SOLUCION

> *restart*

RESPUESTA   1)

> *restart*

> $Ecuacion := \sin(x) + \left(1 + \dfrac{3}{y(x)}\right) \cdot \cos(x) \cdot diff(y(x), x) = 0;$

$$Ecuacion := \sin(x) + \left(1 + \frac{3}{y(x)}\right)\cos(x)\left(\frac{d}{dx}\,y(x)\right) = 0 \tag{1}$$

> *with(DEtools)*:

> *odeadvisor(Ecuacion)*;

$$[\_separable] \tag{2}$$

> $FI := intfactor(Ecuacion);$

$$FI := \frac{1}{\cos(x)} \tag{3}$$

> $M(x, y) := \sin(x); N(x, y) := \left(1 + \dfrac{3}{y}\right)\cos(x);$

$$M(x, y) := \sin(x)$$

$$N(x, y) := \left(1 + \frac{3}{y}\right)\cos(x) \tag{4}$$

> $P(x) := M(x, y) \cdot FI; R(y) := N(x, y) \cdot FI;$

$$P(x) := \frac{\sin(x)}{\cos(x)}$$

$$R(y) := 1 + \frac{3}{y} \tag{5}$$

> `_C:`

> $SolucionGeneral := int(P(x), x) + int(R(y), y) = \_C;$

$$SolucionGeneral := -\ln(\cos(x)) + y + 3\ln(y) = \_C \tag{6}$$

> $parametro := eval(subs(x = 0, y = 1, SolucionGeneral));$

$$parametro := 1 = \_C \tag{7}$$

> $SolucionParticular := subs(\_C = lhs(parametro), SolucionGeneral);$

$$SolucionParticular := -\ln(\cos(x)) + y + 3\ln(y) = 1 \tag{8}$$

> *restart*

FIN RESPUESTA 1)

RESPUESTA   2)

> *restart*

> $Ecuacion := diff(y(x), x\$2) + \left(\dfrac{1}{x}\right) \cdot diff(y(x), x) - \left(\dfrac{1}{x \cdot \cdot 2}\right) \cdot y(x) = 36 \cdot x \cdot \cdot 3$

$$Ecuacion := \frac{d^2}{dx^2}\,y(x) + \frac{\dfrac{d}{dx}\,y(x)}{x} - \frac{y(x)}{x^2} = 36\,x^3 \tag{9}$$

> *EcuacionHomogenea* := *lhs*(*Ecuacion*) = 0;

$$EcuacionHomogenea := \frac{d^2}{dx^2} y(x) + \frac{\frac{d}{dx} y(x)}{x} - \frac{y(x)}{x^2} = 0 \qquad \textbf{(10)}$$

> *Q*(*x*) := *rhs*(*Ecuacion*);

$$Q(x) := 36 x^3 \qquad \textbf{(11)}$$

> **_c:**

> *SolucionHomogenea* := *y*(*x*) = _C1·*x* + $\frac{\_C2}{x}$;

$$SolucionHomogenea := y(x) = \_C1\, x + \frac{\_C2}{x} \qquad \textbf{(12)}$$

> *comprobacion* := *simplify*(*eval*(*subs*(*y*(*x*) = *rhs*(*SolucionHomogenea*), *EcuacionHomogenea*)));

$$comprobacion := 0 = 0 \qquad \textbf{(13)}$$

> *SolucionNoHomogenea* := *y*(*x*) = *A*(*x*)·*x* + $\frac{B(x)}{x}$;

$$SolucionNoHomogenea := y(x) = A(x)\, x + \frac{B(x)}{x} \qquad \textbf{(14)}$$

> *Solucion*$_1$:= *y*(*x*) = *x*; *Solucion*$_2$:= *y*(*x*) = $\frac{1}{x}$;

$$Solucion_1 := y(x) = x$$

$$Solucion_2 := y(x) = \frac{1}{x} \qquad \textbf{(15)}$$

> *AA* := *array*([[*rhs*(*Solucion*$_1$), *rhs*(*Solucion*$_2$)], [*rhs*(*diff* (*Solucion*$_1$, *x*)), *rhs*(*diff* (*Solucion*$_2$, *x*))]]);

$$AA := \begin{bmatrix} x & \dfrac{1}{x} \\[2mm] 1 & -\dfrac{1}{x^2} \end{bmatrix} \qquad \textbf{(16)}$$

> *BB* := *array*([0, *Q*(*x*)]);

$$BB := \begin{bmatrix} 0 & 36 x^3 \end{bmatrix} \qquad \textbf{(17)}$$

> *with*(*linalg*) :
> *SOL* := *linsolve*(*AA*, *BB*);

$$SOL := \begin{bmatrix} 18 x^3 & -18 x^5 \end{bmatrix} \qquad \textbf{(18)}$$

> *Asol* := *SOL*$_1$; *Bsol* := *SOL*$_2$;

$$Asol := 18 x^3$$

$$Bsol := -18 x^5 \qquad \textbf{(19)}$$

> *A*(*x*) := *int*(*Asol*, *x*) + _C1; *B*(*x*) := *int*(*Bsol*, *x*) + _C2;

$$A(x) := \frac{9}{2} x^4 + \_C1$$

$$B(x) := -3\,x^6 + \_C2 \tag{20}$$

> $SolucionGeneral := expand(SolucionNoHomogenea);$

$$SolucionGeneral := y(x) = \frac{3}{2}\,x^5 + \_C1\,x + \frac{\_C2}{x} \tag{21}$$

> $sistema := subs(x = 1, rhs(SolucionGeneral) = 1), subs(x = 1, rhs(diff(SolucionGeneral, x)))$
$= 1;$

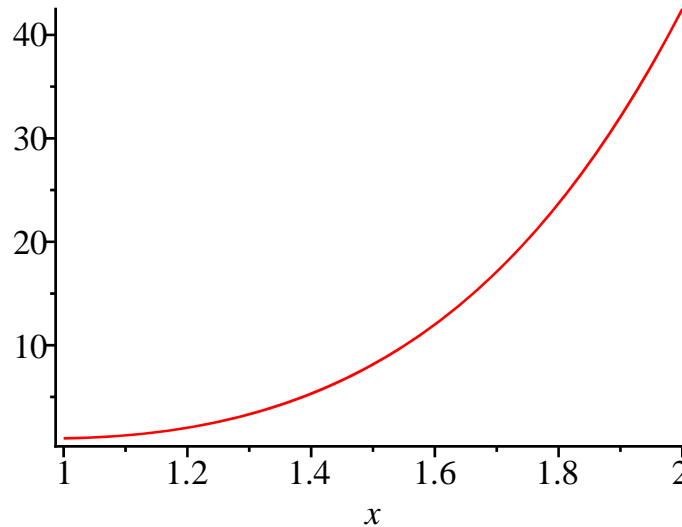$$sistema := \frac{3}{2} + \_C1 + \_C2 = 1,\ \frac{15}{2} + \_C1 - \_C2 = 1 \tag{22}$$

> $parametros := solve(\{sistema\});$

$$parametros := \left\{ \_C1 = -\frac{7}{2},\ \_C2 = 3 \right\} \tag{23}$$

> $SolucionParticular := subs(\_C1 = rhs(parametros_1),\ \_C2 = rhs(parametros_2),$
   $SolucionGeneral);$

$$SolucionParticular := y(x) = \frac{3}{2}\,x^5 - \frac{7}{2}\,x + \frac{3}{x} \tag{24}$$

> $plot(rhs(SolucionParticular), x = 1..2);$

> $restart$

FIN RESPUESTA 2)

RESPUESTA 3)
RUTA 1

> $restart$

> $\_C:$

> $Solucion := y(t) = \_C1 \cdot \exp(t) \cdot \sin(t) + \_C2 \cdot \exp(t) \cdot \cos(t) + 4 \cdot \sin(t) - \cos(t)$

$$Solucion := y(t) = \_C1\,e^t \sin(t) + \_C2\,e^t \cos(t) + 4\sin(t) - \cos(t) \tag{25}$$

> $sistema := diff(Solucion, t), diff(Solucion, t\$2);$

$$sistema := \frac{d}{dt}\,y(t) = \_C1\,e^t \sin(t) + \_C1\,e^t \cos(t) + \_C2\,e^t \cos(t) - \_C2\,e^t \sin(t) + 4\cos(t) \tag{26}$$

$+ \sin(t), \frac{d^2}{dt^2} y(t) = 2 \_C1 \, e^t \cos(t) - 2 \_C2 \, e^t \sin(t) - 4 \sin(t) + \cos(t)$

$>$ $SOL := simplify(solve(\{sistema\}, \{\_C1, \_C2\}))$

$$SOL := \left\{ \_C1 = \frac{1}{2} \, e^{-t} \left(2 \sin(t) \left(\frac{d}{dt} y(t) \right) - 3 \sin(t) \cos(t) - 6 + 5 \cos(t)^2 \right.\right. \tag{27}$$

$$\left. - \sin(t) \left(\frac{d^2}{dt^2} y(t) \right) + \cos(t) \left(\frac{d^2}{dt^2} y(t) \right) \right), \_C2 = -\frac{1}{2} \, e^{-t} \left(-2 \left(\frac{d}{dt} y(t) \right) \cos(t) \right.$$

$$\left.\left. + 3 \cos(t)^2 + 5 \sin(t) \cos(t) + \sin(t) \left(\frac{d^2}{dt^2} y(t) \right) + 4 + \cos(t) \left(\frac{d^2}{dt^2} y(t) \right) \right) \right\}$$

$>$ $EcuacionInicial := simplify\left(subs\left(\_C1 = rhs(SOL_1), \_C2 = rhs(SOL_2), Solucion\right)\right)$

$$EcuacionInicial := y(t) = \frac{d}{dt} y(t) - \frac{9}{2} \cos(t) + \sin(t) - \frac{1}{2} \frac{d^2}{dt^2} y(t) \tag{28}$$

$>$ $EcuacionFinal := simplify\left(2 \cdot \left(lhs(EcuacionInicial) - \left(\left(\frac{d}{dt} y(t) \right) - \frac{1}{2} \frac{d^2}{dt^2} y(t) \right) \right.\right.$

$$\left.\left. = rhs(EcuacionInicial) - \left(\left(\frac{d}{dt} y(t) \right) - \frac{1}{2} \frac{d^2}{dt^2} y(t) \right) \right) \right);$$

$$EcuacionFinal := 2 y(t) - 2 \left(\frac{d}{dt} y(t) \right) + \frac{d^2}{dt^2} y(t) = -9 \cos(t) + 2 \sin(t) \tag{29}$$

$>$ $restart$

RUTA 2

$>$ $Solucion := y(t) = \_C1 \cdot \exp(t) \cdot \sin(t) + \_C2 \cdot \exp(t) \cdot \cos(t) + 4 \cdot \sin(t) - \cos(t)$

$$Solucion := y(t) = \_C1 \, e^t \sin(t) + \_C2 \, e^t \cos(t) + 4 \sin(t) - \cos(t) \tag{30}$$

$>$ $SolucionHomogenea := y(t) = \_C1 \, e^t \sin(t) + \_C2 \, e^t \cos(t)$

$$SolucionHomogenea := y(t) = \_C1 \, e^t \sin(t) + \_C2 \, e^t \cos(t) \tag{31}$$

$>$ $SolucionParticular := y(t) = 4 \sin(t) - \cos(t)$

$$SolucionParticular := y(t) = 4 \sin(t) - \cos(t) \tag{32}$$

$>$ $EcuacionCaracteristica := expand((m - (1 + I)) \cdot (m - (1 - I))) = 0;$

$$EcuacionCaracteristica := m^2 - 2m + 2 = 0 \tag{33}$$

$>$ $EcuacionHomogenea := diff(y(t), t\$2) - 2 \cdot diff(y(t), t) + 2 \cdot y(t) = 0;$

$$EcuacionHomogenea := \frac{d^2}{dt^2} y(t) - 2 \left(\frac{d}{dt} y(t) \right) + 2 y(t) = 0 \tag{34}$$

$>$ $Q(t) := simplify(eval(subs(y(t) = rhs(SolucionParticular), lhs(EcuacionHomogenea))));$

$$Q(t) := 2 \sin(t) - 9 \cos(t) \tag{35}$$

$>$ $EcuacionNoHomogenea := lhs(EcuacionHomogenea) = Q(t);$

$$EcuacionNoHomogenea := \frac{d^2}{dt^2} y(t) - 2 \left(\frac{d}{dt} y(t) \right) + 2 y(t) = 2 \sin(t) - 9 \cos(t) \tag{36}$$

$>$ $restart$

FIN RESPUESTA 3)

RESPUESTA 4)

> *restart*

> *EcuacionIntegral* := $x(t) + int(\text{tau} \cdot \exp(3 \cdot \text{tau}) \cdot x(t - \text{tau}), \text{tau} = 0 .. t) = t \cdot \exp(3 \cdot t)$

$$EcuacionIntegral := x(t) + \int_0^t \tau\, e^{3\tau}\, x(t - \tau)\, d\tau = t\, e^{3t} \tag{37}$$

> *with(inttrans)* :

> *TLecuacion* := *laplace(EcuacionIntegral, t, s)*;

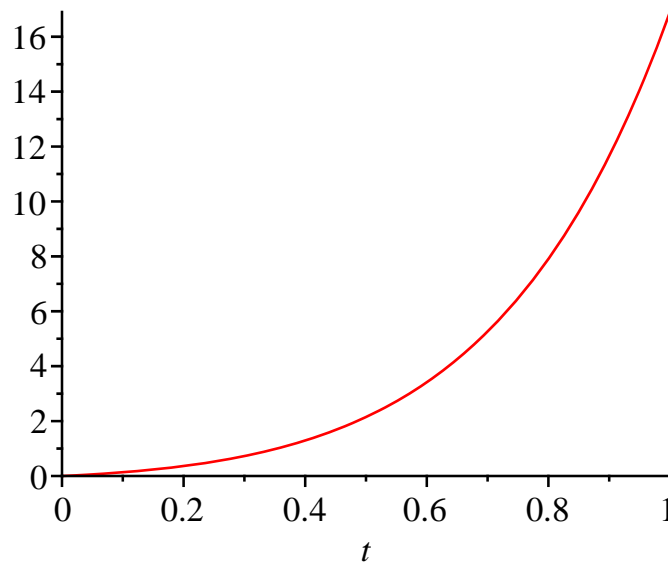$$TLecuacion := laplace(x(t), t, s) + \frac{1}{9}\, \frac{laplace(x(t), t, s)}{\left(\dfrac{1}{3}\, s - 1\right)^2} = \frac{1}{(s-3)^2} \tag{38}$$

> *TLsolucion* := *simplify(isolate(TLecuacion, laplace(x(t), t, s)))*;

$$TLsolucion := laplace(x(t), t, s) = \frac{1}{s^2 - 6s + 10} \tag{39}$$

> *Solucion* := *invlaplace(TLsolucion, s, t)*

$$Solucion := x(t) = e^{3t} \sin(t) \tag{40}$$

> *plot(rhs(Solucion), t = 0 .. 1)*

> *restart*

FIN RESPUESTA 4)

RESPUESTA 5)

> *restart*

> *Sistema* := *diff(x(t), t)* $= x(t) + 2 \cdot y(t) + \exp(t)$, *diff(y(t), t)* $= 2 \cdot x(t) + y(t) + \exp(3 \cdot t)$:
>   *Sistema$_1$; Sistema$_2$;*

$$\frac{d}{dt}\, x(t) = x(t) + 2\, y(t) + e^t$$

$$\frac{d}{dt}\, y(t) = 2\, x(t) + y(t) + e^{3t} \tag{41}$$
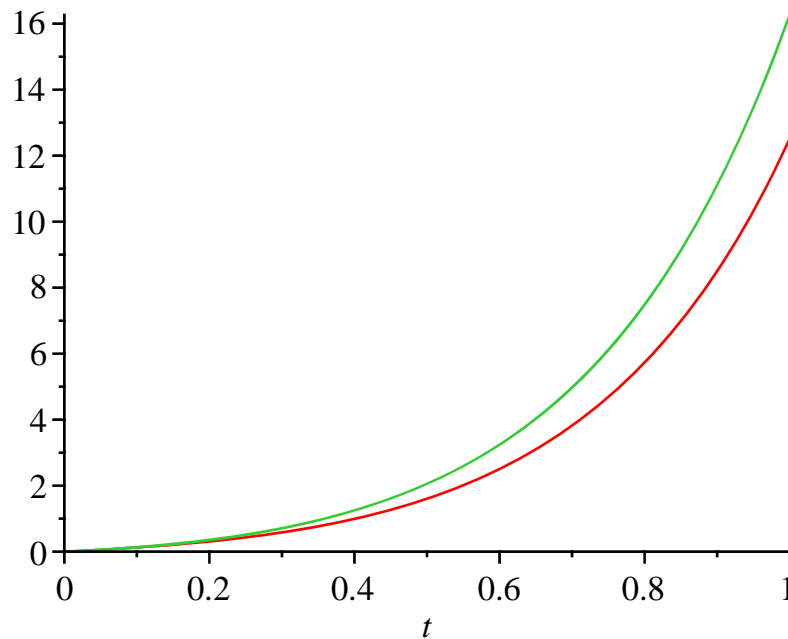
> $Condiciones := x(0) = 0, y(0) = 0;$
$$Condiciones := x(0) = 0, y(0) = 0 \qquad\qquad (42)$$

> $Solucion := dsolve(\{Sistema, Condiciones\}) : Solucion_1; Solucion_2;$
$$x(t) = \frac{1}{8}\,e^{3t} - \frac{1}{8}\,e^{-t} + \frac{1}{2}\,t\,e^{3t}$$
$$y(t) = \frac{3}{8}\,e^{3t} + \frac{1}{8}\,e^{-t} + \frac{1}{2}\,t\,e^{3t} - \frac{1}{2}\,e^{t} \qquad\qquad (43)$$

> $plot([rhs(Solucion_1), rhs(Solucion_2)], t = 0..1)$

> $restart$

FIN RESPUESTA 5)

RESPUESTA 6)

> $restart$

> $Ecuacion := x \cdot diff(T(x, y), x, y) + T(x, y) = 0;$
$$Ecuacion := x\left(\frac{\partial^2}{\partial y\,\partial x}\,T(x, y)\right) + T(x, y) = 0 \qquad\qquad (44)$$

> $EcuacionSeparable := eval(subs(T(x, y) = F(x) \cdot G(y), Ecuacion));$
$$EcuacionSeparable := x\left(\frac{d}{dx}\,F(x)\right)\left(\frac{d}{dy}\,G(y)\right) + F(x)\,G(y) = 0 \qquad\qquad (45)$$

> $EcuacionIntermedia := lhs(EcuacionSeparable) - F(x)G(y) = rhs(EcuacionSeparable)$
$\quad - F(x)G(y)$
$$EcuacionIntermedia := x\left(\frac{d}{dx}\,F(x)\right)\left(\frac{d}{dy}\,G(y)\right) = -F(x)\,G(y) \qquad\qquad (46)$$

>

RUTA 1

> `_c:`

> $EcuacionSeparable_1 := \dfrac{lhs(EcuacionIntermedia)}{x \cdot G(y) \cdot diff(F(x), x)} = \dfrac{rhs(EcuacionIntermedia)}{x \cdot G(y) \cdot diff(F(x), x)}$;

$$EcuacionSeparable_1 := \frac{\dfrac{d}{dy}\,G(y)}{G(y)} = -\frac{F(x)}{x\left(\dfrac{d}{dx}\,F(x)\right)} \tag{47}$$

> $EcuacionPos_x := rhs(EcuacionSeparable_1) = beta \cdot \cdot 2$

$$EcuacionPos_x := -\frac{F(x)}{x\left(\dfrac{d}{dx}\,F(x)\right)} = \beta^2 \tag{48}$$

> $EcuacionPos_y := lhs(EcuacionSeparable_1) = beta \cdot \cdot 2$

$$EcuacionPos_y := \frac{\dfrac{d}{dy}\,G(y)}{G(y)} = \beta^2 \tag{49}$$

> $SolucionPos_x := dsolve(EcuacionPos_x)$;

$$SolucionPos_x := F(x) = \_C1\, x^{-\frac{1}{\beta^2}} \tag{50}$$

> $SolucionPos_y := dsolve(EcuacionPos_y)$

$$SolucionPos_y := G(y) = \_C1\, e^{\beta^2 y} \tag{51}$$

> $SolucionPos := u(x, y) = rhs(SolucionPos_x) \cdot (subs(\_C1 = 1, rhs(SolucionPos_y)))$;

$$SolucionPos := u(x, y) = \_C1\, x^{-\frac{1}{\beta^2}}\, e^{\beta^2 y} \tag{52}$$

> $restart$

RUTA 2

> $Ecuacion := x \cdot diff(T(x, y), x, y) + T(x, y) = 0$;

$$Ecuacion := x\left(\frac{\partial^2}{\partial y\, \partial x}\, T(x, y)\right) + T(x, y) = 0 \tag{53}$$

> $EcuacionSeparable := eval(subs(T(x, y) = F(x) \cdot G(y), Ecuacion))$;

$$EcuacionSeparable := x\left(\frac{d}{dx}\,F(x)\right)\left(\frac{d}{dy}\,G(y)\right) + F(x)\,G(y) = 0 \tag{54}$$

> $EcuacionIntermedia := lhs(EcuacionSeparable) - F(x)G(y) = rhs(EcuacionSeparable) - F(x)G(y)$

$$EcuacionIntermedia := x\left(\frac{d}{dx}\,F(x)\right)\left(\frac{d}{dy}\,G(y)\right) = -F(x)\,G(y) \tag{55}$$

> $EcuacionSeparable_2 := \dfrac{lhs(EcuacionIntermedia)}{diff(G(y), y) \cdot F(x)} = \dfrac{rhs(EcuacionIntermedia)}{diff(G(y), y) \cdot F(x)}$;

$$EcuacionSeparable_2 := \frac{x\left(\dfrac{d}{dx}\,F(x)\right)}{F(x)} = -\frac{G(y)}{\dfrac{d}{dy}\,G(y)} \tag{56}$$

> $EcuacionPos_x := lhs(EcuacionSeparable_2) = beta \cdot \cdot 2$

$$EcuacionPos_x := \frac{x\left(\dfrac{d}{dx}F(x)\right)}{F(x)} = \beta^2 \tag{57}$$

> $EcuacionPos_y := rhs(EcuacionSeparable_2) = beta \cdot \cdot 2$

$$EcuacionPos_y := -\frac{G(y)}{\dfrac{d}{dy}G(y)} = \beta^2 \tag{58}$$

> $SolucionPos_x := dsolve(EcuacionPos_x);$

$$SolucionPos_x := F(x) = \_C1\ x^{\beta^2} \tag{59}$$

> $SolucionPos_y := dsolve(EcuacionPos_y)$

$$SolucionPos_y := G(y) = \_C1\ e^{-\frac{y}{\beta^2}} \tag{60}$$

> $SolucionPos := u(x,y) = rhs(SolucionPos_x) \cdot (subs(\_C1 = 1, rhs(SolucionPos_y)));$

$$SolucionPos := u(x,y) = \_C1\ x^{\beta^2}\ e^{-\frac{y}{\beta^2}} \tag{61}$$

> $restart$

FIN RESPUESTA 6

FIN EXAMEN

>