})) \\ &\text{Para} := \begin{bmatrix} -\sec(x) & \csc(x) \end{bmatrix} \end{aligned} \quad (32)$$

$$\begin{aligned} &> \text{Aprima} := \text{Para}[1] \\ &\text{Aprima} := -\sec(x) \end{aligned} \quad (33)$$

$$\begin{aligned} &> \text{Bprima} := \text{Para}[2] \\ &\text{Bprima} := \csc(x) \end{aligned} \quad (34)$$

$$\begin{aligned} &> \text{SolGral} := y(x) = \text{expand}((\text{int}(\text{Aprima}, x) + _C1) \cdot \text{yy}[1] + (\text{int}(\text{Bprima}, x) + _C2) \cdot \text{yy}[2]) \\ &\text{SolGral} := y(x) = -\cos(x) \ln(\sec(x) + \tan(x)) + \cos(x) _C1 - \sin(x) \ln(\csc(x) + \cot(x)) \\ &\quad + \sin(x) _C2 \end{aligned} \quad (35)$$

$$\begin{aligned} &> \text{Comprobar} := \text{simplify}(\text{eval}(\text{subs}(y(x) = \text{rhs}(\text{SolGral}), \text{Ecua}))) \\ &\text{Comprobar} := \sec(x) \csc(x) = \sec(x) \csc(x) \end{aligned} \quad (36)$$

restart

3) Mediante el método de parámetros variables obtenga la solución general de la ecuación

$$\begin{aligned} &> \\ &\frac{d^2}{dt^2} y(t) - \frac{d}{dt} y(t) - 2 y(t) = -3 e^{-t} + 2 \end{aligned} \quad (37)$$

RESPUESTA

$$\begin{aligned} &> \text{Ecua} := \text{diff}(y(t), t\$2) - \text{diff}(y(t), t) - 2 \cdot y(t) = -3 \cdot \exp(-t) + 2 \\ &\text{Ecua} := \frac{d^2}{dt^2} y(t) - \frac{d}{dt} y(t) - 2 y(t) = -3 e^{-t} + 2 \end{aligned} \quad (38)$$

$$\begin{aligned} &> \text{EcuaHom} := \text{lhs}(\text{Ecua}) = 0 \\ &\text{EcuaHom} := \frac{d^2}{dt^2} y(t) - \frac{d}{dt} y(t) - 2 y(t) = 0 \end{aligned} \quad (39)$$

$$> Q := \text{rhs}(\text{Ecua})$$

$$Q := -3 e^{-t} + 2 \quad (40)$$

$$> \text{EcuaCarac} := m^2 - m - 2 = 0$$

$$\text{EcuaCarac} := m^2 - m - 2 = 0 \quad (41)$$

$$> \text{Raiz} := \text{solve}(\text{EcuaCarac})$$

$$\text{Raiz} := 2, -1 \quad (42)$$

$$> \text{yy}[1] := \exp(\text{Raiz}[1] \cdot t)$$

$$\text{yy}_1 := e^{2t} \quad (43)$$

$$> \text{yy}[2] := \exp(\text{Raiz}[2] \cdot t)$$

$$\text{yy}_2 := e^{-t} \quad (44)$$

$$> \text{with}(\text{linalg}) :$$

$$> \text{WW} := \text{wronskian}([\text{yy}[1], \text{yy}[2]], t)$$

$$\text{WW} := \begin{bmatrix} e^{2t} & e^{-t} \\ 2e^{2t} & -e^{-t} \end{bmatrix} \quad (45)$$

$$> \text{BB} := \text{array}([0, Q])$$

$$\text{BB} := \begin{bmatrix} 0 & -3e^{-t} + 2 \end{bmatrix} \quad (46)$$

$$> \text{Para} := \text{expand}(\text{linsolve}(\text{WW}, \text{BB}))$$

$$\text{Para} := \begin{bmatrix} -\frac{3e^{-t} - 2}{3e^{2t}} & \frac{3e^{-t} - 2}{3e^{-t}} \end{bmatrix} \quad (47)$$

$$> \text{Aprima} := \text{expand}(\text{simplify}(\text{Para}[1]))$$

$$\text{Aprima} := -\frac{1}{(e^t)^3} + \frac{2}{3(e^t)^2} \quad (48)$$

$$> \text{Bprima} := \text{expand}(\text{simplify}(\text{Para}[2]))$$

$$\text{Bprima} := 1 - \frac{2e^t}{3} \quad (49)$$

$$> \text{SolGral} := y(t) = \text{expand}(\text{simplify}((\text{int}(\text{Aprima}, t) + _C1) \cdot \text{yy}[1] + (\text{int}(\text{Bprima}, t) + _C2) \cdot \text{yy}[2]))$$

$$\text{SolGral} := y(t) = \frac{1}{3e^t} - 1 + (e^t)^2 _C1 + \frac{t}{e^t} + \frac{_C2}{e^t} \quad (50)$$

$$> \text{Comprobar} := \text{simplify}(\text{eval}(\text{subs}(y(t) = \text{rhs}(\text{SolGral}), \text{Ecua})))$$

$$\text{Comprobar} := -3e^{-t} + 2 = -3e^{-t} + 2 \quad (51)$$

$$> \text{restart}$$

4) Obtenga la solución general de la ecuación diferencial

>

$$\frac{d^2}{dx^2} y(x) - 4y(x) = e^{2x} - 4\cos(2x) \quad (52)$$

RESPUESTA

$$> \text{Ecua} := y'' - 4y = \exp(2x) - 4\cos(2x)$$

$$Ecua := \frac{d^2}{dx^2} y(x) - 4 y(x) = e^{2x} - 4 \cos(2x) \quad (53)$$

>

$$EcuaHom := lhs(Ecua) = 0$$

$$EcuaHom := \frac{d^2}{dx^2} y(x) - 4 y(x) = 0 \quad (54)$$

$$Q := rhs(Ecua)$$

$$Q := e^{2x} - 4 \cos(2x) \quad (55)$$

$$EcuaCarac := m^2 - 4 = 0$$

$$EcuaCarac := m^2 - 4 = 0 \quad (56)$$

$$Raiz := solve(EcuaCarac)$$

$$Raiz := 2, -2 \quad (57)$$

$$yy[1] := \exp(Raiz[1] \cdot x)$$

$$yy_1 := e^{2x} \quad (58)$$

$$yy[2] := \exp(Raiz[2] \cdot x)$$

$$yy_2 := e^{-2x} \quad (59)$$

$$with(linalg) :$$

$$WW := wronskian([yy[1], yy[2]], x)$$

$$WW := \begin{bmatrix} e^{2x} & e^{-2x} \\ 2e^{2x} & -2e^{-2x} \end{bmatrix} \quad (60)$$

$$BB := array([0, Q])$$

$$BB := \begin{bmatrix} 0 & e^{2x} - 4 \cos(2x) \end{bmatrix} \quad (61)$$

$$Para := linsolve(WW, BB)$$

$$Para := \left[\frac{e^{2x} - 4 \cos(2x)}{4e^{2x}} \quad - \frac{e^{2x} - 4 \cos(2x)}{4e^{-2x}} \right] \quad (62)$$

$$Aprima := Para[1]$$

$$Aprima := \frac{e^{2x} - 4 \cos(2x)}{4e^{2x}} \quad (63)$$

$$Bprima := Para[2]$$

$$Bprima := - \frac{e^{2x} - 4 \cos(2x)}{4e^{-2x}} \quad (64)$$

$$SolGral := y(x) = \text{expand}(\text{simplify}((\text{int}(Aprima, x) + _C1) \cdot yy[1] + (\text{int}(Bprima, x) + _C2) \cdot yy[2]))$$

$$SolGral := y(x) = \cos(x)^2 - \frac{1}{2} + (e^x)^2 _C1 + \frac{(e^x)^2 x}{4} - \frac{(e^x)^2}{16} + \frac{_C2}{(e^x)^2} \quad (65)$$

$$Comprobar := \text{simplify}(\text{eval}(\text{subs}(y(x) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$$

$$(66)$$

$$\text{Comprobar} := 0 = 0 \quad (66)$$

> restart

5) Obtenga la solución de la ecuación diferencial

>

$$\frac{d^4}{dx^4} y(x) - y(x) = 0 \quad (67)$$

RESPUESTA

$$\text{Ecua} := \text{diff}(y(x), x\$4) - y(x) = 0$$

$$\text{Ecua} := \frac{d^4}{dx^4} y(x) - y(x) = 0 \quad (68)$$

>

$$\text{EcuaCarac} := m^4 - 1 = 0$$

$$\text{EcuaCarac} := m^4 - 1 = 0 \quad (69)$$

$$\text{Raiz} := \text{solve}(\text{EcuaCarac})$$

$$\text{Raiz} := 1, -1, i, -i \quad (70)$$

$$\text{yy}[1] := \exp(\text{Raiz}[1] \cdot x)$$

$$\text{yy}_1 := e^x \quad (71)$$

$$\text{yy}[2] := \exp(\text{Raiz}[2] \cdot x)$$

$$\text{yy}_2 := e^{-x} \quad (72)$$

$$\text{yy}[3] := \cos(\text{Im}(\text{Raiz}[3]) \cdot x)$$

$$\text{yy}_3 := \cos(x) \quad (73)$$

$$\text{yy}[4] := \sin(\text{Im}(\text{Raiz}[3]) \cdot x)$$

$$\text{yy}_4 := \sin(x) \quad (74)$$

$$\text{SolGral} := y(x) = _C1 \cdot \text{yy}[1] + _C2 \cdot \text{yy}[2] + _C3 \cdot \text{yy}[3] + _C4 \cdot \text{yy}[4]$$

$$\text{SolGral} := y(x) = _C1 e^x + _C2 e^{-x} + _C3 \cos(x) + _C4 \sin(x) \quad (75)$$

$$\text{Comprobar} := \text{simplify}(\text{eval}(\text{subs}(y(x) = \text{rhs}(\text{SolGral}), \text{Ecua})))$$

$$\text{Comprobar} := 0 = 0 \quad (76)$$

> restart

6) Resuelva la ecuación diferencial

>

$$\frac{d^2}{dx^2} y(x) + 2 \frac{d}{dx} y(x) - 3 y(x) = 15 \cos(3x) \quad (77)$$

RESPUESTA

$$\text{Ecua} := y'' + 2 \cdot y' - 3 \cdot y = 15 \cdot \cos(3 \cdot x)$$

$$\text{Ecua} := \frac{d^2}{dx^2} y(x) + 2 \frac{d}{dx} y(x) - 3 y(x) = 15 \cos(3x) \quad (78)$$

>

$$\text{EcuaHom} := \text{lhs}(\text{Ecua}) = 0$$

$$EcuaHom := \frac{d^2}{dx^2} y(x) + 2 \frac{d}{dx} y(x) - 3 y(x) = 0 \quad (79)$$

> $Q := rhs(Ecua)$

$$Q := 15 \cos(3 x) \quad (80)$$

> $EcuaCarac := m^2 + 2 \cdot m - 3 = 0$

$$EcuaCarac := m^2 + 2 m - 3 = 0 \quad (81)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 1, -3 \quad (82)$$

> $yy[1] := \exp(Raiz[1] \cdot x)$

$$yy_1 := e^x \quad (83)$$

> $yy[2] := \exp(Raiz[2] \cdot x)$

$$yy_2 := e^{-3 x} \quad (84)$$

> $with(linalg) :$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^x & e^{-3 x} \\ e^x & -3 e^{-3 x} \end{bmatrix} \quad (85)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & 15 \cos(3 x) \end{bmatrix} \quad (86)$$

> $Para := linsolve(WW, BB)$

$$Para := \begin{bmatrix} \frac{15 \cos(3 x)}{4 e^x} & -\frac{15 \cos(3 x)}{4 e^{-3 x}} \end{bmatrix} \quad (87)$$

> $Aprima := Para[1]$

$$Aprima := \frac{15 \cos(3 x)}{4 e^x} \quad (88)$$

> $Bprima := Para[2]$

$$Bprima := -\frac{15 \cos(3 x)}{4 e^{-3 x}} \quad (89)$$

> $SolGral := y(x) = \text{expand}(\text{simplify}((\text{int}(Aprima, x) + _C1) \cdot yy[1] + (\text{int}(Bprima, x) + _C2) \cdot yy[2]))$

$$SolGral := y(x) = 2 \sin(x) \cos(x)^2 - \frac{\sin(x)}{2} - 4 \cos(x)^3 + 3 \cos(x) + e^x _C1 + \frac{_C2}{(e^x)^3} \quad (90)$$

> $Comprobar := \text{simplify}(\text{eval}(\text{subs}(y(x) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad (91)$$

> $restart$

7) Resuelva la ecuación diferencial

>

$$(92)$$

$$\frac{d^2}{dx^2} y(x) + y(x) = -\cos(x) \quad (92)$$

RESPUESTA

> $Ecua := y'' + y = -\cos(x)$

$$Ecua := \frac{d^2}{dx^2} y(x) + y(x) = -\cos(x) \quad (93)$$

>

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2} y(x) + y(x) = 0 \quad (94)$$

> $Q := rhs(Ecua)$

$$Q := -\cos(x) \quad (95)$$

> $EcuaCarac := m^2 + 1 = 0$

$$EcuaCarac := m^2 + 1 = 0 \quad (96)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := I, -I \quad (97)$$

> $yy[1] := \cos(\text{Im}(Raiz[1]) \cdot x)$

$$yy_1 := \cos(x) \quad (98)$$

> $yy[2] := \sin(\text{Im}(Raiz[1]) \cdot x)$

$$yy_2 := \sin(x) \quad (99)$$

> $with(linalg) :$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix} \quad (100)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & -\cos(x) \end{bmatrix} \quad (101)$$

> $Para := simplify(linsolve(WW, BB))$

$$Para := \begin{bmatrix} \cos(x) \sin(x) & -\cos(x)^2 \end{bmatrix} \quad (102)$$

> $Aprima := Para[1]$

$$Aprima := \cos(x) \sin(x) \quad (103)$$

> $Bprima := Para[2]$

$$Bprima := -\cos(x)^2 \quad (104)$$

> $SolGral := y(x) = \text{expand}(simplify((\text{int}(Aprima, x) + _C1) \cdot yy[1] + (\text{int}(Bprima, x) + _C2) \cdot yy[2]))$

$$SolGral := y(x) = \sin(x) _C2 - \frac{\sin(x) x}{2} + \cos(x) _C1 \quad (105)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), Ecua)))$

$$Comprobar := -\cos(x) = -\cos(x) \quad (106)$$

> restart

8)

>

$$\frac{d^2}{d\theta^2} y(\theta) + y(\theta) = \sin(\theta)^2 - \cos(\theta)^2 \quad (107)$$

RESPUESTA

> Ecua := diff(y(theta), theta\$2) + y(theta) = sin(theta)^2 - cos(theta)^2

$$Ecua := \frac{d^2}{d\theta^2} y(\theta) + y(\theta) = \sin(\theta)^2 - \cos(\theta)^2 \quad (108)$$

>

> EcuaHom := lhs(Ecua) = 0

$$EcuaHom := \frac{d^2}{d\theta^2} y(\theta) + y(\theta) = 0 \quad (109)$$

> Q := rhs(Ecua)

$$Q := \sin(\theta)^2 - \cos(\theta)^2 \quad (110)$$

> EcuaCarac := m^2 + 1 = 0

$$EcuaCarac := m^2 + 1 = 0 \quad (111)$$

> Raiz := solve(EcuaCarac)

$$Raiz := I, -I \quad (112)$$

> yy[1] := cos(Im(Raiz[1])·theta)

$$yy_1 := \cos(\theta) \quad (113)$$

> yy[2] := sin(Im(Raiz[1])·theta)

$$yy_2 := \sin(\theta) \quad (114)$$

> with(linalg) :

> WW := wronskian([yy[1], yy[2]], theta)

$$WW := \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \quad (115)$$

> BB := array([0, Q])

$$BB := \begin{bmatrix} 0 & \sin(\theta)^2 - \cos(\theta)^2 \end{bmatrix} \quad (116)$$

> Para := simplify(linsolve(WW, BB))

$$Para := \begin{bmatrix} 2 \sin(\theta) \cos(\theta)^2 - \sin(\theta) & -2 \cos(\theta)^3 + \cos(\theta) \end{bmatrix} \quad (117)$$

> Aprima := Para[1]

$$Aprima := 2 \sin(\theta) \cos(\theta)^2 - \sin(\theta) \quad (118)$$

> Bprima := Para[2]

$$Bprima := -2 \cos(\theta)^3 + \cos(\theta) \quad (119)$$

> SolGral := y(theta) = expand(simplify((int(Aprima, theta) + _C1)·yy[1] + (int(Bprima,

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| theta) + _C2)·yy[2]))
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$$\text{SolGral} := y(\theta) = \cos(\theta) _C1 + \sin(\theta) _C2 + \frac{2 \cos(\theta)^2}{3} - \frac{1}{3} \quad (120)$$

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|> Comprobar := simplify(eval(subs(y(theta) = rhs(SolGral), Ecua)))
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$$\text{Comprobar} := -2 \cos(\theta)^2 + 1 = -2 \cos(\theta)^2 + 1 \quad (121)$$

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|> restart
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|FIN DEL EJERCICIO
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