SOLUCIÓN

> *restart*

RESPUESTA 1)

> $Ecuacion := \left(\dfrac{y(x) \cdot \cdot 2}{x} - g \right) + y(x) \cdot diff(y(x), x) = 0;$

$$Ecuacion := \frac{y(x)^2}{x} - g + y(x) \left(\frac{d}{dx} y(x) \right) = 0 \tag{1}$$

> $with(DEtools):$

> $odeadvisor(Ecuacion)$

$$[[\_homogeneous, class\ G], \_rational, \_Bernoulli] \tag{2}$$

> $FI := intfactor(Ecuacion);$

$$FI := x^2 \tag{3}$$

> $M(x, y) := \left(\dfrac{y \cdot \cdot 2}{x} - g \right)$

$$M(x, y) := \frac{y^2}{x} - g \tag{4}$$

> $N(x, y) := y$

$$N(x, y) := y \tag{5}$$

> $MM(x, y) := expand(simplify(FI \cdot M(x, y)));$

$$MM(x, y) := x\, y^2 - g\, x^2 \tag{6}$$

> $NN(x, y) := FI \cdot N(x, y)$

$$NN(x, y) := x^2\, y \tag{7}$$

> $comprobacion := diff(MM(x, y), y) - diff(NN(x, y), x) = 0$

$$comprobacion := 0 = 0 \tag{8}$$

> $IMM := int(MM(x, y), x);$

$$IMM := \frac{1}{2}\, y^2\, x^2 - \frac{1}{3}\, g\, x^3 \tag{9}$$

> `_C:`

> $Solucion := IMM + int((NN(x, y) - diff(IMM, y)), y) = \_C1$

$$Solucion := \frac{1}{2}\, y^2\, x^2 - \frac{1}{3}\, g\, x^3 = \_C1 \tag{10}$$

> $SolucionGeneral := lhs(Solucion) \cdot 2 = \_C1$

$$SolucionGeneral := y^2\, x^2 - \frac{2}{3}\, g\, x^3 = \_C1 \tag{11}$$

> *restart*

FIN RESPUESTA 1)

RESPUESTA 2)

> $OperadorInicial := expand((D-1)\cdot(D+1));$

$$OperadorInicial := D^2 - 1 \tag{12}$$

> $Ecuacion := diff(y(x), x\$2) - y(x) = \dfrac{1}{2} - \exp(-x)$

$$Ecuacion := \frac{d^2}{dx^2} y(x) - y(x) = \frac{1}{2} - e^{-x} \tag{13}$$

> $condiciones := y(0) = 1, D(y)(0) = -1;$

$$condiciones := y(0) = 1, D(y)(0) = -1 \tag{14}$$

> $EcuacionHomogenea := lhs(Ecuacion) = 0;$

$$EcuacionHomogenea := \frac{d^2}{dx^2} y(x) - y(x) = 0 \tag{15}$$

> $Q(x) := rhs(Ecuacion);$

$$Q(x) := \frac{1}{2} - e^{-x} \tag{16}$$

> $EcuacionCaracteritica := m\cdot\cdot2 - 1 = 0;$

$$EcuacionCaracteritica := m^2 - 1 = 0 \tag{17}$$

> $Raiz := solve(EcuacionCaracteritica);$

$$Raiz := 1, -1 \tag{18}$$

> $Solucion_1 := y(x) = \exp(Raiz_1 \cdot x); Solucion_2 := y(x) = \exp(Raiz_2 \cdot x);$

$$Solucion_1 := y(x) = e^x$$

$$Solucion_2 := y(x) = e^{-x} \tag{19}$$

> `_C:`

> $SolucionHomogenea := y(x) = \_C1 \cdot rhs(Solucion_1) + \_C2 \cdot rhs(Solucion_2);$

$$SolucionHomogenea := y(x) = \_C1\, e^x + \_C2\, e^{-x} \tag{20}$$

> $SolucionNoHomogenea := y(x) = A(x) \cdot rhs(Solucion_1) + B(x) \cdot rhs(Solucion_2);$

$$SolucionNoHomogenea := y(x) = A(x)\, e^x + B(x)\, e^{-x} \tag{21}$$

> $AA := array([[rhs(Solucion_1), rhs(Solucion_2)], [rhs(diff(Solucion_1, x)), rhs(diff(Solucion_2, x))]]);$

$$AA := \begin{bmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{bmatrix} \tag{22}$$

> $BB := array([0, Q(x)]);$

$$BB := \begin{bmatrix} 0 & \dfrac{1}{2} - e^{-x} \end{bmatrix} \tag{23}$$

> $with(linalg):$

> $SOL := simplify(linsolve(AA, BB));$

$$\tag{24}$$

$$SOL := \left[-\frac{1}{4}\left(-1 + 2\,e^{-x}\right)e^{-x} \quad \frac{1}{4}\left(-1 + 2\,e^{-x}\right)e^{x} \right] \tag{24}$$

> $Aprima := SOL_1; Bprima := SOL_2;$

$$Aprima := -\frac{1}{4}\left(-1 + 2\,e^{-x}\right)e^{-x}$$

$$Bprima := \frac{1}{4}\left(-1 + 2\,e^{-x}\right)e^{x} \tag{25}$$

> $A(x) := int(Aprima, x) + \_C1; B(x) := int(Bprima, x) + \_C2;$

$$A(x) := -\frac{1}{4}\,e^{-x} + \frac{1}{4}\left(e^{-x}\right)^2 + \_C1$$

$$B(x) := -\frac{1}{4}\,e^{x} + \frac{1}{2}\,x + \_C2 \tag{26}$$

> $SolucionGeneral := expand(SolucionNoHomogenea)$

$$SolucionGeneral := y(x) = -\frac{1}{2} + \frac{1}{4\,e^{x}} + \_C1\,e^{x} + \frac{1}{2}\,\frac{x}{e^{x}} + \frac{\_C2}{e^{x}} \tag{27}$$

> $SistemaCondiciones := eval(subs(x = 0, rhs(SolucionGeneral) = 1)), eval(subs(x = 0,$
  $rhs(diff(SolucionGeneral, x)) = -1));$

$$SistemaCondiciones := -\frac{1}{4} + \_C1 + \_C2 = 1, \; \frac{1}{4} + \_C1 - \_C2 = -1 \tag{28}$$
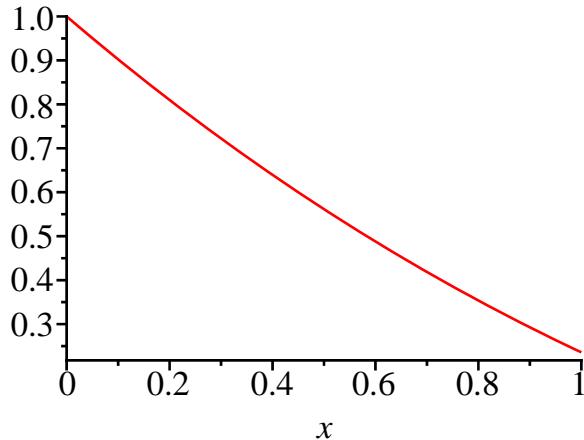
> $parametros := solve(\{SistemaCondiciones\}, \{\_C1, \_C2\})$

$$parametros := \left\{ \_C1 = 0, \_C2 = \frac{5}{4} \right\} \tag{29}$$
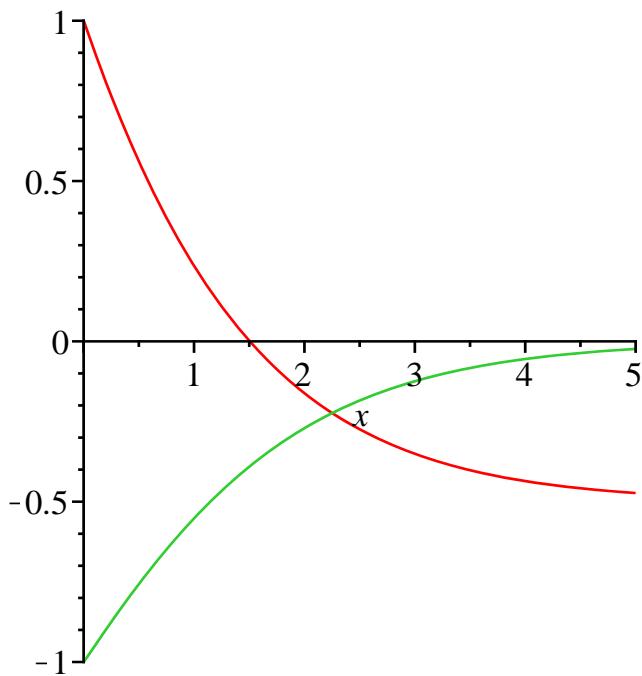
> $SolucionParticular := simplify(subs(\_C1 = rhs(parametros_1), \_C2 = rhs(parametros_2),$
  $SolucionGeneral));$

$$SolucionParticular := y(x) = -\frac{1}{2} + \frac{3}{2}\,e^{-x} + \frac{1}{2}\,e^{-x}x \tag{30}$$

> $plot(rhs(SolucionParticular), x = 0..1);$

> $plot([rhs(SolucionParticular), rhs(diff(SolucionParticular, x))], x = 0..5);$

> $comprobacion := simplify(dsolve(\{Ecuacion, condiciones\}));$

$$comprobacion := y(x) = -\frac{1}{2} + \frac{3}{2}\,e^{-x} + \frac{1}{2}\,e^{-x}\,x \qquad (31)$$

> $restart$

FIN RESPUESTA 2)

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RESPUESTA 3)

> $Ecuacion := diff(y(x), x\$2) - 2 \cdot diff(y(x), x) + y(x) = \frac{\exp(x)}{(1 + x \cdot\cdot 2)};$

$$Ecuacion := \frac{d^2}{dx^2}\,y(x) - 2\left(\frac{d}{dx}\,y(x)\right) + y(x) = \frac{e^x}{1 + x^2} \qquad (32)$$

> $EcuacionHomogenea := lhs(Ecuacion) = 0;$

$$EcuacionHomogenea := \frac{d^2}{dx^2}\,y(x) - 2\left(\frac{d}{dx}\,y(x)\right) + y(x) = 0 \qquad (33)$$

> $Q(x) := rhs(Ecuacion);$

$$Q(x) := \frac{e^x}{1 + x^2} \qquad (34)$$

> $EcuacionCaracteristica := m \cdot\cdot 2 - 2 \cdot m + 1 = 0;$

$$EcuacionCaracteristica := m^2 - 2\,m + 1 = 0 \qquad (35)$$

> $Raiz := solve(EcuacionCaracteristica);$

$$Raiz := 1, 1 \qquad (36)$$

> $Solucion_1 := y(x) = \exp(Raiz_1 \cdot x); Solucion_2 := y(x) = x \cdot \exp(Raiz_1 \cdot x);$

$$Solucion_1 := y(x) = e^x$$

$$Solucion_2 := y(x) = x\,e^x \qquad (37)$$

> _c:

> $SolucionHomogenea := y(x) = \_C1 \cdot rhs(Solucion_1) + \_C2 \cdot rhs(Solucion_2);$

$$SolucionHomogenea := y(x) = \_C1\,e^x + \_C2\,x\,e^x \qquad (38)$$

> $SolucionNoHomogenea := y(x) = A(x) \cdot rhs(Solucion_1) + B(x) \cdot rhs(Solucion_2);$

$$SolucionNoHomogenea := y(x) = A(x)\,e^x + B(x)\,x\,e^x \qquad (39)$$

> $AA := array([[rhs(Solucion_1), rhs(Solucion_2)], [rhs(diff(Solucion_1, x)),$
$rhs(diff(Solucion_2, x))]]);$

$$AA := \begin{bmatrix} e^x & x\,e^x \\ e^x & e^x + x\,e^x \end{bmatrix} \qquad (40)$$

> $BB := array([0, Q(x)]);$

$$BB := \begin{bmatrix} 0 & \dfrac{e^x}{1+x^2} \end{bmatrix} \qquad (41)$$

> $with(linalg):$
> $SOL := linsolve(AA, BB);$

$$SOL := \begin{bmatrix} -\dfrac{x}{1+x^2} & \dfrac{1}{1+x^2} \end{bmatrix} \qquad (42)$$

> $Aprima := SOL_1; Bprima := SOL_2;$

$$Aprima := -\frac{x}{1+x^2}$$

$$Bprima := \frac{1}{1+x^2} \qquad (43)$$

> $A(x) := int(Aprima, x) + \_C1; B(x) := int(Bprima, x) + \_C2;$

$$A(x) := -\frac{1}{2}\ln(1+x^2) + \_C1$$

$$B(x) := \arctan(x) + \_C2 \qquad (44)$$

> $SolucionGeneral := expand(SolucionNoHomogenea);$

$$SolucionGeneral := y(x) = -\frac{1}{2}\,e^x\ln(1+x^2) + \_C1\,e^x + x\,e^x\arctan(x) + \_C2\,x\,e^x \qquad (45)$$

> $restart$

FIN RESPUESTA 3)

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RESPUESTA 4)

> $Sistema := diff(x(t), t\$2) + diff(y(t), t\$2) + 4\cdot x(t) = t\cdot\exp(-t), diff(y(t), t\$2) + 4\cdot x(t)$
$= 0 : Sistema_1; Sistema_2;$

$$\frac{d^2}{dt^2}\,x(t) + \frac{d^2}{dt^2}\,y(t) + 4\,x(t) = t\,e^{-t}$$
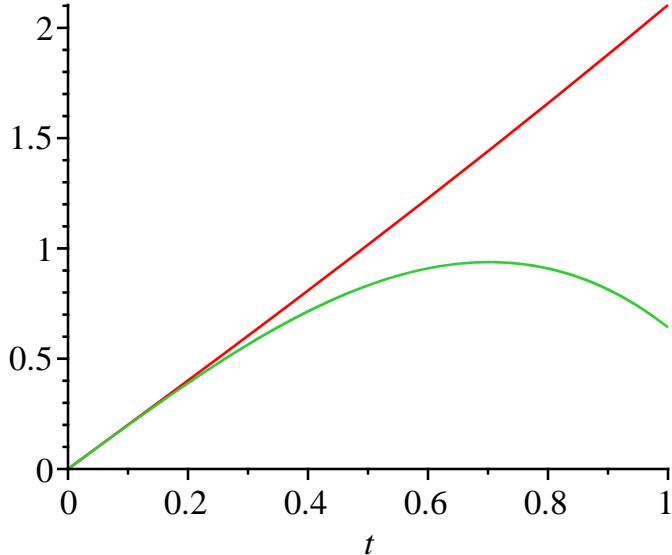
$$\frac{d^2}{dt^2}\,y(t) + 4\,x(t) = 0 \qquad (46)$$

> $Condiciones := x(0) = 0, D(x)(0) = 2, y(0) = 0, D(y)(0) = 2;$

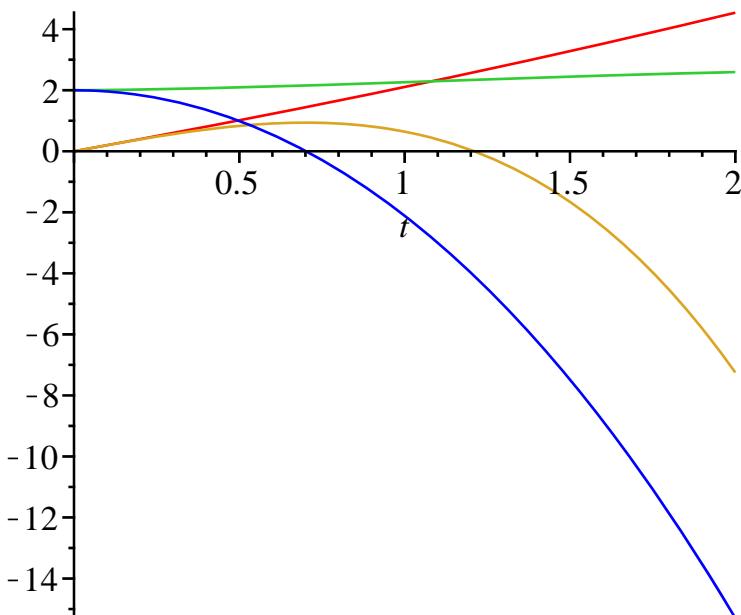$$Condiciones := x(0) = 0, D(x)(0) = 2, y(0) = 0, D(y)(0) = 2 \tag{47}$$

> $Solucion := dsolve(\{Sistema, Condiciones\}) : Solucion_1; Solucion_2;$

$$x(t) = 2\,e^{-t} + t\,e^{-t} + 3\,t - 2$$

$$y(t) = -16\,e^{-t} - 4\,t\,e^{-t} - 2\,t^3 + 4\,t^2 - 10\,t + 16 \tag{48}$$

> $plot([rhs(Solucion_1), rhs(Solucion_2)], t = 0..1);$

> $plot([rhs(Solucion_1), rhs(diff(Solucion_1, t)), rhs(Solucion_2), rhs(diff(Solucion_2, t))], t = 0$
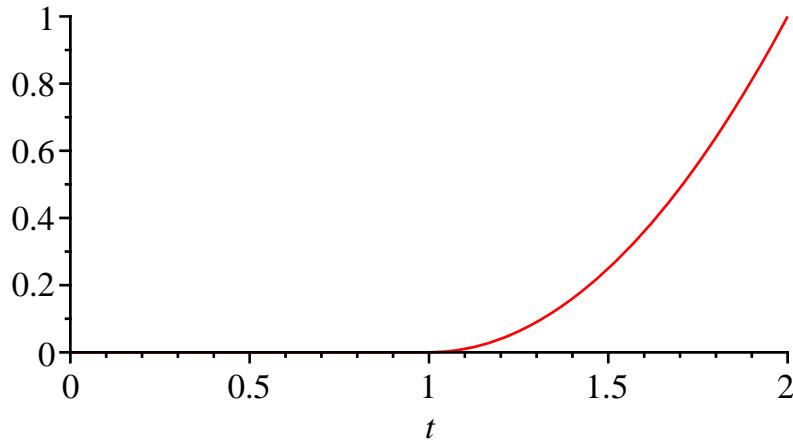 $..2)$

> $restart$

FIN RESPUESTA 4)

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RESPUESTA 5)

> $f(t) := (t - 1)\cdot\cdot2\cdot\text{Heaviside}(t - 1);$

$$f(t) := (t-1)^2 \, \text{Heaviside}(t-1) \tag{49}$$

> $plot(f(t), t = 0..2)$

> $with(inttrans):$
> $F(s) := laplace(f(t), t, s)$

$$F(s) := \frac{2 \, e^{-s}}{s^3} \tag{50}$$

> $restart$

FIN RESPUESTA 5

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RESPUESTA 6)

> $with(inttrans):$
> $f(t) := \exp(-t) \cdot \cos(2 \cdot t); \, g(t) := \text{Dirac}(t-1);$

$$f(t) := e^{-t} \cos(2t)$$
$$g(t) := \text{Dirac}(t-1) \tag{51}$$

> $F(s) := laplace(f(t), t, s); \, G(s) := laplace(g(t), t, s);$

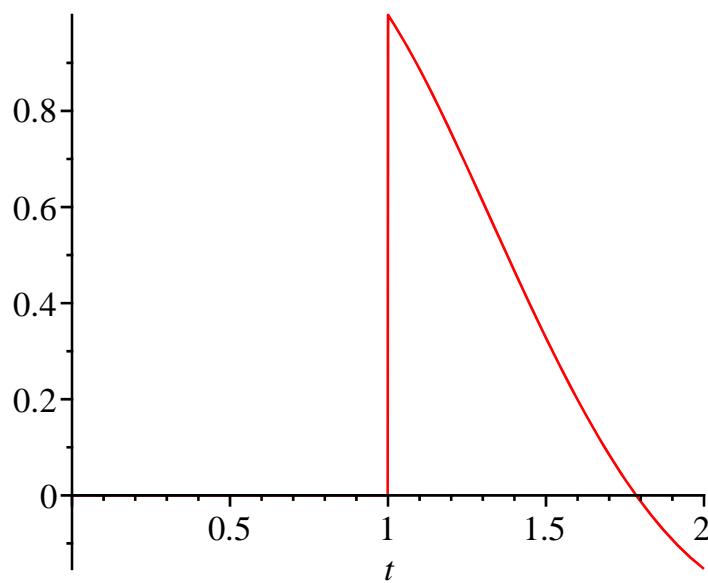$$F(s) := \frac{s+1}{(s+1-2\,I)\,(s+1+2\,I)}$$

$$G(s) := e^{-s} \tag{52}$$

> $H(s) := F(s) \cdot G(s);$

$$H(s) := \frac{(s+1)\,e^{-s}}{(s+1-2\,I)\,(s+1+2\,I)} \tag{53}$$

> $h(t) := invlaplace(H(s), s, t);$

$$h(t) := \text{Heaviside}(t-1)\,e^{1-t}\cos(2t-2) \tag{54}$$
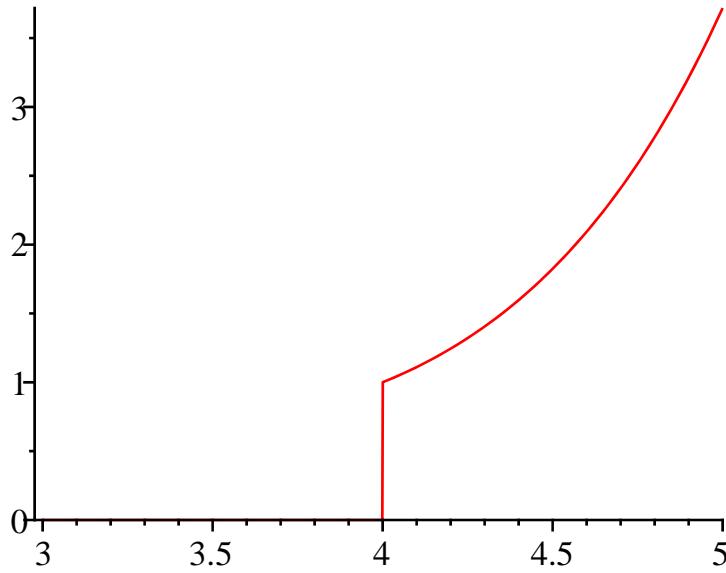
> $plot(h(t), t = 0..2)$

> $R(s) := \dfrac{\exp(-4\,s)\cdot(s\cdot\cdot 2 - s + 1)}{(s\cdot\cdot 3 - 2\cdot s\cdot\cdot 2 + s)};$

$$R(s) := \frac{e^{-4s}\left(s^2 - s + 1\right)}{s^3 - 2\,s^2 + s} \tag{55}$$

> $r(t) := invlaplace(R(s), s, t);$

$$r(t) := \text{Heaviside}(t-4)\left((t-4)\,e^{t-4} + 1\right) \tag{56}$$

> $plot(r(t), t = 3..5)$

> *restart*

FIN RESPUESTA 6)

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RESPUESTA 7)

> $f(t) := t\cdot\text{Heaviside}(t) - (t-1)\cdot\text{Heaviside}(t-1) - \text{Heaviside}(t-1); plot(f(t), t = 0..2);$

$$f(t) := t\,\text{Heaviside}(t) - (t-1)\,\text{Heaviside}(t-1) - \text{Heaviside}(t-1)$$
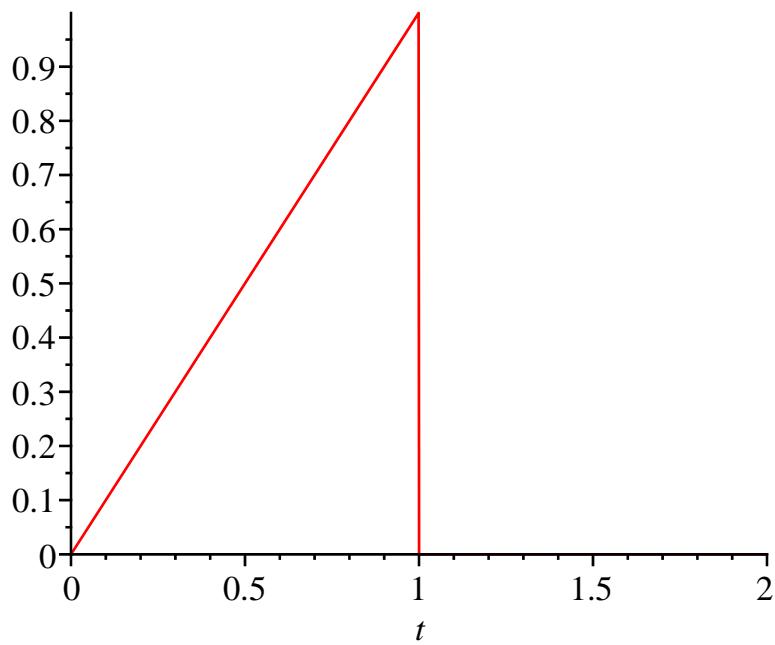
> $g(t) := \text{Heaviside}(t+1) - (t+1) \cdot \text{Heaviside}(t+1) + t \cdot \text{Heaviside}(t) + f(t); plot(g(t), t = -2..2)$

$g(t) := \text{Heaviside}(t+1) - (t+1)\,\text{Heaviside}(t+1) + 2\,t\,\text{Heaviside}(t) - (t-1)\,\text{Heaviside}(t-1) - \text{Heaviside}(t-1)$
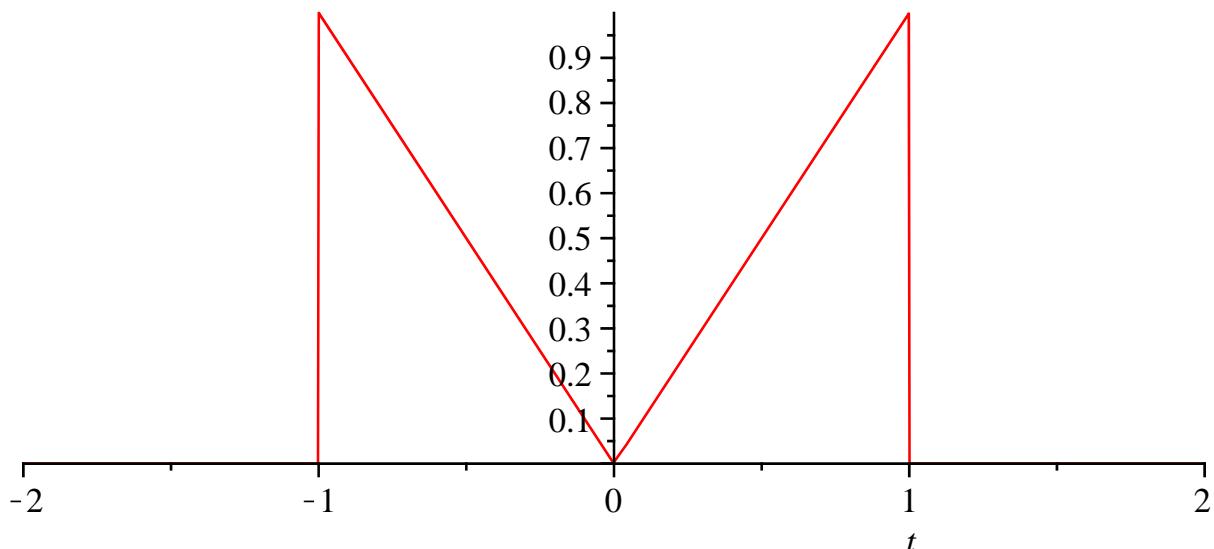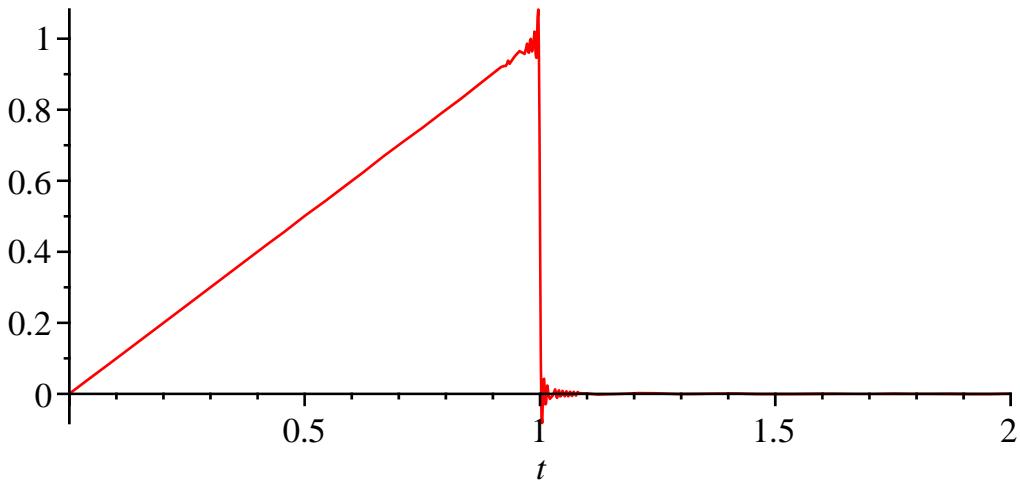

> $L := 2;$

$$L := 2 \tag{57}$$

> $a_0 := \left(\dfrac{1}{L}\right) \cdot int(g(t), t = -L..L);$

$$a_0 := \frac{1}{2} \tag{58}$$

> $C := \dfrac{a_0}{2};$

$$C := \frac{1}{4} \tag{59}$$

> $a_n := \left(\dfrac{1}{L}\right) \cdot int\left(g(t) \cdot \cos\left(\dfrac{n \cdot \text{Pi} \cdot t}{L}\right), t = -L..L\right);$

$$a_n := \frac{4\left(\cos\left(\dfrac{1}{2}\, n\, \pi\right) + \dfrac{1}{2}\, n\, \pi \sin\left(\dfrac{1}{2}\, n\, \pi\right)\right)}{n^2\, \pi^2} - \frac{4}{n^2\, \pi^2} \tag{60}$$

> $STF := C + Sum\left(a_n \cdot \cos\left(\dfrac{n \cdot \text{Pi} \cdot t}{L}\right), n = 1..infinity\right);$

$$STF := \frac{1}{4} + \sum_{n=1}^{\infty}\left(\frac{4\left(\cos\left(\dfrac{1}{2}\, n\, \pi\right) + \dfrac{1}{2}\, n\, \pi \sin\left(\dfrac{1}{2}\, n\, \pi\right)\right)}{n^2\, \pi^2} - \frac{4}{n^2\, \pi^2}\right) \cos\left(\dfrac{1}{2}\, n\, \pi\, t\right) \tag{61}$$

> $STF_{500} := C + sum\left(a_n \cdot \cos\left(\dfrac{n \cdot \text{Pi} \cdot t}{L}\right), n = 1..500\right):$

> $plot\left(STF_{500}, t = 0..2\right);$

> $restart$

FIN RESPUESTA 7)

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> $restart$

FIN EXAMEN

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