>

SOLUCIÓN

>

ECUACIONES DIFERENCIALES
PRIMER EXAMEN FINAL COLEGIADO
SEMESTRE 2011-2

2011 JUNIO 02
TIPO "A"

> *restart*

1) Resuelva la ecuación diferencial

$$x\sin(y(x)) + x\,y(x)\sin(x) + \left(\frac{1}{2}x^2\cos(y(x)) - x\cos(x) + \sin(x)\right)\left(\frac{d}{dx}y(x)\right) = 0 \qquad (1)$$

> _C:

RESPUESTA 1)

> $Ecuacion := x\cdot\sin(y(x)) + x\cdot y(x)\cdot\sin(x) + \left(\frac{x\cdot\cdot 2}{2}\cdot\cos(y(x)) - x\cdot\cos(x) + \sin(x)\right)\cdot diff(y(x), x) = 0$

$$Ecuacion := x\sin(y(x)) + x\,y(x)\sin(x) + \left(\frac{1}{2}x^2\cos(y(x)) - x\cos(x) + \sin(x)\right)\left(\frac{d}{dx}y(x)\right) = 0 \qquad (2)$$

> *with*(*DEtools*) :

> *odeadvisor*(*Ecuacion*)

$$[\_exact] \qquad (3)$$

> $M(x, y) := x\cdot\sin(y) + x\cdot y\cdot\sin(x); N(x, y) := \frac{x\cdot\cdot 2}{2}\cdot\cos(y) - x\cdot\cos(x) + \sin(x)$

$$M(x, y) := x\sin(y) + x\,y\sin(x)$$
$$N(x, y) := \frac{1}{2}x^2\cos(y) - x\cos(x) + \sin(x) \qquad (4)$$

> $comprobacion1 := simplify(diff(M(x, y), y) - diff(N(x, y), x) = 0)$

$$comprobacion1 := 0 = 0 \qquad (5)$$

> $IM_x := int(M(x, y), x)$

$$IM_x := \frac{1}{2}x^2\sin(y) + y(\sin(x) - x\cos(x)) \qquad (6)$$

> $SolucionGeneral := IM_x + int((N(x, y) - diff(IM_x, y)), y) = \_C1$

$$SolucionGeneral := \frac{1}{2}x^2\sin(y) + y(\sin(x) - x\cos(x)) = \_C1 \qquad (7)$$

>

COMPROBACIÓN

> $Solucion := \frac{1}{2}x^2\sin(y(x)) + y(x)\cdot(\sin(x) - x\cos(x)) = \_C1$

$$Solucion := \frac{1}{2}x^2\sin(y(x)) + y(x)(\sin(x) - x\cos(x)) = \_C1 \qquad (8)$$

> $Derivada1 := isolate(diff(Solucion, x), diff(y(x), x))$

(9)

$$Derivada1 := \frac{d}{dx} y(x) = \frac{-x \sin(y(x)) - x\, y(x) \sin(x)}{\frac{1}{2} x^2 \cos(y(x)) - x \cos(x) + \sin(x)} \quad (9)$$

> Derivada2 := isolate(Ecuacion, diff(y(x), x))

$$Derivada2 := \frac{d}{dx} y(x) = \frac{-x \sin(y(x)) - x\, y(x) \sin(x)}{\frac{1}{2} x^2 \cos(y(x)) - x \cos(x) + \sin(x)} \quad (10)$$

> comprobacion2 := simplify(rhs(Derivada1) − rhs(Derivada2) = 0)

$$comprobacion2 := 0 = 0 \quad (11)$$

>

FIN RESPUESTA 1)

> restart

2) Resuelva la ecuación diferencial

$$\frac{d^3}{dx^3} y(x) - \left(\frac{d}{dx} y(x)\right) = \frac{1}{2} e^x - \frac{1}{2} \quad (12)$$

>

RESPUESTA 2)

> Ecuacion := $\frac{d^3}{dx^3} y(x) - \left(\frac{d}{dx} y(x)\right) = \frac{1}{2} e^x - \frac{1}{2}$

$$Ecuacion := \frac{d^3}{dx^3} y(x) - \left(\frac{d}{dx} y(x)\right) = \frac{1}{2} e^x - \frac{1}{2} \quad (13)$$

> SolucionGeneral := dsolve(Ecuacion)

$$SolucionGeneral := y(x) = e^x \_C2 - e^{-x} \_C1 + \frac{1}{4} e^x x - \frac{3}{8} e^x + \frac{1}{2} x + \_C3 \quad (14)$$

>

COMPROBACIÓN

> comprobacion1 := simplify(eval(subs(y(x) = rhs(SolucionGeneral), lhs(Ecuacion) − rhs(Ecuacion) = 0)))

$$comprobacion1 := 0 = 0 \quad (15)$$

>

FIN RESPUESTA 2)

> restart

3) Resuelva la ecuacion diferencial (por el método de Parámetros Variables)

$$\frac{d^2}{dx^2} y(x) + y(x) = \frac{1}{\cos(x)} \quad (16)$$

> _C:

RESPUESTA 3)

> EcuacionNoHom := $\frac{d^2}{dx^2} y(x) + y(x) = \frac{1}{\cos(x)}$

$$EcuacionNoHom := \frac{d^2}{dx^2} y(x) + y(x) = \frac{1}{\cos(x)} \quad (17)$$

> EcuacionHom := lhs(EcuacionNoHom) = 0

$$EcuacionHom := \frac{d^2}{dx^2} y(x) + y(x) = 0 \quad \textbf{(18)}$$

> $Q(x) := rhs(EcuacionNoHom);$

$$Q(x) := \frac{1}{\cos(x)} \quad \textbf{(19)}$$

> $EcuacionCaracteristica := m \cdot \cdot 2 + 1 = 0$

$$EcuacionCaracteristica := m^2 + 1 = 0 \quad \textbf{(20)}$$

> $Raiz := solve(EcuacionCaracteristica)$

$$Raiz := I, -I \quad \textbf{(21)}$$

> $Solucion1 := y(x) = \cos(\mathrm{Im}(Raiz_1) \cdot x); Solucion2 := y(x) = \sin(\mathrm{Im}(Raiz_1) \cdot x)$

$$Solucion1 := y(x) = \cos(x)$$

$$Solucion2 := y(x) = \sin(x) \quad \textbf{(22)}$$

> $SolucionHom := y(x) = \_C1 \cdot rhs(Solucion1) + \_C2 \cdot rhs(Solucion2)$

$$SolucionHom := y(x) = \_C1 \cos(x) + \_C2 \sin(x) \quad \textbf{(23)}$$

> $SolucionNoHom := y(x) = A(x) \cdot rhs(Solucion1) + B(x) \cdot rhs(Solucion2)$

$$SolucionNoHom := y(x) = A(x) \cos(x) + B(x) \sin(x) \quad \textbf{(24)}$$

> $WW := array([[rhs(Solucion1), rhs(Solucion2)], [rhs(diff(Solucion1, x)), rhs(diff(Solucion2, x))]])$

$$WW := \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix} \quad \textbf{(25)}$$

> $BB := array([0, Q(x)])$

$$BB := \begin{bmatrix} 0 & \frac{1}{\cos(x)} \end{bmatrix} \quad \textbf{(26)}$$

> $with(linalg):$

> $SOL := linsolve(WW, BB)$

$$SOL := \begin{bmatrix} -\frac{\sin(x)}{\cos(x)\,(\sin(x)^2 + \cos(x)^2)} & \frac{1}{\sin(x)^2 + \cos(x)^2} \end{bmatrix} \quad \textbf{(27)}$$

> $Aprima := SOL_1; Bprima := SOL_2;$

$$Aprima := -\frac{\sin(x)}{\cos(x)\,(\sin(x)^2 + \cos(x)^2)}$$

$$Bprima := \frac{1}{\sin(x)^2 + \cos(x)^2} \quad \textbf{(28)}$$

> $A(x) := int(Aprima, x) + \_C1; B(x) := int(Bprima, x) + \_C2;$

$$A(x) := \ln(\cos(x)) + \_C1$$

$$B(x) := x + \_C2 \quad \textbf{(29)}$$

> $simplify(SolucionNoHom);$

$$y(x) = \cos(x) \ln(\cos(x)) + \_C1 \cos(x) + \sin(x)\, x + \_C2 \sin(x) \quad \textbf{(30)}$$

>

COMPROBACIÓN

> $comprobacion1 := simplify(eval(subs(y(x) = rhs(SolucionNoHom), lhs(EcuacionNoHom) - rhs(EcuacionNoHom) = 0)))$

$$comprobacion1 := 0 = 0 \quad (31)$$

>

FIN RESPUESTA 3)

> *restart*

4) Obtenga la solución del sistema de ecuaciones diferenciales

$$\frac{d}{dt} x(t) = x(t) + 2\,y(t) + e^{-t}$$
$$\frac{d}{dt} y(t) = 2\,x(t) + y(t) \quad (32)$$

sujeta a las condiciones iniciales

$$x(0) = 0$$
$$y(0) = 1 \quad (33)$$

> *restart*

RESPUESTA 4)

> $Sistema := \frac{d}{dt} x(t) = x(t) + 2\,y(t) + e^{-t}, \frac{d}{dt} y(t) = 2\,x(t) + y(t)$

$$Sistema := \frac{d}{dt} x(t) = x(t) + 2\,y(t) + e^{-t}, \frac{d}{dt} y(t) = 2\,x(t) + y(t) \quad (34)$$

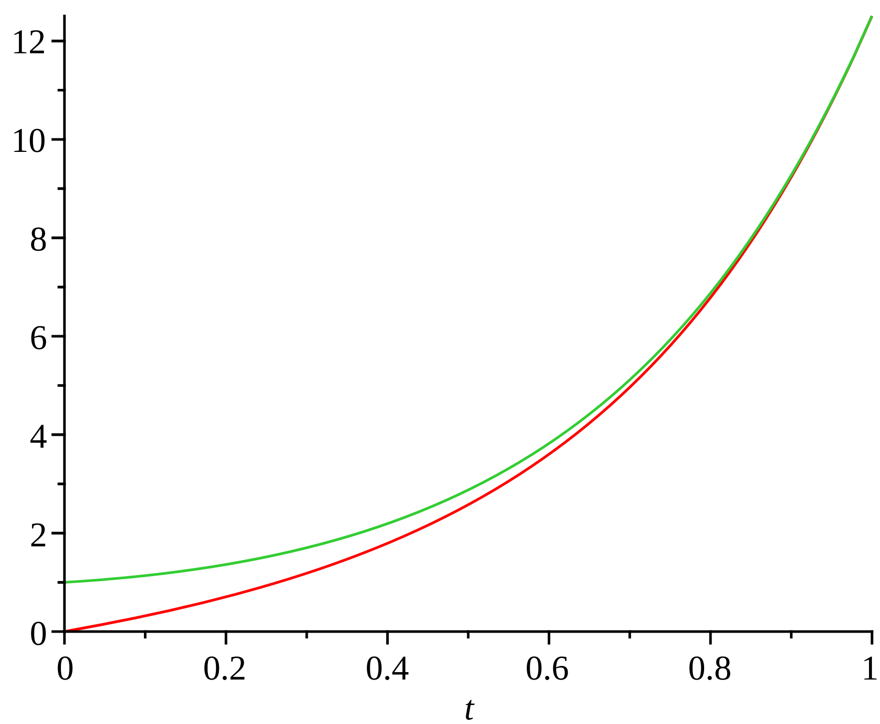
> $Condiciones := x(0) = 0, y(0) = 1$

$$Condiciones := x(0) = 0, y(0) = 1 \quad (35)$$

> $Solucion := dsolve(\{Sistema, Condiciones\})$

$$Solucion := \left\{x(t) = \frac{5}{8}\,e^{3t} - \frac{5}{8}\,e^{-t} + \frac{1}{2}\,t\,e^{-t}, y(t) = \frac{5}{8}\,e^{3t} + \frac{3}{8}\,e^{-t} - \frac{1}{2}\,t\,e^{-t}\right\} \quad (36)$$

> $plot([rhs(Solucion_1), rhs(Solucion_2)], t = 0 ..1)$




>

COMPROBACIONES

> $comprobacion1 := simplify(eval(subs(x(t) = rhs(Solucion_1), y(t) = rhs(Solucion_2), lhs(Sistema_1) - rhs(Sistema_1) = 0)))$

$$comprobacion1 := 0 = 0 \tag{37}$$

> $comprobacion2 := simplify(eval(subs(x(t) = rhs(Solucion_1), y(t) = rhs(Solucion_2), lhs(Sistema_2) - rhs(Sistema_2) = 0)))$

$$comprobacion2 := 0 = 0 \tag{38}$$

>

FIN RESPUESTA 4)

> *restart*

5) Utilice el teorema de convolución para obtener la transformada inversa de Laplace de

$$\frac{1}{s\,(s^2+9)} \tag{39}$$

>

RESPUESTA 5)

> $F(s) := \frac{1}{s};\ G(s) := \frac{1}{s\cdot\cdot 2+9}$

$$F(s) := \frac{1}{s}$$

$$G(s) := \frac{1}{s^2+9} \tag{40}$$

> $H(s) := F(s)\cdot G(s)$

$$H(s) := \frac{1}{s\,(s^2+9)} \tag{41}$$

> $with(inttrans):$

> $f(t) := invlaplace(F(s), s, t);\ g(t) := invlaplace(G(s), s, t)$

$$f(t) := 1$$

$$g(t) := \frac{1}{3}\sin(3\,t) \tag{42}$$

> $h(t) := int((subs(t = t - \mathrm{tau}, f(t))\cdot subs(t = \mathrm{tau}, g(t))), \mathrm{tau} = 0..t)$

$$h(t) := \frac{1}{9} - \frac{1}{9}\cos(3\,t) \tag{43}$$

> $hh(t) := int((subs(t = \mathrm{tau}, f(t))\cdot subs(t = t - \mathrm{tau}, g(t))), \mathrm{tau} = 0..t)$

$$hh(t) := \frac{1}{9} - \frac{1}{9}\cos(3\,t) \tag{44}$$

>

COMPROBACION

> $hhh(t) := invlaplace(H(s), s, t)$

$$hhh(t) := \frac{1}{9} - \frac{1}{9}\cos(3\,t) \tag{45}$$

>

FIN RESPUESTA 5)

> *restart*

6) Resuelva el problema del valor inicial

$$\frac{d^2}{dt^2}\, y(t) - y(t) = \mathrm{Dirac}(t - 2\pi)$$
$$y(0) = 1$$
$$D(y)(0) = 0 \qquad \textbf{(46)}$$

>

RESPUESTA 6)

> $Ecuacion := \frac{d^2}{dt^2}\, y(t) - y(t) = \mathrm{Dirac}(t - 2\pi)$

$$Ecuacion := \frac{d^2}{dt^2}\, y(t) - y(t) = \mathrm{Dirac}(t - 2\pi) \qquad \textbf{(47)}$$

> $Condiciones := y(0) = 1, D(y)(0) = 0;$

$$Condiciones := y(0) = 1, D(y)(0) = 0 \qquad \textbf{(48)}$$

> $with(inttrans):$

> $TLecuacion := simplify(subs(Condiciones, laplace(Ecuacion, t, s)))$

$$TLecuacion := s^2\, laplace(y(t), t, s) - s - laplace(y(t), t, s) = e^{-2 s \pi} \qquad \textbf{(49)}$$

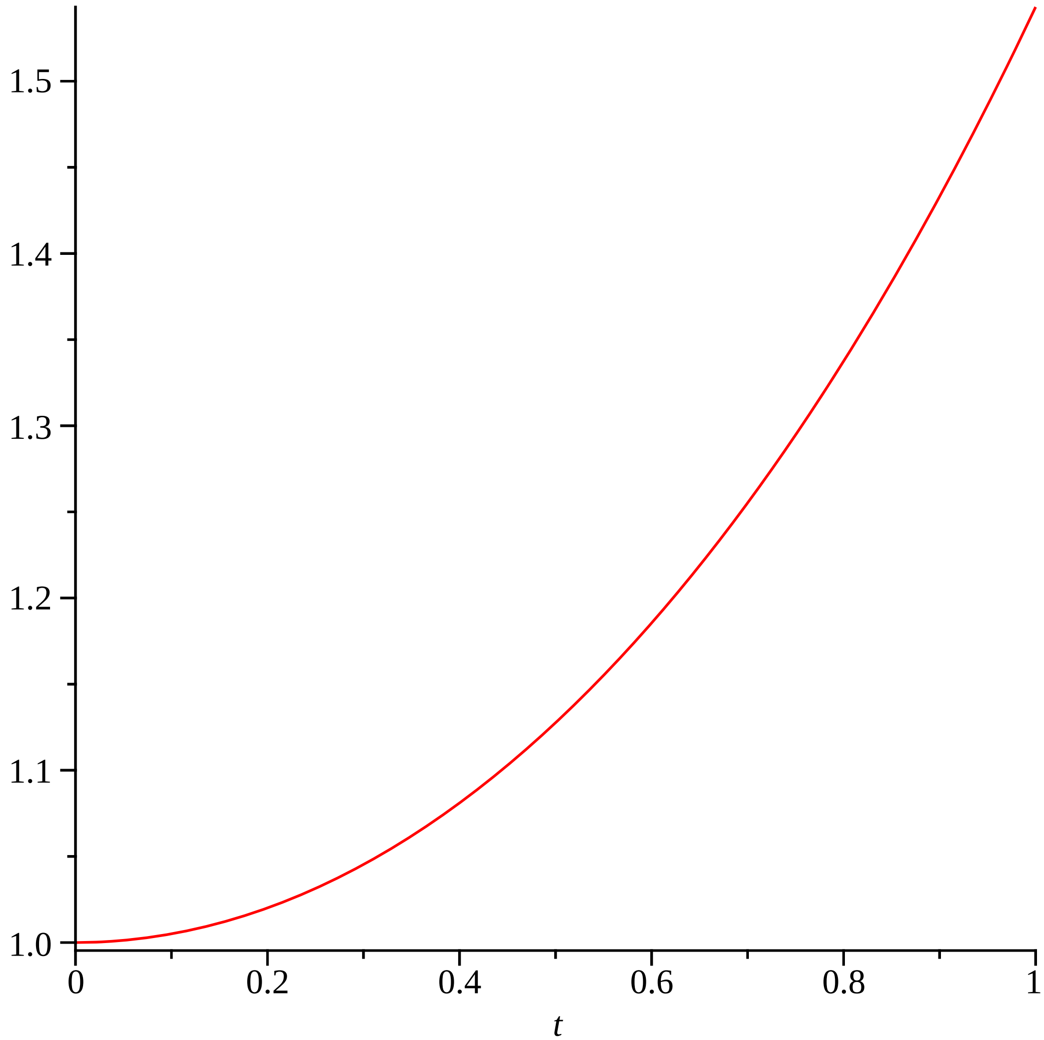
> $TLsolucion := isolate(TLecuacion, laplace(y(t), t, s))$

$$TLsolucion := laplace(y(t), t, s) = \frac{e^{-2 s \pi} + s}{s^2 - 1} \qquad \textbf{(50)}$$

> $Solucion := invlaplace(TLsolucion, s, t)$

$$Solucion := y(t) = \mathrm{Heaviside}(t - 2\pi)\, \sinh(t - 2\pi) + \cosh(t) \qquad \textbf{(51)}$$

> $plot(rhs(Solucion), t = 0..1)$




>

COMPROBACIÓN

> $comprobacion1 := simplify(eval(subs(y(t) = rhs(Solucion), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$comprobacion1 := 0 = 0 \quad (52)$$

>

FIN RESPUESTA 6)

> *restart*

7) Obtenga la Serie Trigonométrica de Fourier de la función en el intervalo -Pi < x < Pi

$$f(x) := x + \pi \quad (53)$$

>

RESPUESTA 7)

> $f(x) := x + \pi$

$$f(x) := x + \pi \quad (54)$$

> $L := \text{Pi}$

$$L := \pi \quad (55)$$

$$> a_0 := \left(\frac{1}{L}\right) \cdot int(f(x), x = -L..L)$$

$$a_0 := 2\pi \qquad (56)$$

$$> C := \frac{a_0}{2}$$

$$C := \pi \qquad (57)$$

$$> a_n := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^{\wedge}n, \left(\frac{1}{L}\right) \cdot int\left(f(x) \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), x = -L..L\right)\right)$$

$$a_n := 0 \qquad (58)$$

$$> b_n := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^{\wedge}n, \left(\frac{1}{L}\right) \cdot int\left(f(x) \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), x = -L..L\right)\right)$$

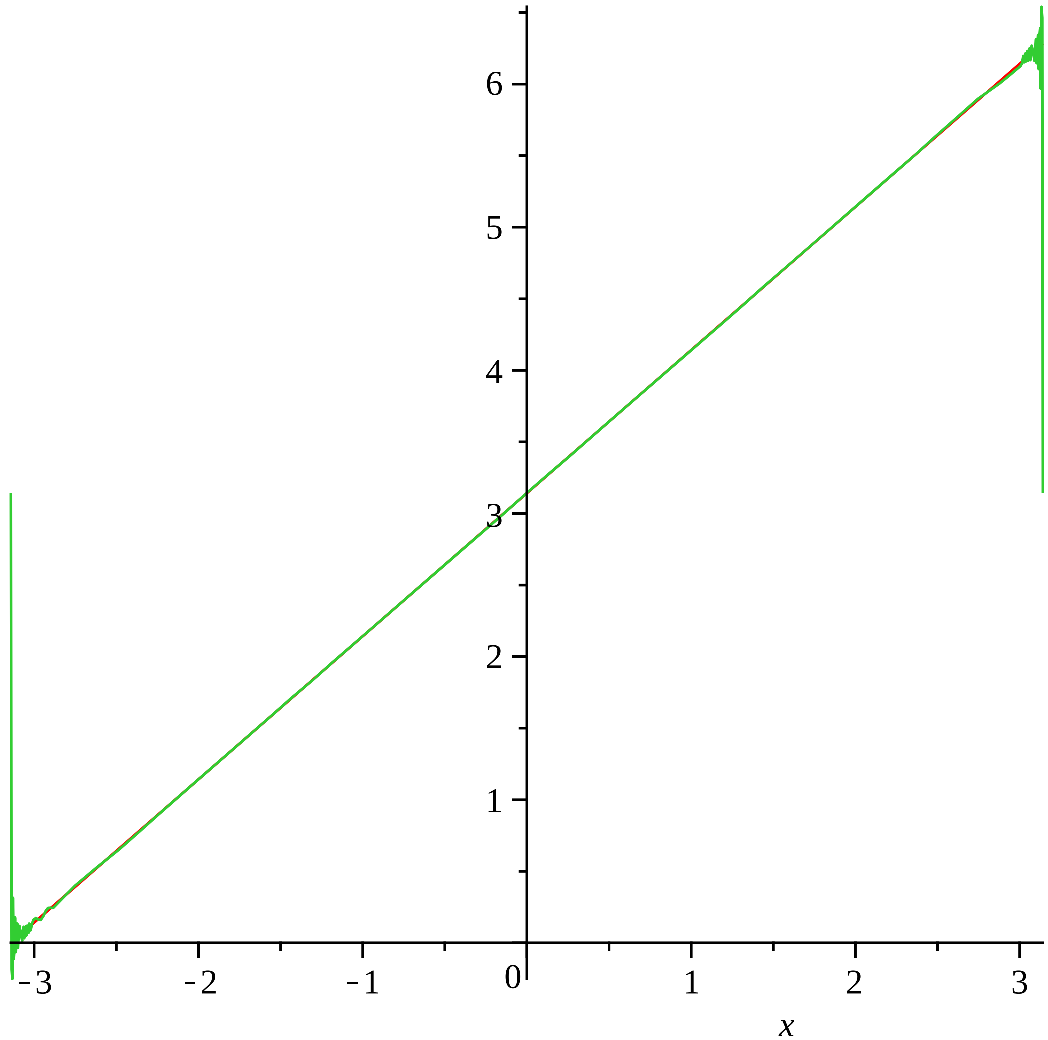
$$b_n := -\frac{2(-1)^n}{n} \qquad (59)$$

$$> STF := C + Sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right) + b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), n = 1..infinity\right)$$

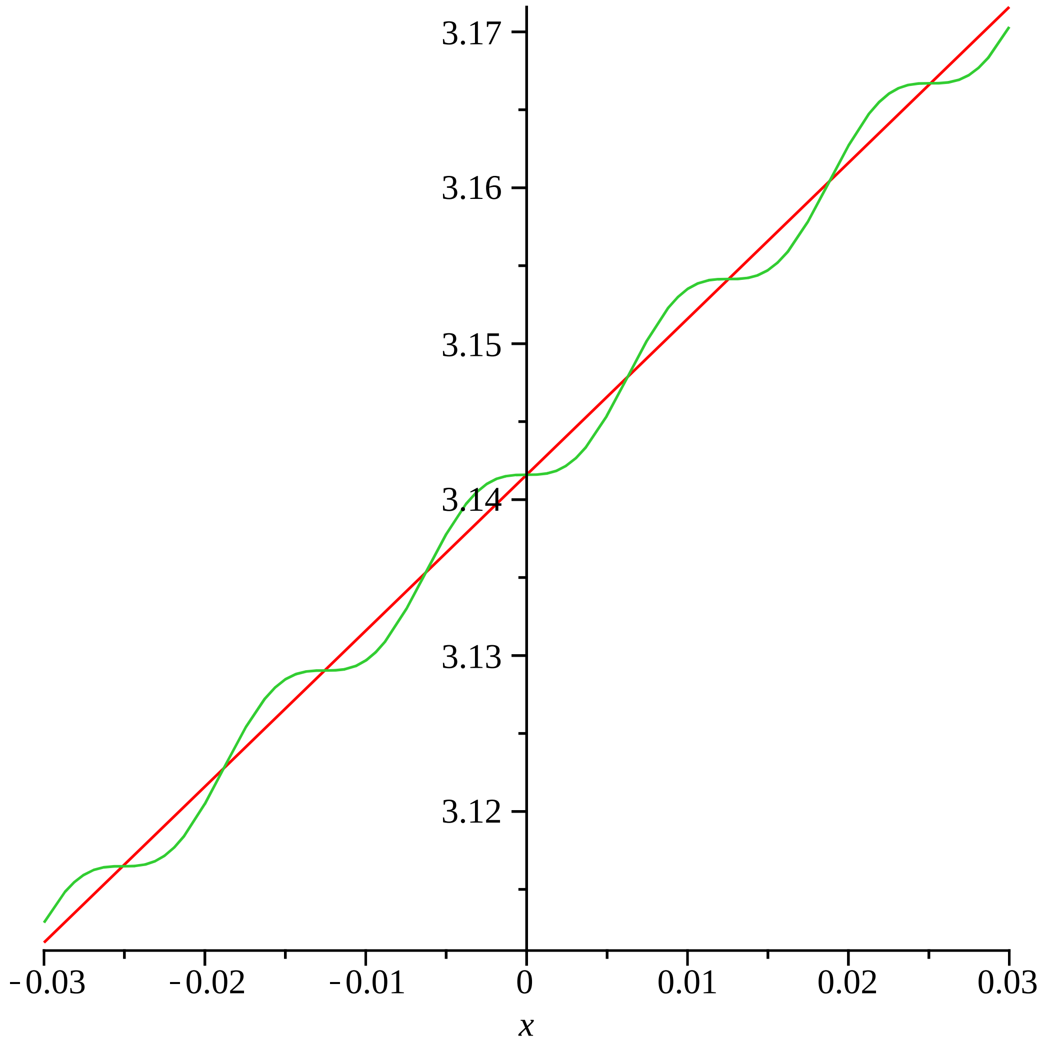
$$STF := \pi + \sum_{n=1}^{\infty}\left(-\frac{2(-1)^n \sin(n\,x)}{n}\right) \qquad (60)$$

$$> STF_{500} := C + Sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right) + b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), n = 1..500\right) :$$

$$> plot([f(x), STF_{500}], x = -L..L)$$




> $plot([f(x), STF_{500}], x = -0.03..0.03)$




>

FIN RESPUESTA 7)

>

FIN EXAMEN

>