

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
TERCER EXAMEN PARCIAL

2011 MAYO 23

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1) UTILIZANDO EXCLUSIVAMENTE TRANSFORMADA DE LAPLACE (sin usar dsolve):

a) (15/100 puntos) OBTENER LA SOLUCIÓN PARTICULAR DE LA ECUACIÓN DADA CON LAS CONDICIONES INICIALES DADAS

b) (15/100 puntos) GRAFICAR - JUNTAS - LA SOLUCIÓN OBTENIDA EN EL INCISO a) Y SU PRIMERA DERIVADA PARA UN INTERVALO DE $0 < t < 15$

> diff(y(t), t, 2) + 6 * diff(y(t), t) + 9 * y(t) = 36 * Heaviside(t-4) * cos(2 * t - 8); y(0) = 1; D(y)(0) = 0

$$\frac{d^2}{dt^2} y(t) + 6 \left(\frac{d}{dt} y(t) \right) + 9 y(t) = 36 \text{Heaviside}(t-4) \cos(2 t - 8)$$

$$y(0) = 1$$

$$D(y)(0) = 0$$

(1)

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RESPUESTA 1a)

> Ecuacion := $\frac{d^2}{dt^2} y(t) + 6 \left(\frac{d}{dt} y(t) \right) + 9 y(t) = 36 \text{Heaviside}(t-4) \cos(2 t - 8)$

$$\text{Ecuacion} := \frac{d^2}{dt^2} y(t) + 6 \left(\frac{d}{dt} y(t) \right) + 9 y(t) = 36 \text{Heaviside}(t-4) \cos(2 t - 8) \quad (2)$$

> Condiciones := y(0) = 1, D(y)(0) = 0;

$$\text{Condiciones} := y(0) = 1, D(y)(0) = 0$$

(3)

> with(inttrans) :

> TransfLaplaceEcuacion := subs(Condiciones, laplace(Ecuacion, t, s))

$$\text{TransfLaplaceEcuacion} := s^2 \text{laplace}(y(t), t, s) - 6 - s + 6 s \text{laplace}(y(t), t, s) \quad (4)$$

$$+ 9 \text{laplace}(y(t), t, s) = \frac{36 e^{-4s} s}{s^2 + 4}$$

> TransfLaplaceSolucion := simplify(isolate(TransfLaplaceEcuacion, laplace(y(t), t, s)))

$$\text{TransfLaplaceSolucion} := \text{laplace}(y(t), t, s) = \frac{36 e^{-4s} s + 6 s^2 + 24 + s^3 + 4 s}{(s^2 + 4) (s^2 + 6 s + 9)} \quad (5)$$

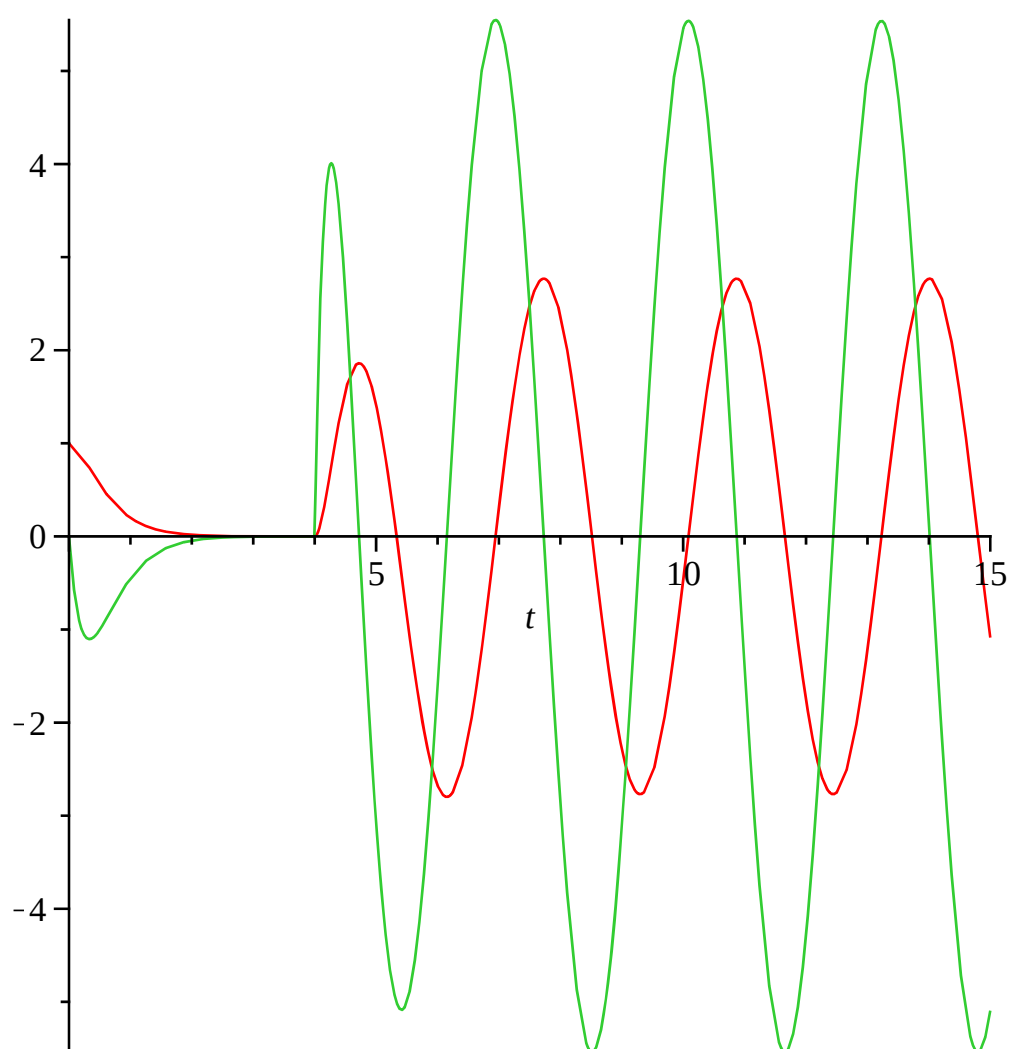
> SolucionParticular := invlaplace(TransfLaplaceSolucion, s, t)

$$\text{SolucionParticular} := y(t) = e^{-3t} (3 t + 1) + \frac{36}{169} (12 \sin(2 t - 8) + 5 \cos(2 t - 8)) - e^{-3t + 12} (39 t - 151) \text{Heaviside}(t - 4) \quad (6)$$

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RESPUESTA 2b)

> plot([rhs(SolucionParticular), rhs(diff(SolucionParticular, t))], t = 0..15)



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FIN RESPUESTA 1)

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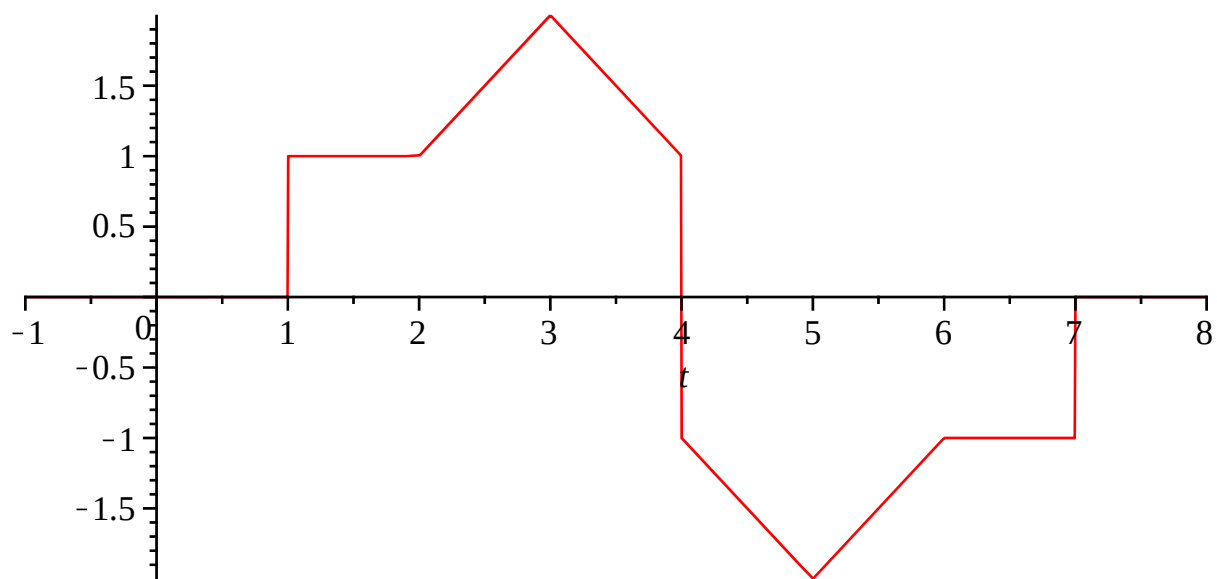
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2) DE LA FUNCIÓN DIBUJADA:

a) (15/100 puntos) OBTENER SU TRANSFORMADA DE LAPLACE

b) (25/100 puntos) GRAFICAR EN EL INTERVALO $1.8 < x < 2.2$ A LA FUNCIÓN Y SU SERIE SENO OBTENIDA CALCULANDO SUS PRIMEROS 500 TÉRMINOS

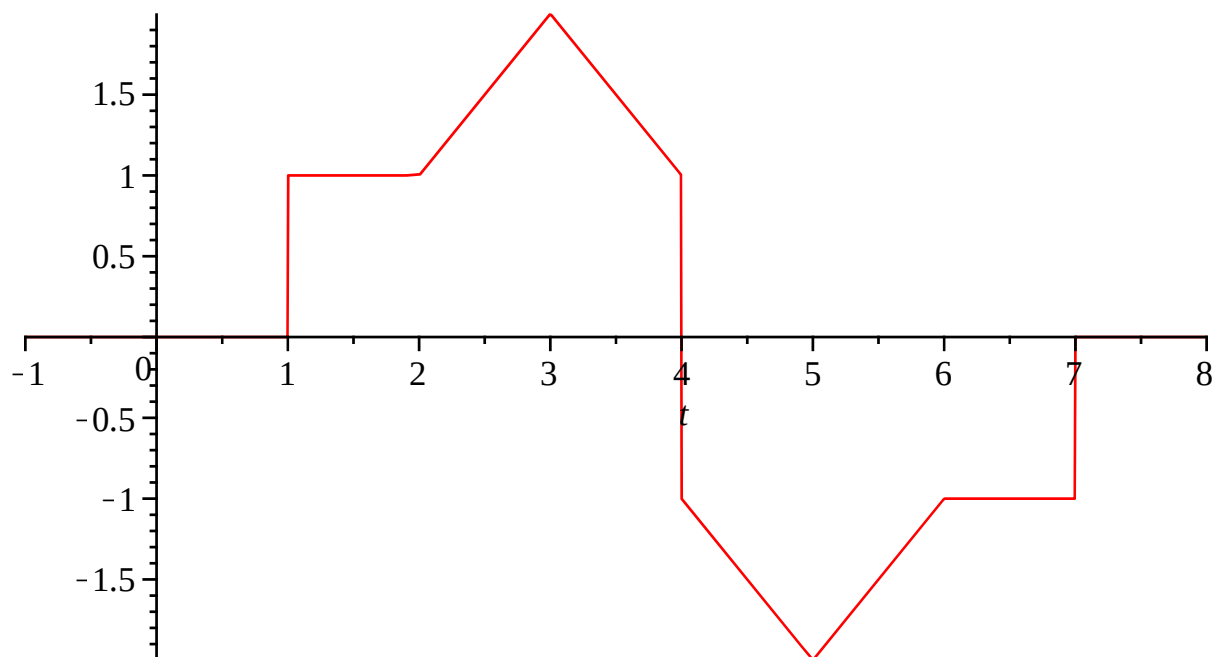


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RESPUESTA 2a)

> $f(t) := \text{Heaviside}(t-1) + (t-2) \cdot \text{Heaviside}(t-2) - 2 \cdot (t-3) \cdot \text{Heaviside}(t-3) + (t-4) \cdot \text{Heaviside}(t-4) - 2 \cdot \text{Heaviside}(t-4) - (t-4) \cdot \text{Heaviside}(t-4) + 2 \cdot (t-5) \cdot \text{Heaviside}(t-5) - (t-6) \cdot \text{Heaviside}(t-6) + \text{Heaviside}(t-7); \text{plot}(f(t), t=-1..8)$

$f(t) := \text{Heaviside}(t-1) + (t-2) \text{Heaviside}(t-2) - 2 (t-3) \text{Heaviside}(t-3) - 2 \text{Heaviside}(t-4) + 2 (t-5) \text{Heaviside}(t-5) - (t-6) \text{Heaviside}(t-6) + \text{Heaviside}(t-7)$



> with(inttrans) :

> $F(s) := \text{laplace}(f(t), t, s)$

$$F(s) := \frac{e^{-2s} - e^{-6s} + 2e^{-5s} - 2e^{-3s}}{s^2} + \frac{e^{-s} + e^{-7s} - 2e^{-4s}}{s}$$

(7)

RESPUESTA 2b) ALTERNATIVA 1

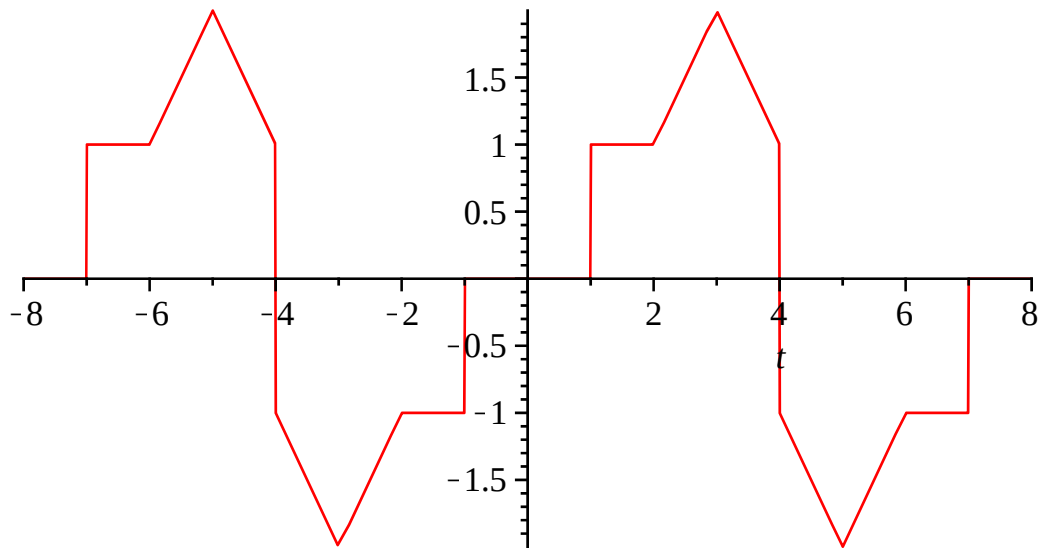
> L := 8;

L := 8

(8)

> r(t) := Heaviside(t + 7) + (t + 6) · Heaviside(t + 6) - 2 · (t + 5) · Heaviside(t + 5) + (t + 4) · Heaviside(t + 4) - 2 · Heaviside(t + 4) - (t + 4) · Heaviside(t + 4) + 2 · (t + 3) · Heaviside(t + 3) - (t + 2) · Heaviside(t + 2) + Heaviside(t + 1) + f(t); plot(r(t), t = -8 .. 8)

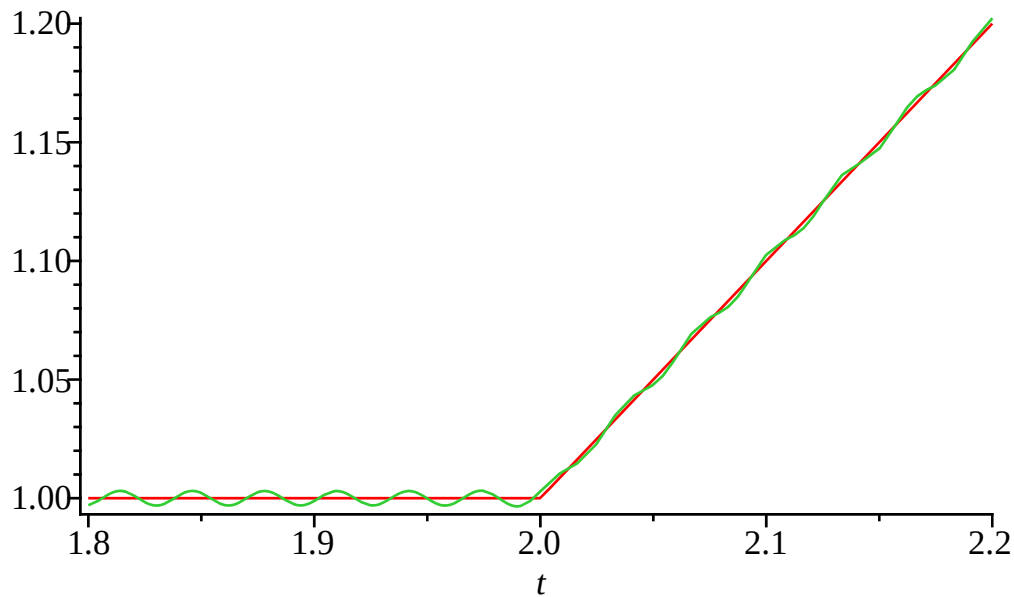
r(t) := Heaviside(t + 7) + (t + 6) Heaviside(t + 6) - 2 (t + 5) Heaviside(t + 5) - 2 Heaviside(t + 4) + 2 (t + 3) Heaviside(t + 3) - (t + 2) Heaviside(t + 2) + Heaviside(t + 1) + Heaviside(t - 1) + (t - 2) Heaviside(t - 2) - 2 (t - 3) Heaviside(t - 3) - 2 Heaviside(t - 4) + 2 (t - 5) Heaviside(t - 5) - (t - 6) Heaviside(t - 6) + Heaviside(t - 7)



> b_n := $\left(\frac{1}{L}\right) \cdot \text{int}\left(r(t) \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), t = -L .. L\right) :$

> STF500 := sum $\left(b_n \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), n = 1 .. 500\right) :$

> plot([f(t), STF500], t = 1.8 .. 2.2)



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RESPUESTA 2b) ALTERNATIVA 2

> LL := 4;

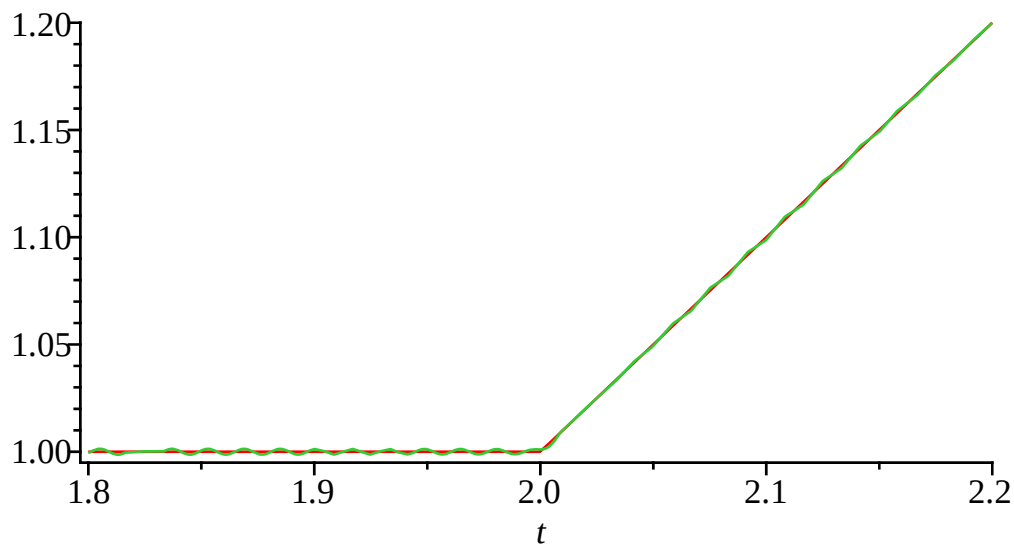
LL := 4

(9)

> $bb_n := \left(\frac{1}{LL} \right) \cdot \text{int} \left(f(t) \cdot \sin \left(\frac{n \cdot \text{Pi} \cdot t}{LL} \right), t = 0 .. 2 \cdot LL \right) :$

> $STFF500 := \text{sum} \left(bb_n \cdot \sin \left(\frac{n \cdot \text{Pi} \cdot t}{LL} \right), n = 1 .. 500 \right) :$

> plot([f(t), STFF500], t = 1.8 .. 2.2)



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FIN RESPUESTA 2)

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3) (30/100 puntos) OBTENER LA SOLUCIÓN DE LA SIGUIENTE ECUACIÓN EN DERIVADAS

PARCIALES, UTILIZANDO EL MÉTODO DE SEPARACIÓN DE VARIABLES CON UNA CONSTANTE DE SEPARACIÓN IGUAL A MENOS SEIS UNIDADES

$$\frac{\partial^2}{\partial x^2} z(x, y) - 8 \left(\frac{\partial}{\partial y} z(x, y) \right) = z(x, y) \quad (10)$$

> restart

RESPUESTA 3)

> Ecuacion := diff(z(x, y), x, 2) - 8 * diff(z(x, y), y) = z(x, y)

$$Ecuacion := \frac{\partial^2}{\partial x^2} z(x, y) - 8 \left(\frac{\partial}{\partial y} z(x, y) \right) = z(x, y) \quad (11)$$

> EcuacionSeparable := simplify(eval(subs(z(x, y) = F(x) * G(y), Ecuacion)))

$$EcuacionSeparable := \left(\frac{d^2}{dx^2} F(x) \right) G(y) - 8 F(x) \left(\frac{d}{dy} G(y) \right) = F(x) G(y) \quad (12)$$

> _C:

ALTERNATIVA UNO

> EcuacionSeparadaUno :=
$$\frac{\left(lhs(EcuacionSeparable) + 8 F(x) \left(\frac{d}{dy} G(y) \right) \right)}{F(x) \cdot G(y)}$$

$$= simplify \left(\frac{\left(rhs(EcuacionSeparable) + 8 F(x) \left(\frac{d}{dy} G(y) \right) \right)}{F(x) \cdot G(y)} \right)$$

$$EcuacionSeparadaUno := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \frac{G(y) + 8 \left(\frac{d}{dy} G(y) \right)}{G(y)} \quad (13)$$

> EcuacionUno_x := lhs(EcuacionSeparadaUno) = alpha; EcuacionUno_y
:= rhs(EcuacionSeparadaUno) = alpha;

$$EcuacionUno_x := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \alpha$$

$$EcuacionUno_y := \frac{G(y) + 8 \left(\frac{d}{dy} G(y) \right)}{G(y)} = \alpha \quad (14)$$

> SolucionUno_x := dsolve(subs(alpha = -6, EcuacionUno_x))

$$SolucionUno_x := F(x) = _C1 \sin(\sqrt{6} x) + _C2 \cos(\sqrt{6} x) \quad (15)$$

> SolucionUno_y := dsolve(subs(alpha = -6, EcuacionUno_y))

$$SolucionUno_y := G(y) = _C1 e^{-\frac{7}{8} y} \quad (16)$$

> SolucionGeneralUno := z(x, y) = subs(_C1 = 1, rhs(SolucionUno_y)) * rhs(SolucionUno_x)

$$SolucionGeneralUno := z(x, y) = e^{-\frac{7}{8} y} (_C1 \sin(\sqrt{6} x) + _C2 \cos(\sqrt{6} x)) \quad (17)$$

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ALTERNATIVA DOS

> EcuacionSeparable;

$$\left(\frac{d^2}{dx^2} F(x) \right) G(y) - 8 F(x) \left(\frac{d}{dy} G(y) \right) = F(x) G(y) \quad (18)$$

> EcuacionSeparadaDos

$$\begin{aligned} &:= \text{simplify} \left(\frac{\left(\text{lhs}(\text{EcuacionSeparable}) + 8 F(x) \left(\frac{d}{dy} G(y) \right) - F(x) \cdot G(y) \right)}{F(x) \cdot G(y)} \right) \\ &= \text{simplify} \left(\frac{\left(\text{rhs}(\text{EcuacionSeparable}) + 8 F(x) \left(\frac{d}{dy} G(y) \right) - F(x) \cdot G(y) \right)}{F(x) \cdot G(y)} \right) \\ \text{EcuacionSeparadaDos} &:= \frac{\frac{d^2}{dx^2} F(x) - F(x)}{F(x)} = \frac{8 \left(\frac{d}{dy} G(y) \right)}{G(y)} \end{aligned} \quad (19)$$

> EcuacionDos_x := lhs(EcuacionSeparadaDos) = alpha; EcuacionDos_y
:= rhs(EcuacionSeparadaDos) = alpha;

$$\begin{aligned} \text{EcuacionDos}_x &:= \frac{\frac{d^2}{dx^2} F(x) - F(x)}{F(x)} = \alpha \\ \text{EcuacionDos}_y &:= \frac{8 \left(\frac{d}{dy} G(y) \right)}{G(y)} = \alpha \end{aligned} \quad (20)$$

> SolucionDos_x := dsolve(subs(alpha=-6, EcuacionDos_x))

$$\text{SolucionDos}_x := F(x) = _C1 \sin(\sqrt{5} x) + _C2 \cos(\sqrt{5} x) \quad (21)$$

> SolucionDos_y := dsolve(subs(alpha=-6, EcuacionDos_y))

$$\text{SolucionDos}_y := G(y) = _C1 e^{-\frac{3}{4} y} \quad (22)$$

> SolucionGeneralDos := z(x, y) = subs(_C1 = 1, rhs(SolucionDos_y)) · rhs(SolucionDos_x)

$$\text{SolucionGeneralDos} := z(x, y) = e^{-\frac{3}{4} y} (_C1 \sin(\sqrt{5} x) + _C2 \cos(\sqrt{5} x)) \quad (23)$$

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COMPROBACIONES

> comprobacionUno := simplify(eval(subs(z(x, y) = rhs(SolucionGeneralUno), lhs(Ecuacion) - rhs(Ecuacion) = 0)))

$$\text{comprobacionUno} := 0 = 0 \quad (24)$$

> comprobacionDos := simplify(eval(subs(z(x, y) = rhs(SolucionGeneralDos), lhs(Ecuacion) - rhs(Ecuacion) = 0)))

$$\text{comprobacionDos} := 0 = 0 \quad (25)$$

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FIN RESPUESTA 3)

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FIN EXAMEN