

**SOLUCION****PRIMER EXAMEN FINAL COLEGIADO  
SEMESTRE 2011-1**

TIPO "A"

```
[> restart
RESPUESTA 1)
[> restart
> Ecuacion := cos(x) + (1 + 2/y(x)) * sin(x) * diff(y(x), x) = 0;
      Ecuacion := cos(x) + (1 + 2/y(x)) sin(x) (d/dx y(x)) = 0 (1)
[> with(DEtools) :
> odeadvisor(Ecuacion);
      [_separable] (2)
[> FI := intfactor(Ecuacion);
      FI := 1/sin(x) (3)
[> M(x, y) := cos(x); N(x, y) := (1 + 2/y) sin(x);
      M(x, y) := cos(x)
      N(x, y) := (1 + 2/y) sin(x) (4)
[> P(x) := M(x, y) * FI; R(y) := N(x, y) * FI;
      P(x) := cos(x)/sin(x)
      R(y) := 1 + 2/y (5)
[> _C:
> SolucionGeneral := int(P(x), x) + int(R(y), y) = _C;
      SolucionGeneral := ln(sin(x)) + y + 2 ln(y) = _C (6)
[> parametro := eval(subs(x = Pi/2, y = 1, SolucionGeneral));
      parametro := 1 = _C (7)
[> SolucionParticular := subs(_C = lhs(parametro), SolucionGeneral);
      SolucionParticular := ln(sin(x)) + y + 2 ln(y) = 1 (8)
[> restart
FIN RESPUESTA 1)
```

**RESPUESTA 2)**

```
[> restart
[> Ecuacion := diff(y(x), x$2) + (1/x) * diff(y(x), x) - (1/(x*2)) * y(x) = 72 * x * 3
(9)
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$$\text{Ecuacion} := \frac{d^2}{dx^2} y(x) + \frac{\frac{d}{dx} y(x)}{x} - \frac{y(x)}{x^2} = 72 x^3 \quad (9)$$

> *EcuacionHomogenea* := lhs(*Ecuacion*) = 0;

$$\text{EcuacionHomogenea} := \frac{d^2}{dx^2} y(x) + \frac{\frac{d}{dx} y(x)}{x} - \frac{y(x)}{x^2} = 0 \quad (10)$$

> *Q(x)* := rhs(*Ecuacion*);

$$Q(x) := 72 x^3 \quad (11)$$

> **\_c:**

> *SolucionHomogenea* :=  $y(x) = \_C1 \cdot x + \frac{\_C2}{x}$ ;

$$\text{SolucionHomogenea} := y(x) = \_C1 x + \frac{\_C2}{x} \quad (12)$$

> *comprobacion* := simplify(eval(subs(y(x) = rhs(*SolucionHomogenea*), *EcuacionHomogenea*)));

$$\text{comprobacion} := 0 = 0 \quad (13)$$

> *SolucionNoHomogenea* :=  $y(x) = A(x) \cdot x + \frac{B(x)}{x}$ ;

$$\text{SolucionNoHomogenea} := y(x) = A(x) x + \frac{B(x)}{x} \quad (14)$$

> *Solucion*<sub>1</sub> :=  $y(x) = x$ ; *Solucion*<sub>2</sub> :=  $y(x) = \frac{1}{x}$ ;

$$\text{Solucion}_1 := y(x) = x$$

$$\text{Solucion}_2 := y(x) = \frac{1}{x} \quad (15)$$

> *AA* := array([[rhs(*Solucion*<sub>1</sub>), rhs(*Solucion*<sub>2</sub>)], [rhs(diff(*Solucion*<sub>1</sub>, x)), rhs(diff(*Solucion*<sub>2</sub>, x))]]);

$$AA := \begin{bmatrix} x & \frac{1}{x} \\ 1 & -\frac{1}{x^2} \end{bmatrix} \quad (16)$$

> *BB* := array([0, *Q(x)*]);

$$BB := \begin{bmatrix} 0 & 72 x^3 \end{bmatrix} \quad (17)$$

> with(linalg) :

> *SOL* := linsolve(*AA*, *BB*);

$$SOL := \begin{bmatrix} 36 x^3 & -36 x^5 \end{bmatrix} \quad (18)$$

> *Asol* := *SOL*<sub>1</sub>; *Bsol* := *SOL*<sub>2</sub>;

$$\text{Asol} := 36 x^3$$

$$\text{Bsol} := -36 x^5 \quad (19)$$

```
> A(x) := int(Asol, x) + _C1; B(x) := int(Bsol, x) + _C2;
      A(x) := 9 x^4 + _C1
      B(x) := -6 x^6 + _C2
```

(20)

```
> SolucionGeneral := expand(SolucionNoHomogenea);
      SolucionGeneral := y(x) = 3 x^5 + _C1 x +  $\frac{-C2}{x}$ 
```

(21)

```
> sistema := subs(x = 1, rhs(SolucionGeneral) = 1), subs(x = 1, rhs(diff(SolucionGeneral, x))
      = 1);
      sistema := 3 + _C1 + _C2 = 1, 15 + _C1 - _C2 = 1
```

(22)

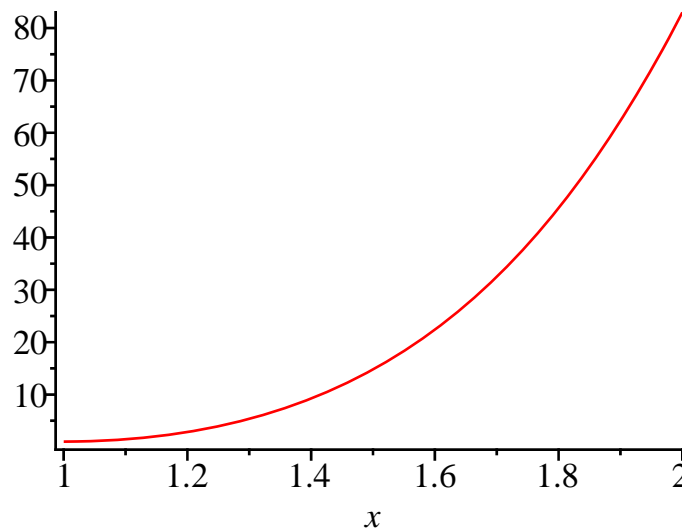
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> parametros := solve({sistema});
      parametros := {_C1 = -8, _C2 = 6}
```

(23)

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> SolucionParticular := subs(_C1 = rhs(parametros_1), _C2 = rhs(parametros_2),
      SolucionGeneral);
      SolucionParticular := y(x) = 3 x^5 - 8 x +  $\frac{6}{x}$ 
```

(24)

```
> plot(rhs(SolucionParticular), x = 1 .. 2);
```



```
> restart
```

**FIN RESPUESTA 2)**

### RESPUESTA 3)

RUTA 1

```
> restart
```

```
> _c:
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```
> Solucion := y(t) = _C1 * exp(-t) * cos(t) + _C2 * exp(-t) * sin(t) + 3 * sin(t) - cos(t)
      Solucion := y(t) = _C1 e^{-t} cos(t) + _C2 e^{-t} sin(t) + 3 sin(t) - cos(t)
```

(25)

```
> sistema := diff(Solucion, t), diff(Solucion, t$2);
```

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sistema :=  $\frac{d}{dt} y(t) = -_C1 e^{-t} \cos(t) - _C1 e^{-t} \sin(t) - _C2 e^{-t} \sin(t) + _C2 e^{-t} \cos(t)$ 
```

(26)

$$+ 3 \cos(t) + \sin(t), \frac{d^2}{dt^2} y(t) = 2\_C1 e^{-t} \sin(t) - 2\_C2 e^{-t} \cos(t) - 3 \sin(t) + \cos(t)$$

> SOL := simplify(solve({sistema}, {\_C1, \_C2}))

$$SOL := \left\{ \begin{aligned} & -C1 = \frac{1}{2} e^t \left( -2 \cos(t) \left( \frac{d}{dt} y(t) \right) + 4 \cos(t)^2 - 2 \cos(t) \sin(t) \right. \\ & \left. - \cos(t) \left( \frac{d^2}{dt^2} y(t) \right) + \sin(t) \left( \frac{d^2}{dt^2} y(t) \right) + 3 \right), -C2 = \frac{1}{2} e^t \left( -2 \left( \frac{d}{dt} y(t) \right) \sin(t) \right. \\ & \left. + 4 \cos(t) \sin(t) - 1 + 2 \cos(t)^2 - \cos(t) \left( \frac{d^2}{dt^2} y(t) \right) - \sin(t) \left( \frac{d^2}{dt^2} y(t) \right) \right) \end{aligned} \right\} \quad (27)$$

> EcuacionInicial := simplify(subs(\_C1 = rhs(SOL<sub>1</sub>), \_C2 = rhs(SOL<sub>2</sub>), Solucion))

$$EcuacionInicial := y(t) = \frac{5}{2} \cos(t) - \left( \frac{d}{dt} y(t) \right) + \frac{5}{2} \sin(t) - \frac{1}{2} \frac{d^2}{dt^2} y(t) \quad (28)$$

> EcuacionFinal := simplify(2 \* (lhs(EcuacionInicial) + ((d/dt y(t)) + 1/2 d^2/dt^2 y(t))) = rhs(EcuacionInicial) + ((d/dt y(t)) + 1/2 d^2/dt^2 y(t)));

$$EcuacionFinal := 2 y(t) + 2 \left( \frac{d}{dt} y(t) \right) + \frac{d^2}{dt^2} y(t) = 5 \cos(t) + 5 \sin(t) \quad (29)$$

> restart

ruta 2

> Solucion := y(t) = \_C1 \* exp(-t) \* cos(t) + \_C2 \* exp(-t) \* sin(t) + 3 \* sin(t) - cos(t)

$$Solucion := y(t) = _C1 e^{-t} \cos(t) + _C2 e^{-t} \sin(t) + 3 \sin(t) - \cos(t) \quad (30)$$

> SolucionHomogenea := y(t) = \_C1 e^{-t} cos(t) + \_C2 e^{-t} sin(t)

$$SolucionHomogenea := y(t) = _C1 e^{-t} \cos(t) + _C2 e^{-t} \sin(t) \quad (31)$$

> SolucionParticular := y(t) = 3 sin(t) - cos(t)

$$SolucionParticular := y(t) = 3 \sin(t) - \cos(t) \quad (32)$$

> EcuacionCaracteristica := expand((m - (-1 + I)) \* (m - (-1 - I))) = 0;

$$EcuacionCaracteristica := m^2 + 2m + 2 = 0 \quad (33)$$

> EcuacionHomogenea := diff(y(t), t\$2) + 2 \* diff(y(t), t) + 2 \* y(t) = 0;

$$EcuacionHomogenea := \frac{d^2}{dt^2} y(t) + 2 \left( \frac{d}{dt} y(t) \right) + 2 y(t) = 0 \quad (34)$$

> Q(t) := simplify(eval(subs(y(t) = rhs(SolucionParticular), lhs(EcuacionHomogenea))));

$$Q(t) := 5 \sin(t) + 5 \cos(t) \quad (35)$$

> EcuacionNoHomogenea := lhs(EcuacionHomogenea) = Q(t);

$$EcuacionNoHomogenea := \frac{d^2}{dt^2} y(t) + 2 \left( \frac{d}{dt} y(t) \right) + 2 y(t) = 5 \sin(t) + 5 \cos(t) \quad (36)$$

> restart

FIN RESPUESTA 3)

**RESPUESTA 4)**

```
> restart
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$$\text{EcuacionIntegral} := y(t) + \int_0^t \tau e^{2\tau} y(t - \tau) d\tau = t e^{2t} \quad (37)$$

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> with(inttrans) :
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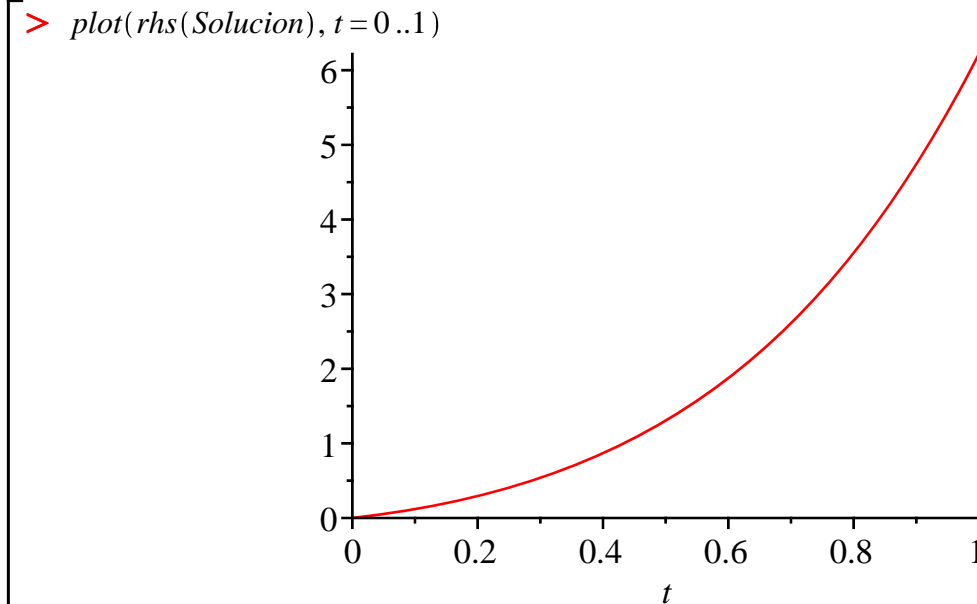
$$\text{TLecuacion} := \text{laplace}(y(t), t, s) + \frac{1}{4} \frac{\text{laplace}(y(t), t, s)}{\left(\frac{1}{2}s - 1\right)^2} = \frac{1}{(s - 2)^2} \quad (38)$$

```
> Tlsolucion := simplify(isolate(TLecuacion, laplace(y(t), t, s)));
```

$$\text{Tlsolucion} := \text{laplace}(y(t), t, s) = \frac{1}{s^2 - 4s + 5} \quad (39)$$

```
> Solucion := invlaplace(Tlsolucion, s, t)
```

$$\text{Solucion} := y(t) = e^{2t} \sin(t) \quad (40)$$



```
> restart
```

**FIN RESPUESTA 4)**
**RESPUESTA 5)**

```
> restart
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$$\text{Sistema} := \text{diff}(x(t), t) = x(t) + 2 \cdot y(t) + \exp(t), \text{diff}(y(t), t) = 2 \cdot x(t) + y(t) + \exp(3 \cdot t) :$$

$$\text{Sistema}_1; \text{Sistema}_2;$$

$$\frac{d}{dt} x(t) = x(t) + 2 y(t) + e^t$$

$$\frac{d}{dt} y(t) = 2 x(t) + y(t) + e^{3t} \quad (41)$$



$$\begin{aligned} > \text{EcuacionSeparable}_1 := \frac{\text{lhs}(\text{EcuacionIntermedia})}{G(y) \cdot \text{diff}(F(x), x)} = \frac{\text{rhs}(\text{EcuacionIntermedia})}{G(y) \cdot \text{diff}(F(x), x)}; \\ \text{EcuacionSeparable}_1 := \frac{y \left( \frac{d}{dy} G(y) \right)}{G(y)} = - \frac{F(x)}{\frac{d}{dx} F(x)} \end{aligned} \quad (47)$$

$$\begin{aligned} > \text{EcuacionPos}_x := \text{rhs}(\text{EcuacionSeparable}_1) = \text{beta} \cdot 2 \\ \text{EcuacionPos}_x := - \frac{F(x)}{\frac{d}{dx} F(x)} = \beta^2 \end{aligned} \quad (48)$$

$$\begin{aligned} > \text{EcuacionPos}_y := \text{lhs}(\text{EcuacionSeparable}_1) = \text{beta} \cdot 2 \\ \text{EcuacionPos}_y := \frac{y \left( \frac{d}{dy} G(y) \right)}{G(y)} = \beta^2 \end{aligned} \quad (49)$$

$$\begin{aligned} > \text{SolucionPos}_x := \text{dsolve}(\text{EcuacionPos}_x); \\ \text{SolucionPos}_x := F(x) = \_C1 e^{-\frac{x}{\beta^2}} \end{aligned} \quad (50)$$

$$\begin{aligned} > \text{SolucionPos}_y := \text{dsolve}(\text{EcuacionPos}_y) \\ \text{SolucionPos}_y := G(y) = \_C1 y^{\beta^2} \end{aligned} \quad (51)$$

$$\begin{aligned} > \text{SolucionPos} := u(x, y) = \text{rhs}(\text{SolucionPos}_x) \cdot (\text{subs}(\_C1 = 1, \text{rhs}(\text{SolucionPos}_y))); \\ \text{SolucionPos} := u(x, y) = \_C1 e^{-\frac{x}{\beta^2}} y^{\beta^2} \end{aligned} \quad (52)$$

> restart

RUTA 2

$$\begin{aligned} > \text{EcuacionIntermedia} := \text{lhs}(\text{EcuacionSeparable}) - F(x)G(y) = \text{rhs}(\text{EcuacionSeparable}) \\ - F(x)G(y) \\ \text{EcuacionIntermedia} := y \left( \frac{d}{dx} F(x) \right) \left( \frac{d}{dy} G(y) \right) = -F(x) G(y) \end{aligned} \quad (53)$$

$$\begin{aligned} > \text{EcuacionSeparable}_2 := \frac{\text{lhs}(\text{EcuacionIntermedia})}{y \cdot \text{diff}(G(y), y) \cdot F(x)} = \frac{\text{rhs}(\text{EcuacionIntermedia})}{y \cdot \text{diff}(G(y), y) \cdot F(x)}; \\ \text{EcuacionSeparable}_2 := \frac{\frac{d}{dx} F(x)}{F(x)} = - \frac{G(y)}{y \left( \frac{d}{dy} G(y) \right)} \end{aligned} \quad (54)$$

$$\begin{aligned} > \text{EcuacionPos}_x := \text{lhs}(\text{EcuacionSeparable}_2) = \text{beta} \cdot 2 \\ \text{EcuacionPos}_x := \frac{\frac{d}{dx} F(x)}{F(x)} = \beta^2 \end{aligned} \quad (55)$$

$$\begin{aligned} > \text{EcuacionPos}_y := \text{rhs}(\text{EcuacionSeparable}_2) = \text{beta} \cdot 2 \\ \text{EcuacionPos}_y := - \frac{G(y)}{y \left( \frac{d}{dy} G(y) \right)} = \beta^2 \end{aligned} \quad (56)$$

$$\begin{aligned} &> \text{SolucionPos}_x := \text{dsolve}(\text{EcuacionPos}_x); \\ & \qquad \qquad \qquad \text{SolucionPos}_x := F(x) = \_C1 e^{\beta^2 x} \end{aligned} \tag{57}$$

$$\begin{aligned} &> \text{SolucionPos}_y := \text{dsolve}(\text{EcuacionPos}_y) \\ & \qquad \qquad \qquad \text{SolucionPos}_y := G(y) = \_C1 y^{-\frac{1}{\beta^2}} \end{aligned} \tag{58}$$

$$\begin{aligned} &> \text{SolucionPos} := u(x, y) = \text{rhs}(\text{SolucionPos}_x) \cdot (\text{subs}(\_C1 = 1, \text{rhs}(\text{SolucionPos}_y))); \\ & \qquad \qquad \qquad \text{SolucionPos} := u(x, y) = \_C1 e^{\beta^2 x} y^{-\frac{1}{\beta^2}} \end{aligned} \tag{59}$$

[> restart  
**FIN RESPUESTA 6**

[> restart  
 [>  
**FIN EXAMEN**