>

SOLUCIÓN

>

ECUACIONES DIFERENCIALES
PRIMER EXAMEN FINAL COLEGIADO
SEMESTRE 2011-2

2011 JUNIO 02
TIPO "B"

> *restart*

1) Resuelva la ecuación diferencial

$$\frac{1}{2}\, y(x)^2 \cos(x) - y(x)\, \cos(y(x)) + \sin(y(x)) + (y(x)\, \sin(x) + x\, y(x)\, \sin(y(x)))\left(\frac{\mathrm{d}}{\mathrm{d}x}\, y(x)\right) = 0 \quad (1)$$

> _C:

RESPUESTA 1)

> $Ecuacion := \frac{1}{2}\, y(x)^2 \cos(x) - y(x)\, \cos(y(x)) + \sin(y(x)) + (y(x)\, \sin(x) + x\, y(x)\, \sin(y(x)))\left(\frac{\mathrm{d}}{\mathrm{d}x}\, y(x)\right) = 0$

$$Ecuacion := \frac{1}{2}\, y(x)^2 \cos(x) - y(x)\, \cos(y(x)) + \sin(y(x)) + (y(x)\, \sin(x) + x\, y(x)\, \sin(y(x)))\left(\frac{\mathrm{d}}{\mathrm{d}x}\, y(x)\right) = 0 \quad (2)$$

> *with*(*DEtools*) :

> *odeadvisor*(*Ecuacion*)

$$[\_exact, \_dAlembert] \quad (3)$$

> $M(x, y) := \frac{1}{2}\, y^2 \cos(x) - y \cdot \cos(y) + \sin(y); N(x, y) := y \cdot \sin(x) + x \cdot y \cdot \sin(y)$

$$M(x, y) := \frac{1}{2}\, y^2 \cos(x) - y\cos(y) + \sin(y)$$

$$N(x, y) := y\sin(x) + x\, y\sin(y) \quad (4)$$

> $comprobacion1 := simplify(diff(M(x, y), y) - diff(N(x, y), x) = 0)$

$$comprobacion1 := 0 = 0 \quad (5)$$

> $IM_x := int(M(x, y), x)$

$$IM_x := \frac{1}{2}\, y^2 \sin(x) - y\cos(y)\, x + \sin(y)\, x \quad (6)$$

> $SolucionGeneral := IM_x + int\left(\left(N(x, y) - diff\left(IM_x, y\right)\right), y\right) = \_C1$

$$SolucionGeneral := \frac{1}{2}\, y^2 \sin(x) - y\cos(y)\, x + \sin(y)\, x = \_C1 \quad (7)$$

>

COMPROBACIÓN

> $Solucion := \frac{1}{2}\, y(x)^2 \sin(x) - y(x) \cdot \cos(y(x))\, x + \sin(y(x))\, x = \_C1$

(8)

$$Solucion := \frac{1}{2}\, y(x)^2 \sin(x) - y(x)\, \cos(y(x))\, x + \sin(y(x))\, x = \_C1 \quad (8)$$

> Derivada1 := isolate(diff(Solucion, x), diff(y(x), x))

$$Derivada1 := \frac{d}{dx}\, y(x) = \frac{-\frac{1}{2}\, y(x)^2 \cos(x) + y(x)\, \cos(y(x)) - \sin(y(x))}{y(x)\, \sin(x) + x\, y(x)\, \sin(y(x))} \quad (9)$$

> Derivada2 := isolate(Ecuacion, diff(y(x), x))

$$Derivada2 := \frac{d}{dx}\, y(x) = \frac{-\frac{1}{2}\, y(x)^2 \cos(x) + y(x)\, \cos(y(x)) - \sin(y(x))}{y(x)\, \sin(x) + x\, y(x)\, \sin(y(x))} \quad (10)$$

> comprobacion2 := simplify(rhs(Derivada1) − rhs(Derivada2) = 0)

$$comprobacion2 := 0 = 0 \quad (11)$$

>

FIN RESPUESTA 1)

> restart

2) Resuelva la ecuación diferencial

$$\frac{d^3}{dx^3}\, y(x) - \left(\frac{d}{dx}\, y(x)\right) = \frac{1}{2}\, e^{-x} - \frac{1}{2} \quad (12)$$

RESPUESTA 2)

> Ecuacion := $\frac{d^3}{dx^3}\, y(x) - \left(\frac{d}{dx}\, y(x)\right) = \frac{1}{2}\, e^{-x} - \frac{1}{2}$

$$Ecuacion := \frac{d^3}{dx^3}\, y(x) - \left(\frac{d}{dx}\, y(x)\right) = \frac{1}{2}\, e^{-x} - \frac{1}{2} \quad (13)$$

> SolucionGeneral := dsolve(Ecuacion)

$$SolucionGeneral := y(x) = -e^{-x}\, \_C2 + e^{x}\, \_C1 + \frac{1}{2}\, x + \frac{1}{4}\, e^{-x}\, x + \frac{3}{8}\, e^{-x} + \_C3 \quad (14)$$

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COMPROBACIÓN

> comprobacion1 := simplify(eval(subs(y(x) = rhs(SolucionGeneral), lhs(Ecuacion)) − rhs(Ecuacion) = 0)))

$$comprobacion1 := 0 = 0 \quad (15)$$

>

FIN RESPUESTA 2)

> restart

3) Resuelva la ecuacion diferencial (<u>por el método de Parámetros Variables</u>)

$$\frac{d^2}{dx^2}\, y(x) + y(x) = \frac{1}{\sin(x)} \quad (16)$$

> **_C:**

RESPUESTA 3)

> EcuacionNoHom := $\frac{d^2}{dx^2}\, y(x) + y(x) = \frac{1}{\sin(x)}$

(17)

$$EcuacionNoHom := \frac{d^2}{dx^2} y(x) + y(x) = \frac{1}{\sin(x)} \quad \textbf{(17)}$$

> $EcuacionHom := lhs(EcuacionNoHom) = 0$

$$EcuacionHom := \frac{d^2}{dx^2} y(x) + y(x) = 0 \quad \textbf{(18)}$$

> $Q(x) := rhs(EcuacionNoHom);$

$$Q(x) := \frac{1}{\sin(x)} \quad \textbf{(19)}$$

> $EcuacionCaracteristica := m \cdot \cdot 2 + 1 = 0$

$$EcuacionCaracteristica := m^2 + 1 = 0 \quad \textbf{(20)}$$

> $Raiz := solve(EcuacionCaracteristica)$

$$Raiz := I, -I \quad \textbf{(21)}$$

> $Solucion1 := y(x) = \cos(\mathrm{Im}(Raiz_1) \cdot x); Solucion2 := y(x) = \sin(\mathrm{Im}(Raiz_1) \cdot x)$

$$Solucion1 := y(x) = \cos(x)$$

$$Solucion2 := y(x) = \sin(x) \quad \textbf{(22)}$$

> $SolucionHom := y(x) = \_C1 \cdot rhs(Solucion1) + \_C2 \cdot rhs(Solucion2)$

$$SolucionHom := y(x) = \_C1 \cos(x) + \_C2 \sin(x) \quad \textbf{(23)}$$

> $SolucionNoHom := y(x) = A(x) \cdot rhs(Solucion1) + B(x) \cdot rhs(Solucion2)$

$$SolucionNoHom := y(x) = A(x) \cos(x) + B(x) \sin(x) \quad \textbf{(24)}$$

> $WW := array([[rhs(Solucion1), rhs(Solucion2)], [rhs(diff(Solucion1, x)), rhs(diff(Solucion2, x))]])$

$$WW := \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix} \quad \textbf{(25)}$$

> $BB := array([0, Q(x)])$

$$BB := \begin{bmatrix} 0 & \frac{1}{\sin(x)} \end{bmatrix} \quad \textbf{(26)}$$

> $with(linalg):$

> $SOL := linsolve(WW, BB)$

$$SOL := \begin{bmatrix} -\frac{1}{\sin(x)^2 + \cos(x)^2} & \frac{\cos(x)}{\sin(x)\,(\sin(x)^2 + \cos(x)^2)} \end{bmatrix} \quad \textbf{(27)}$$

> $Aprima := SOL_1; Bprima := SOL_2;$

$$Aprima := -\frac{1}{\sin(x)^2 + \cos(x)^2}$$

$$Bprima := \frac{\cos(x)}{\sin(x)\,(\sin(x)^2 + \cos(x)^2)} \quad \textbf{(28)}$$

> $A(x) := int(Aprima, x) + \_C1; B(x) := int(Bprima, x) + \_C2;$

$$A(x) := -x + \_C1$$

$$B(x) := \ln(\sin(x)) + \_C2 \quad \textbf{(29)}$$

> $simplify(SolucionNoHom);$

$$y(x) = -\cos(x)\, x + \_C1 \cos(x) + \sin(x) \ln(\sin(x)) + \_C2 \sin(x) \quad \textbf{(30)}$$

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COMPROBACIÓN

> $comprobacion1 := simplify(eval(subs(y(x) = rhs(SolucionNoHom), lhs(EcuacionNoHom) - rhs(EcuacionNoHom) = 0)))$

$$comprobacion1 := 0 = 0 \quad \textbf{(31)}$$

>

FIN RESPUESTA 3)

> restart

4) Obtenga la solución del sistema de ecuaciones diferenciales

$$\frac{d}{dt} x(t) = x(t) + 2\, y(t) + e^{-t}$$
$$\frac{d}{dt} y(t) = 2\, x(t) + y(t) \quad \textbf{(32)}$$

sujeta a las condiciones iniciales

$$x(0) = 0$$
$$y(0) = 1 \quad \textbf{(33)}$$

> restart

RESPUESTA 4)

> $Sistema := \frac{d}{dt} x(t) = x(t) + 2\, y(t) + e^{-t}, \frac{d}{dt} y(t) = 2\, x(t) + y(t)$

$$Sistema := \frac{d}{dt} x(t) = x(t) + 2\, y(t) + e^{-t}, \frac{d}{dt} y(t) = 2\, x(t) + y(t) \quad \textbf{(34)}$$

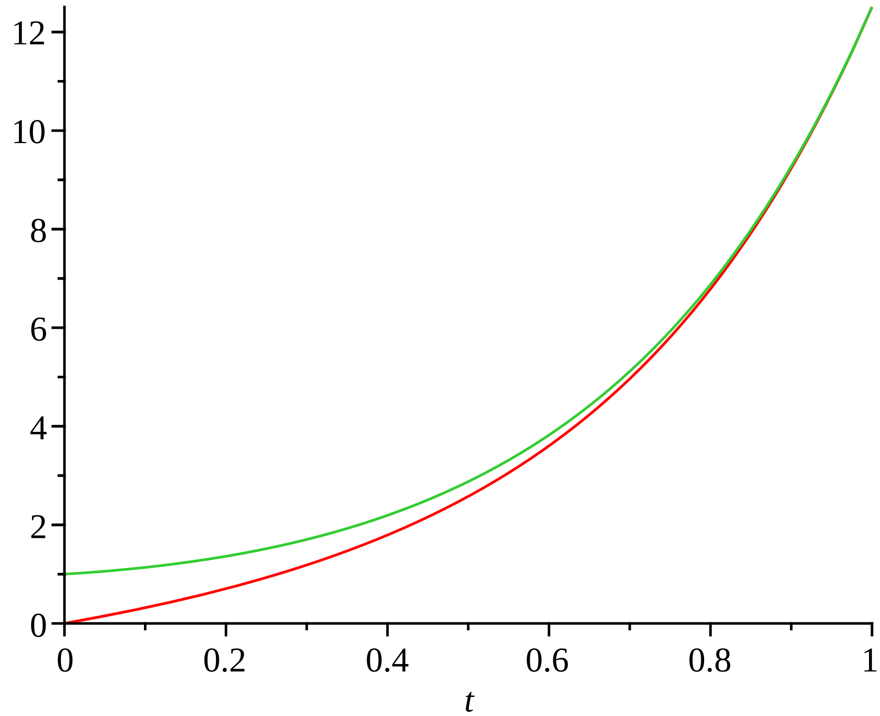
> $Condiciones := x(0) = 0, y(0) = 1$

$$Condiciones := x(0) = 0, y(0) = 1 \quad \textbf{(35)}$$

> $Solucion := dsolve(\{Sistema, Condiciones\})$

$$Solucion := \left\{x(t) = -\frac{5}{8}\, e^{-t} + \frac{5}{8}\, e^{3t} + \frac{1}{2}\, t\, e^{-t}, y(t) = \frac{3}{8}\, e^{-t} + \frac{5}{8}\, e^{3t} - \frac{1}{2}\, t\, e^{-t}\right\} \quad \textbf{(36)}$$

> $plot([rhs(Solucion_1), rhs(Solucion_2)], t = 0..1)$



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COMPROBACIONES

> $comprobacion1 := simplify\left(eval\left(subs\left(x(t) = rhs\left(Solucion_1\right), y(t) = rhs\left(Solucion_2\right), lhs\left(Sistema_1\right) - rhs\left(Sistema_1\right) = 0\right)\right)\right)$

$$comprobacion1 := 0 = 0 \tag{37}$$

> $comprobacion2 := simplify\left(eval\left(subs\left(x(t) = rhs\left(Solucion_1\right), y(t) = rhs\left(Solucion_2\right), lhs\left(Sistema_2\right) - rhs\left(Sistema_2\right) = 0\right)\right)\right)$

$$comprobacion2 := 0 = 0 \tag{38}$$

>

FIN RESPUESTA 4)

> *restart*

5) Utilice el teorema de convolución para obtener la transformada inversa de Laplace de

$$\frac{1}{s\left(s^2+4\right)} \tag{39}$$

RESPUESTA 5)

> $F(s) := \frac{1}{s}; G(s) := \frac{1}{s \cdot \cdot 2 + 4}$

$$F(s) := \frac{1}{s}$$

$$G(s) := \frac{1}{s^2+4} \tag{40}$$

> $H(s) := F(s) \cdot G(s)$

$$H(s) := \frac{1}{s\left(s^2+4\right)} \tag{41}$$

> $with(inttrans):$

> $f(t) := invlaplace(F(s), s, t); g(t) := invlaplace(G(s), s, t)$

$$f(t) := 1$$

$$g(t) := \frac{1}{2}\sin(2\,t) \tag{42}$$

> $h(t) := int((subs(t = t - \tau, f(t)) \cdot subs(t = \tau, g(t))), \tau = 0..t)$

$$h(t) := \frac{1}{4} - \frac{1}{4}\cos(2\,t) \tag{43}$$

> $hh(t) := int((subs(t = \tau, f(t)) \cdot subs(t = t - \tau, g(t))), \tau = 0..t)$

$$hh(t) := \frac{1}{4} - \frac{1}{4}\cos(2\,t) \tag{44}$$

>

COMPROBACION

> $hhh(t) := invlaplace(H(s), s, t)$

$$hhh(t) := \frac{1}{4} - \frac{1}{4}\cos(2\,t) \tag{45}$$

>

FIN RESPUESTA 5)

> *restart*

6) Resuelva el problema del valor inicial

$$\frac{d^2}{dt^2}y(t) - y(t) = \mathrm{Dirac}(t - 3\,\pi)$$

$$y(0) = 1$$

$$\mathrm{D}(y)(0) = 0 \tag{46}$$

RESPUESTA 6)

> $Ecuacion := \frac{d^2}{dt^2}y(t) - y(t) = \mathrm{Dirac}(t - 3\,\pi)$

$$Ecuacion := \frac{d^2}{dt^2}y(t) - y(t) = \mathrm{Dirac}(t - 3\,\pi) \tag{47}$$

> $Condiciones := y(0) = 1, \mathrm{D}(y)(0) = 0;$

$$Condiciones := y(0) = 1, \mathrm{D}(y)(0) = 0 \tag{48}$$

> $with(inttrans):$

> $TLecuacion := simplify(subs(Condiciones, laplace(Ecuacion, t, s)))$

$$TLecuacion := s^2\,laplace(y(t), t, s) - s - laplace(y(t), t, s) = \mathrm{e}^{-3\,s\,\pi} \tag{49}$$

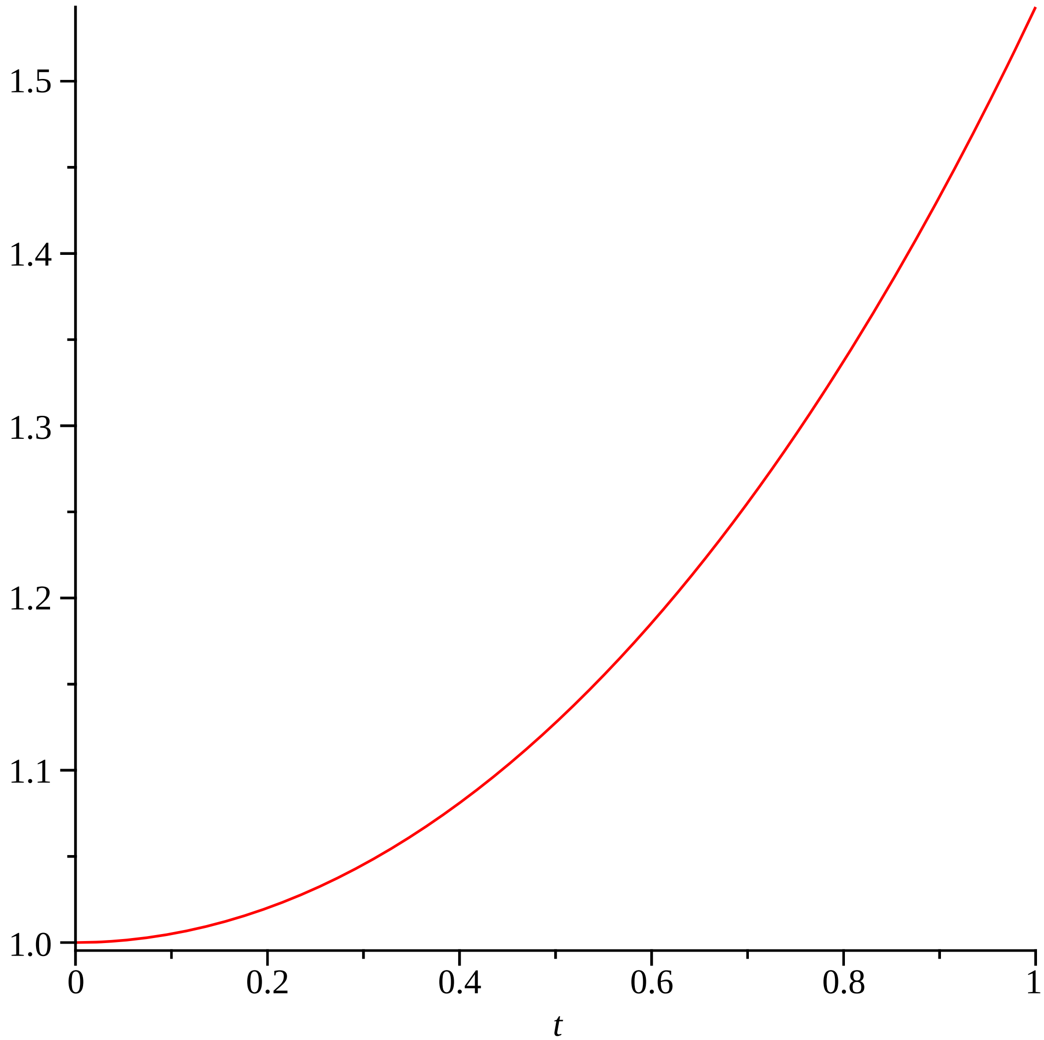
> $TLsolucion := isolate(TLecuacion, laplace(y(t), t, s))$

$$TLsolucion := laplace(y(t), t, s) = \frac{\mathrm{e}^{-3\,s\,\pi} + s}{s^2 - 1} \tag{50}$$

> $Solucion := invlaplace(TLsolucion, s, t)$

$$Solucion := y(t) = \mathrm{Heaviside}(t - 3\,\pi)\,\sinh(t - 3\,\pi) + \cosh(t) \tag{51}$$

> $plot(rhs(Solucion), t = 0..1)$



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COMPROBACIÓN

> $comprobacion1 := simplify(eval(subs(y(t) = rhs(Solucion), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$comprobacion1 := 0 = 0 \quad (52)$$

>

FIN RESPUESTA 6)

> *restart*

7) Obtenga la Serie Trigonométrica de Fourier de la función en el intervalo -Pi < x < Pi

$$f(x) := x + \pi \quad (53)$$

>

RESPUESTA 7)

> $f(x) := x + \pi$

$$f(x) := x + \pi \quad (54)$$

> $L :=$ Pi

$$L := \pi \quad (55)$$

> $a_0 := \left(\frac{1}{L}\right) \cdot int(f(x), x = -L..L)$

$$a_0 := 2\,\pi \tag{56}$$

> $C := \frac{a_0}{2}$

$$C := \pi \tag{57}$$

> $a_n := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^{\cdot} \cdot n, \left(\frac{1}{L}\right) \cdot int\left(f(x) \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), x = -L..L\right)\right)$

$$a_n := 0 \tag{58}$$

> $b_n := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^{\cdot} \cdot n, \left(\frac{1}{L}\right) \cdot int\left(f(x) \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), x = -L..L\right)\right)$

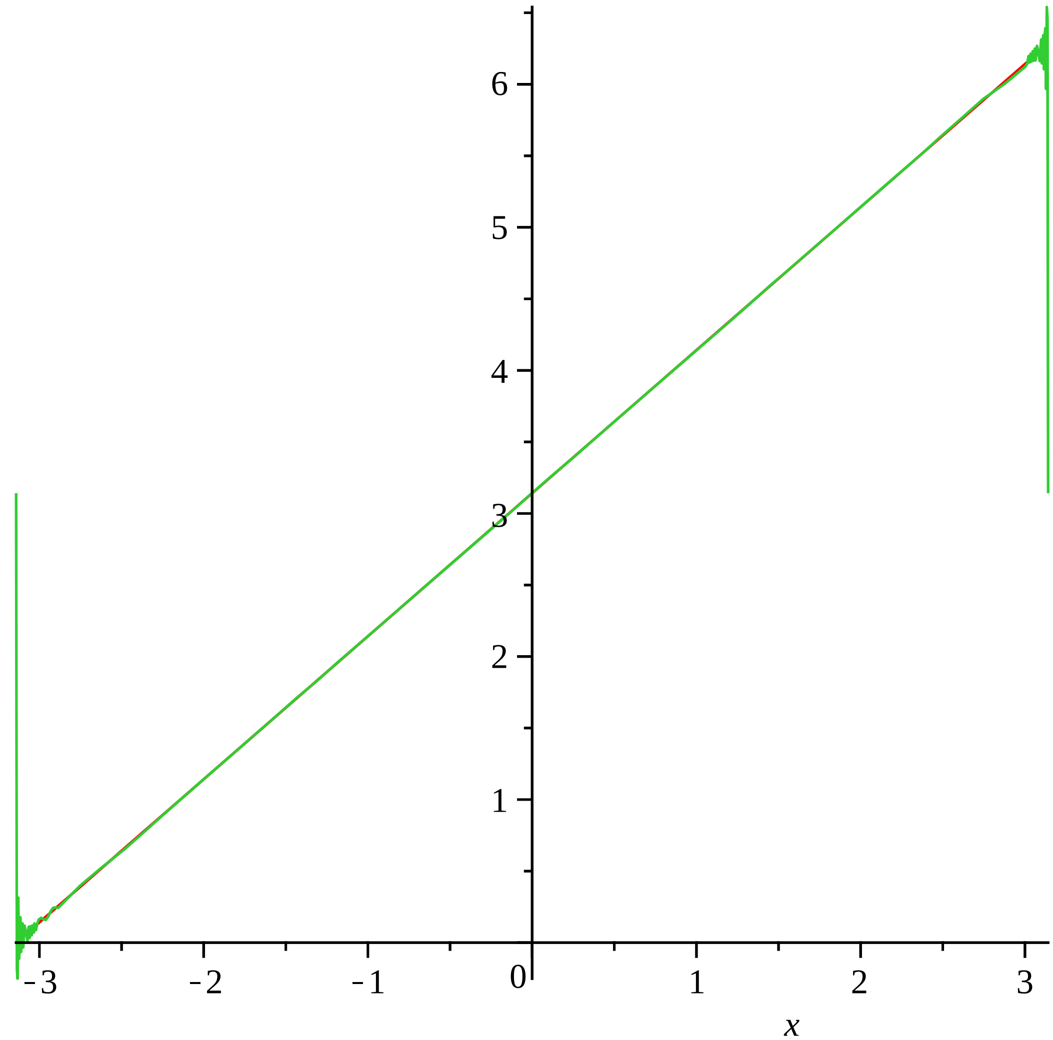
$$b_n := -\frac{2\,(-1)^n}{n} \tag{59}$$

> $STF := C + Sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right) + b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), n = 1..infinity\right)$

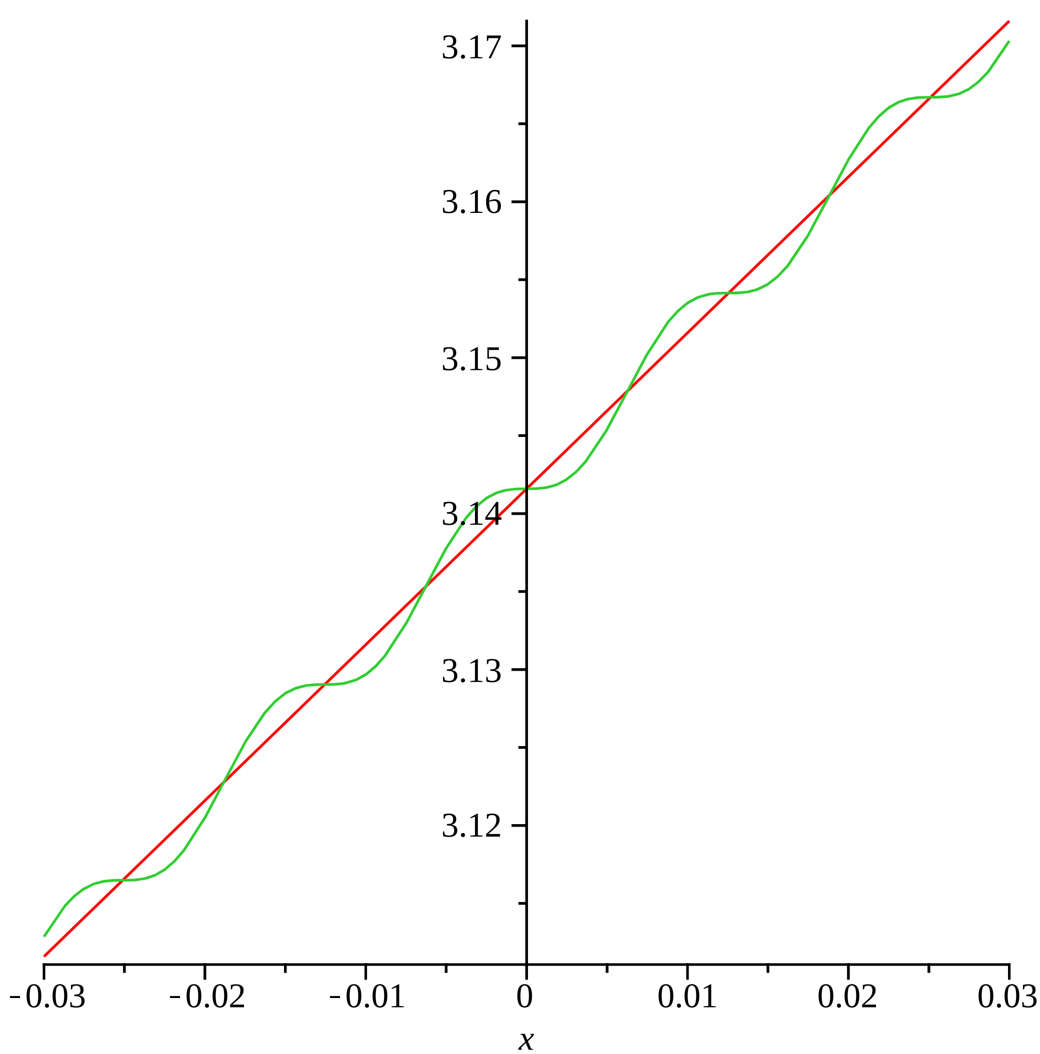
$$STF := \pi + \sum_{n=1}^{\infty}\left(-\frac{2\,(-1)^n \sin(n\,x)}{n}\right) \tag{60}$$

> $STF_{500} := C + Sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right) + b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), n = 1..500\right)$:

> $plot([f(x), STF_{500}], x = -L..L)$



> $plot([f(x), STF_{500}], x=-0.03..0.03)$



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FIN RESPUESTA 7)

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FIN EXAMEN

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