>

SOLUCIÓN

>

ECUACIONES DIFERENCIALES
SEGUNDO EXAMEN FINAL COLEGIADO
SEMESTRE 2011-2

2011 JUNIO 09
TIPO "A

> *restart*

1) Resuelva el problema del valor inicial

> $-y(x)+x\cdot diff(y(x),x)=2\cdot x\cdot\cdot 2\cdot y(x)\cdot\cdot 2\cdot diff(y(x),x); y(1)=-2;$

$$-y(x)+x\left(\frac{d}{dx}y(x)\right)=2x^2y(x)^2\left(\frac{d}{dx}y(x)\right)$$
$$y(1)=-2 \quad (1)$$

> **_C:**

RESPUESTA 1)

> $Ecuacion := -y(x)+x\cdot diff(y(x),x)=2\cdot x\cdot\cdot 2\cdot y(x)\cdot\cdot 2\cdot diff(y(x),x);$

$$Ecuacion := -y(x)+x\left(\frac{d}{dx}y(x)\right)=2x^2y(x)^2\left(\frac{d}{dx}y(x)\right) \quad (2)$$

> $EcuacionDespejada := isolate(Ecuacion, diff(y(x),x))$

$$EcuacionDespejada := \frac{d}{dx}y(x)=\frac{y(x)}{x-2x^2y(x)^2} \quad (3)$$

> $EcuacionNoExacta := -y(x)+(x-2\cdot x\cdot\cdot 2\cdot y(x)\cdot\cdot 2)\cdot diff(y(x),x)=0$

$$EcuacionNoExacta := -y(x)+\left(x-2x^2y(x)^2\right)\left(\frac{d}{dx}y(x)\right)=0 \quad (4)$$

> *with*(*DEtools*) :

> *odeadvisor*(*EcuacionNoExacta*)

[[_homogeneous, class G], _rational] (5)

> $FI := intfactor(EcuacionNoExacta)$

$$FI := \frac{1}{x^2} \quad (6)$$

> $M(x,y) := y; N(x,y) := -x+2x^2y^2$

$$M(x,y) := y$$
$$N(x,y) := -x+2x^2y^2 \quad (7)$$

> $diff(M(x,y),y); diff(N(x,y),x)$

$$1$$
$$-1+4xy^2 \quad (8)$$

>

> $MM(x,y) := simplify(FI\cdot M(x,y))$

$$MM(x,y) := \frac{y}{x^2} \quad (9)$$

> $NN(x,y) := expand(N(x,y)\cdot FI)$

(10)

$$NN(x, y) := -\frac{1}{x} + 2y^2 \quad \mathbf{(10)}$$

> $comprobacion := simplify(diff(MM(x, y), y) - diff(NN(x, y), x)) = 0;$

$$comprobacion := 0 = 0 \quad \mathbf{(11)}$$

> $IntMMx := int(MM(x, y), x)$

$$IntMMx := -\frac{y}{x} \quad \mathbf{(12)}$$

>

> $h(y) := expand(int((NN(x, y) - diff(IntMMx, y)), y))$

$$h(y) := \frac{2}{3}y^3 \quad \mathbf{(13)}$$

> $SolucionGeneral := IntMMx + h(y) = \_C1$

$$SolucionGeneral := -\frac{y}{x} + \frac{2}{3}y^3 = \_C1 \quad \mathbf{(14)}$$

> $parametro := subs(x = 1, y = -2, SolucionGeneral)$

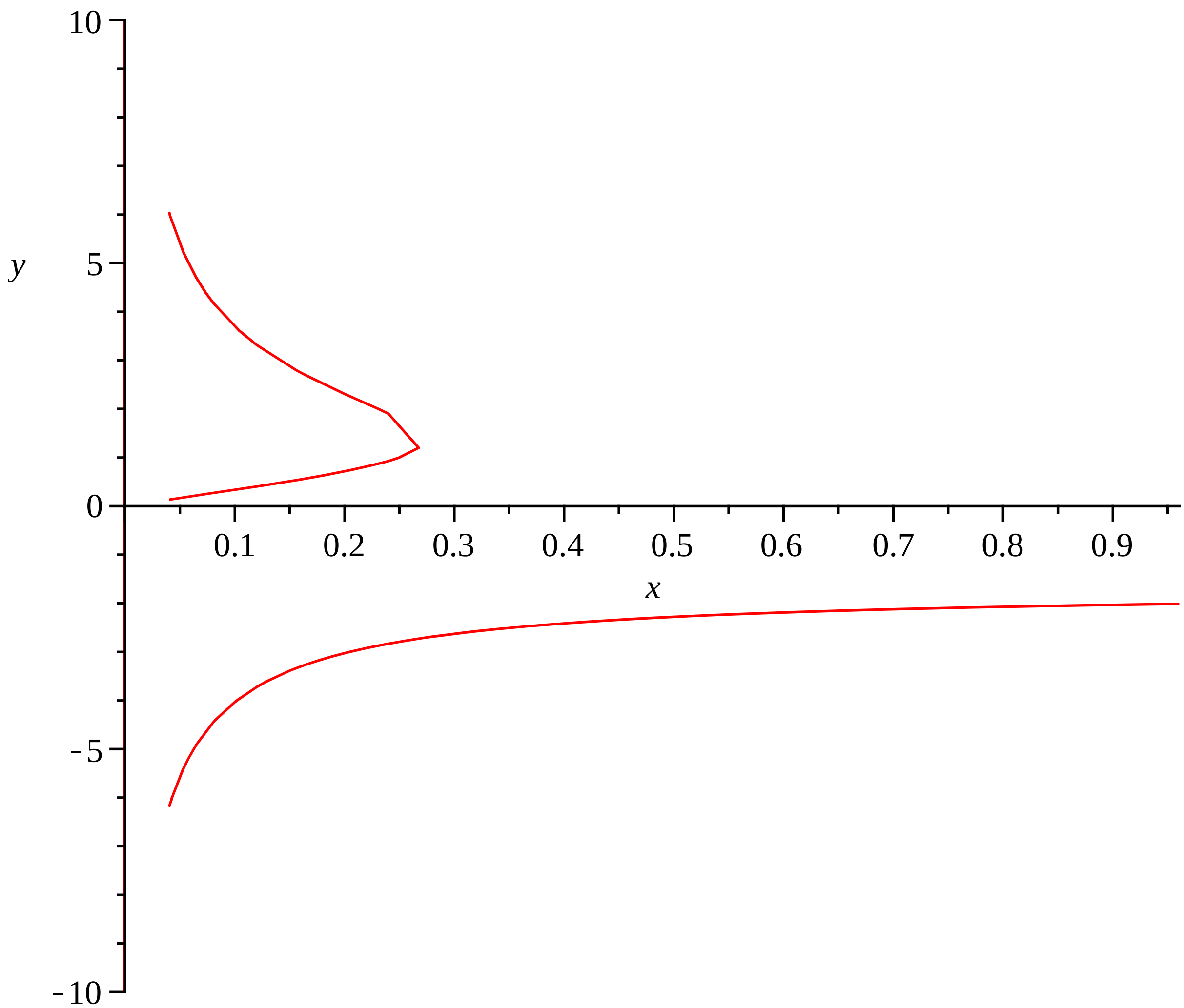
$$parametro := -\frac{10}{3} = \_C1 \quad \mathbf{(15)}$$

> $SolucionParticular := subs(\_C1 = lhs(parametro), SolucionGeneral)$

$$SolucionParticular := -\frac{y}{x} + \frac{2}{3}y^3 = -\frac{10}{3} \quad \mathbf{(16)}$$

> $with(plots):$

> $implicitplot(SolucionParticular, x = 0..1, y = -10..10, rational)$



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FIN RESPUESTA 1)

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> *restart*

2) Resuelva la ecuación diferencial

> $diff(x \cdot y' - y, x) + (x \cdot y' - y) = x \cdot \exp(-x) - y$

$$x\left(\frac{d^2}{dx^2}y(x)\right) + x\left(\frac{d}{dx}y(x)\right) - y(x) = x\,e^{-x} - y(x) \quad \textbf{(17)}$$

> **_C:**

RESPUESTA 2)

> $Ecuacion := x\left(\frac{d^2}{dx^2}y(x)\right) + x\left(\frac{d}{dx}y(x)\right) - y(x) = x\,e^{-x} - y(x)$

$$Ecuacion := x\left(\frac{d^2}{dx^2}y(x)\right) + x\left(\frac{d}{dx}y(x)\right) - y(x) = x\,e^{-x} - y(x) \quad \textbf{(18)}$$

> $EcuacionNoHomogenea := simplify\left(\frac{(lhs(Ecuacion) + y(x))}{x}\right) = \frac{(rhs(Ecuacion) + y(x))}{x}$

(19)

$$EcuacionNoHomogenea := \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) = e^{-x} \quad (19)$$

> $EcuacionHomogenea := lhs(EcuacionNoHomogenea) = 0$

$$EcuacionHomogenea := \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) = 0 \quad (20)$$

> $Q(x) := rhs(EcuacionNoHomogenea);$

$$Q(x) := e^{-x} \quad (21)$$

> $EcuacionCaracteristica := m \cdot \cdot 2 + m = 0$

$$EcuacionCaracteristica := m^2 + m = 0 \quad (22)$$

> $Raiz := solve(EcuacionCaracteristica)$

$$Raiz := 0, -1 \quad (23)$$

> $Solucion1 := y(x) = \exp(Raiz_1 \cdot x)$

$$Solucion1 := y(x) = 1 \quad (24)$$

> $Solucion2 := y(x) = \exp(Raiz_2 \cdot x)$

$$Solucion2 := y(x) = e^{-x} \quad (25)$$

> $SolucionHomogenea := y(x) = \_C1 \cdot rhs(Solucion1) + \_C2 \cdot rhs(Solucion2)$

$$SolucionHomogenea := y(x) = \_C1 + \_C2\, e^{-x} \quad (26)$$

> $SolucionNoHomogenea := y(x) = A(x) \cdot rhs(Solucion1) + B(x) \cdot rhs(Solucion2)$

$$SolucionNoHomogenea := y(x) = A(x) + B(x)\, e^{-x} \quad (27)$$

> $WW := array([[rhs(Solucion1), rhs(Solucion2)], [rhs(diff(Solucion1, x)), rhs(diff(Solucion2, x))]])$

$$WW := \begin{bmatrix} 1 & e^{-x} \\ 0 & -e^{-x} \end{bmatrix} \quad (28)$$

> $BB := array([0, Q(x)])$

$$BB := \begin{bmatrix} 0 & e^{-x} \end{bmatrix} \quad (29)$$

> $with(linalg):$

> $SOL := linsolve(WW, BB)$

$$SOL := \begin{bmatrix} e^{-x} & -1 \end{bmatrix} \quad (30)$$

> $Aprima := SOL_1; Bprima := SOL_2;$

$$Aprima := e^{-x}$$
$$Bprima := -1 \quad (31)$$

> $A(x) := int(Aprima, x) + \_C1; B(x) := int(Bprima, x) + \_C2;$

$$A(x) := -e^{-x} + \_C1$$
$$B(x) := -x + \_C2 \quad (32)$$

> $SolucionFinal := factor(SolucionNoHomogenea);$

$$SolucionFinal := y(x) = -e^{-x} + \_C1 - x\, e^{-x} + \_C2\, e^{-x} \quad (33)$$

> $Solucion2 := dsolve(Ecuacion)$

$$Solucion2 := y(x) = -(1 + \_C1 + x)\, e^{-x} + \_C2 \quad (34)$$

FIN RESPUESTA 2)

> *restart*

3) Sea

> $Solucion1 := y(x) = x^{\left(-\frac{1}{2}\right)}; Solucion2 := y(x) = x^{(-1)}$

$$Solucion1 := y(x) = \frac{1}{\sqrt{x}}$$

$$Solucion2 := y(x) = \frac{1}{x} \quad \textbf{(35)}$$

un conjunto fundamental de soluciones de la ecuacion diferencial

> $2 \cdot x^2 \cdot y'' + 5 \cdot x \cdot y' + y = 0$

$$2\,x^2 \left(\frac{d^2}{dx^2}\,y(x)\right) + 5\,x\left(\frac{d}{dx}\,y(x)\right) + y(x) = 0 \quad \textbf{(36)}$$

obtenga la solucion general de la ecuacion diferencial no homogenea

> $2 \cdot x^2 \cdot y'' + 5 \cdot x \cdot y' + y = x^2 - x$

$$2\,x^2 \left(\frac{d^2}{dx^2}\,y(x)\right) + 5\,x\left(\frac{d}{dx}\,y(x)\right) + y(x) = x^2 - x \quad \textbf{(37)}$$

> `_C:`

RESPUESTA 3)

> $SolucionHomogenea := y(x) = \_C1 \cdot rhs(Solucion1) + \_C2 \cdot rhs(Solucion2)$

$$SolucionHomogenea := y(x) = \frac{\_C1}{\sqrt{x}} + \frac{\_C2}{x} \quad \textbf{(38)}$$

> $EcuacionHomogenea := 2\,x^2 \left(\frac{d^2}{dx^2}\,y(x)\right) + 5\,x\left(\frac{d}{dx}\,y(x)\right) + y(x) = 0$

$$EcuacionHomogenea := 2\,x^2 \left(\frac{d^2}{dx^2}\,y(x)\right) + 5\,x\left(\frac{d}{dx}\,y(x)\right) + y(x) = 0 \quad \textbf{(39)}$$

> $comprobacion1 := simplify(eval(subs(y(x) = rhs(SolucionHomogenea), EcuacionHomogenea)));$

$$comprobacion1 := 0 = 0 \quad \textbf{(40)}$$

> $EcuacionHomogeneaNormalizada := expand\left(\frac{lhs(EcuacionHomogenea)}{2 \cdot x^2}\right) = 0$

$$EcuacionHomogeneaNormalizada := \frac{d^2}{dx^2}\,y(x) + \frac{5}{2}\,\frac{\frac{d}{dx}\,y(x)}{x} + \frac{1}{2}\,\frac{y(x)}{x^2} = 0 \quad \textbf{(41)}$$

> $comprobacion2 := simplify(eval(subs(y(x) = rhs(SolucionHomogenea), EcuacionHomogeneaNormalizada)));$

$$comprobacion2 := 0 = 0 \quad \textbf{(42)}$$

> $EcuacionNoHomogenea := 2\,x^2 \left(\frac{d^2}{dx^2}\,y(x)\right) + 5\,x\left(\frac{d}{dx}\,y(x)\right) + y(x) = x^2 - x$

$$EcuacionNoHomogenea := 2\,x^2 \left(\frac{d^2}{dx^2}\,y(x)\right) + 5\,x\left(\frac{d}{dx}\,y(x)\right) + y(x) = x^2 - x \quad \textbf{(43)}$$

> EcuacionNoHomogeneaNormalizada := expand(lhs(EcuacionNoHomogenea) / (2·x··2)) = expand(rhs(EcuacionNoHomogenea) / (2·x··2))

$$EcuacionNoHomogeneaNormalizada := \frac{d^2}{dx^2}\,y(x) + \frac{5}{2}\,\frac{\frac{d}{dx}\,y(x)}{x} + \frac{1}{2}\,\frac{y(x)}{x^2} = \frac{1}{2} - \frac{1}{2\,x} \quad (44)$$

> Q(x) := rhs(EcuacionNoHomogeneaNormalizada)

$$Q(x) := \frac{1}{2} - \frac{1}{2\,x} \quad (45)$$

> WW := array([[rhs(Solucion1), rhs(Solucion2)], [rhs(diff (Solucion1, x)), rhs(diff (Solucion2, x))]])

$$WW := \begin{bmatrix} \frac{1}{\sqrt{x}} & \frac{1}{x} \\ -\frac{1}{2\,x^{3/2}} & -\frac{1}{x^2} \end{bmatrix} \quad (46)$$

> BB := array([0, Q(x)])

$$BB := \begin{bmatrix} 0 & \frac{1}{2} - \frac{1}{2\,x} \end{bmatrix} \quad (47)$$

> with(linalg) :

> SOL := linsolve(WW, BB)

$$SOL := \begin{bmatrix} \sqrt{x}\,(x-1) & -x^2 + x \end{bmatrix} \quad (48)$$

> Aprima := SOL[1]; Bprima := SOL[2];

$$Aprima := \sqrt{x}\,(x-1)$$
$$Bprima := -x^2 + x \quad (49)$$

> A(x) := int(Aprima, x) + _C1; B(x) := int(Bprima, x) + _C2;

$$A(x) := \frac{2}{15}\,x^{3/2}\,(-5 + 3\,x) + \_C1$$
$$B(x) := -\frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + \_C2 \quad (50)$$

> SolucionGeneralNoHomogenea := y(x) = A(x)·rhs(Solucion1) + B(x)·rhs(Solucion2)

$$SolucionGeneralNoHomogenea := y(x) = \frac{\frac{2}{15}\,x^{3/2}\,(-5 + 3\,x) + \_C1}{\sqrt{x}} + \frac{-\frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + \_C2}{x} \quad (51)$$

> SolucionFinal := expand(SolucionGeneralNoHomogenea)

$$SolucionFinal := y(x) = -\frac{1}{6}\,x + \frac{1}{15}\,x^2 + \frac{\_C1}{\sqrt{x}} + \frac{\_C2}{x} \quad (52)$$

> Solucion2 := dsolve(EcuacionNoHomogenea)

$$Solucion2 := y(x) = \frac{\_C1}{x} + \frac{\_C2}{\sqrt{x}} + \frac{1}{30} x(-5 + 2x) \quad (53)$$

FIN RESPUESTA 3)

> *restart*

4) Resuelva el sistema de ecuaciones diferenciales

> $diff(x(t), t) + 2 \cdot diff(y(t), t) = 4 \cdot x(t) + 5 \cdot y(t); 2 \cdot diff(x(t), t) - diff(y(t), t) = 3 \cdot x(t); x(0) = 1; y(0) = -1;$

$$\frac{d}{dt} x(t) + 2\left(\frac{d}{dt} y(t)\right) = 4 x(t) + 5 y(t)$$

$$2\left(\frac{d}{dt} x(t)\right) - \left(\frac{d}{dt} y(t)\right) = 3 x(t)$$

$$x(0) = 1$$

$$y(0) = -1 \quad (54)$$

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RESPUESTA 4)

> $Sistema := \frac{d}{dt} x(t) + 2\left(\frac{d}{dt} y(t)\right) = 4 x(t) + 5 y(t), 2\left(\frac{d}{dt} x(t)\right) - \left(\frac{d}{dt} y(t)\right) = 3 x(t);$

$$Sistema := \frac{d}{dt} x(t) + 2\left(\frac{d}{dt} y(t)\right) = 4 x(t) + 5 y(t), 2\left(\frac{d}{dt} x(t)\right) - \left(\frac{d}{dt} y(t)\right) = 3 x(t) \quad (55)$$

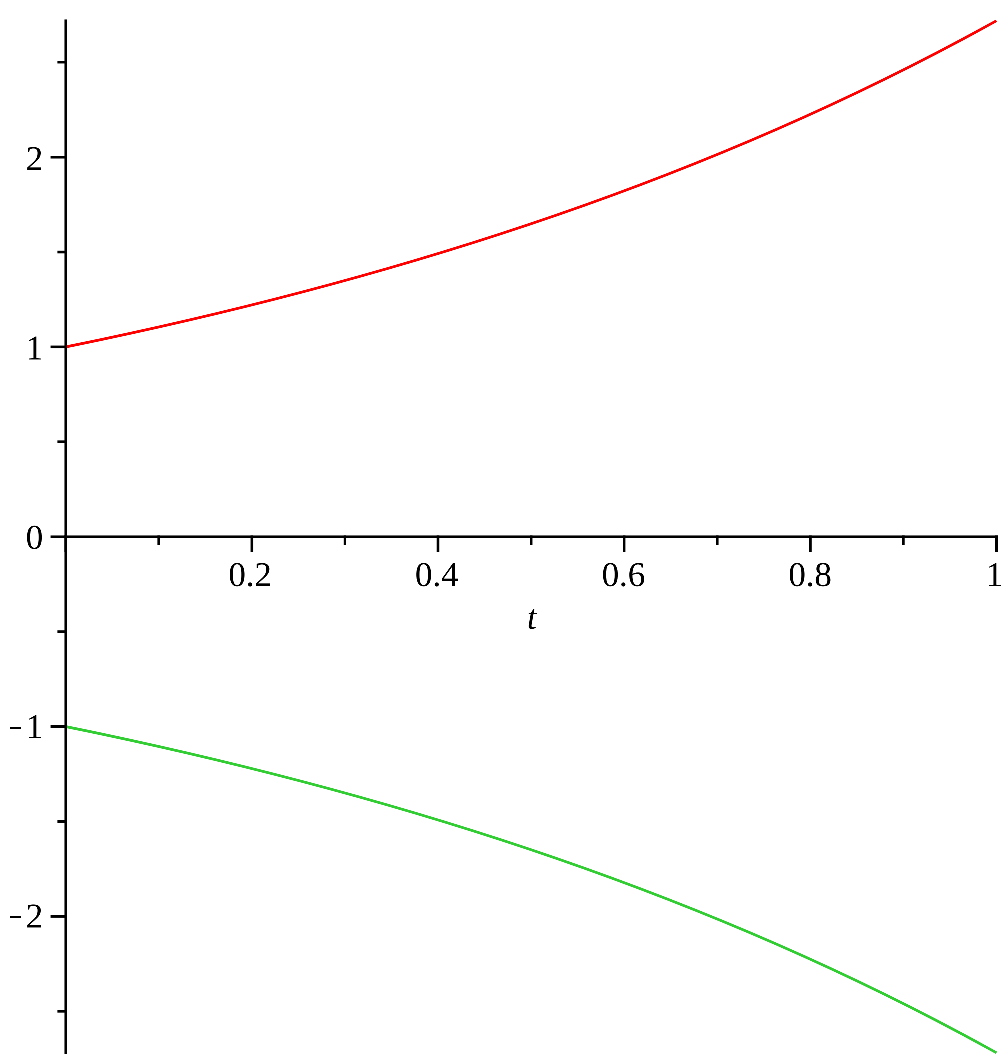
> $Condiciones := x(0) = 1, y(0) = -1;$

$$Condiciones := x(0) = 1, y(0) = -1 \quad (56)$$

> $Solucion := dsolve(\{Sistema, Condiciones\});$

$$Solucion := \{x(t) = e^{t}, y(t) = -e^{t}\} \quad (57)$$

> $plot([rhs(Solucion_1), rhs(Solucion_2)], t = 0..1)$



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SEGUNDA RESPUESTA 4)

> $Ecuacion_1 := \frac{d}{dt}x(t) + 2\left(\frac{d}{dt}y(t)\right) = 4x(t) + 5y(t);$

$$Ecuacion_1 := \frac{d}{dt}x(t) + 2\left(\frac{d}{dt}y(t)\right) = 4x(t) + 5y(t) \quad (58)$$

> $Ecuacion_2 := 2\left(\frac{d}{dt}x(t)\right) - \left(\frac{d}{dt}y(t)\right) = 3x(t);$

$$Ecuacion_2 := 2\left(\frac{d}{dt}x(t)\right) - \left(\frac{d}{dt}y(t)\right) = 3x(t) \quad (59)$$

> *Condiciones*;

$$x(0) = 1, y(0) = -1 \quad (60)$$

> *with*(*inttrans*) :

> $TLecuacion1 := subs(Condiciones, laplace(Ecuacion_1, t, s))$

$$TLecuacion1 := s\,laplace(x(t), t, s) + 1 + 2\,s\,laplace(y(t), t, s) = 4\,laplace(x(t), t, s) + 5\,laplace(y(t), t, s) \quad (61)$$

> $TLecuacion2 := subs(Condiciones, laplace(Ecuacion_2, t, s))$

$$TLecuacion2 := 2\,s\,laplace(x(t), t, s) - 3 - s\,laplace(y(t), t, s) = 3\,laplace(x(t), t, s) \quad (62)$$

> $TLsoluciones := solve(\{TLecuacion1, TLecuacion2\}, \{laplace(x(t), t, s), laplace(y(t), t, s)\})$

$$TLsoluciones := \left\{laplace(x(t), t, s) = \frac{1}{s-1}, laplace(y(t), t, s) = -\frac{1}{s-1}\right\} \quad \textbf{(63)}$$

> $Solucion10 := invlaplace(TLsoluciones_1, s, t)$

$$Solucion10 := x(t) = e^t \quad \textbf{(64)}$$

> $Solucion20 := invlaplace(TLsoluciones_2, s, t)$

$$Solucion20 := y(t) = -e^t \quad \textbf{(65)}$$

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FIN RESPUESTA 4)

> *restart*

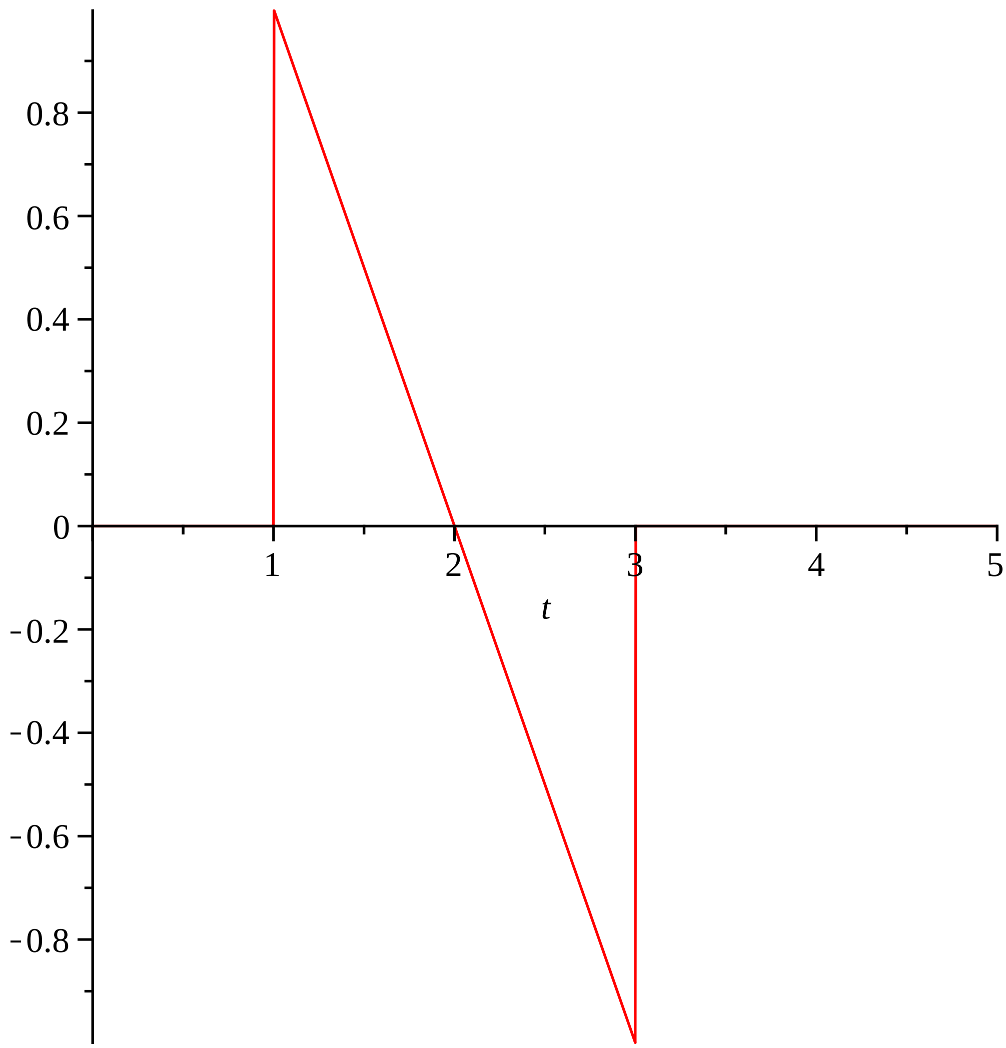
5) Calcular

a) F(s)

> $f(t) := \text{Heaviside}(t-1)\cdot(2-t) - \text{Heaviside}(t-3)\cdot(2-t)$

$$f(t) := \text{Heaviside}(t-1)\ (2-t) - \text{Heaviside}(t-3)\ (2-t) \quad \textbf{(66)}$$

> $plot(f(t), t = 0..5)$



> $with(inttrans):$

> $F(s) := laplace(f(t), t, s)$

$$F(s) := \frac{(s+1)\,e^{-3s} + (s-1)\,e^{-s}}{s^2} \tag{67}$$

>

b) g(t)

> $G(s) := \frac{\exp(-2\cdot s)\cdot(s+1)}{s\cdot\cdot 2 - 4\cdot s + 5};$

$$G(s) := \frac{e^{-2s}\,(s+1)}{s^2 - 4s + 5} \tag{68}$$

> $g(t) := invlaplace(G(s), s, t)$

$$g(t) := \mathrm{Heaviside}(t-2)\,e^{2t-4}\,(\cos(t-2) + 3\sin(t-2)) \tag{69}$$

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FIN RESPUESTA 5)

> *restart*

6) Resuelva la ecuacion integral

> $f(t) = 4\cdot t - 3 + int(f(\mathrm{tau})\cdot\sin(t - \mathrm{tau}), \mathrm{tau} = 0..t)$

$$f(t) = 4t - 3 + \int_0^t f(\tau)\,\sin(t-\tau)\,d\tau \tag{70}$$

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RESPUESTA 6)

> $EcuacionIntegral := f(t) = 4t - 3 + \int_0^t f(\tau)\,\sin(t-\tau)\,d\tau$

$$EcuacionIntegral := f(t) = 4t - 3 + \int_0^t f(\tau)\,\sin(t-\tau)\,d\tau \tag{71}$$

> *with(inttrans)* :

> $TLecuacion := laplace(EcuacionIntegral, t, s)$

$$TLecuacion := laplace(f(t), t, s) = \frac{4}{s^2} - \frac{3}{s} + \frac{laplace(f(t), t, s)}{s^2+1} \tag{72}$$

> $TLsolucion := isolate(TLecuacion, laplace(f(t), t, s))$

$$TLsolucion := laplace(f(t), t, s) = \frac{\frac{4}{s^2} - \frac{3}{s}}{1 - \frac{1}{s^2+1}} \tag{73}$$

> $Solucion := invlaplace(TLsolucion, s, t)$

$$Solucion := f(t) = -\frac{3}{2}t^2 + \frac{2}{3}t^3 + 4t - 3 \tag{74}$$

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FIN RESPUESTA 6)

> *restart*

7) Utilice el método de separación de variables para resolver la ecuación diferencial en derivadas parciales

> $diff(u(x,t), x\$2) - R\cdot C\cdot diff(u(x,t), t) - R\cdot G\cdot u(x,t) = 0$

$$\frac{\partial^2}{\partial x^2} u(x,t) - R\,C\left(\frac{\partial}{\partial t} u(x,t)\right) - R\,G\,u(x,t) = 0 \quad \textbf{(75)}$$

Considerando una constante de separación igual 1

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RESPUESTA 7)

> $Ecuacion := \frac{\partial^2}{\partial x^2} u(x,t) - R\,C\left(\frac{\partial}{\partial t} u(x,t)\right) - R\,G\,u(x,t) = 0$

$$Ecuacion := \frac{\partial^2}{\partial x^2} u(x,t) - R\,C\left(\frac{\partial}{\partial t} u(x,t)\right) - R\,G\,u(x,t) = 0 \quad \textbf{(76)}$$

> $EcuacionSeparable := eval(subs(u(x,t) = F(x)\cdot H(t), Ecuacion))$

$$EcuacionSeparable := \left(\frac{d^2}{dx^2} F(x)\right) H(t) - R\,C\,F(x)\left(\frac{d}{dt} H(t)\right) - R\,G\,F(x)\,H(t) = 0 \quad \textbf{(77)}$$

> $EcuacionSeparada := \frac{\left(lhs(EcuacionSeparable) + R\,C\,F(x)\left(\frac{d}{dt} H(t)\right) + R\,G\,F(x)\,H(t)\right)}{F(x)\cdot H(t)} = simplify\left(\frac{\left(rhs(EcuacionSeparable) + R\,C\,F(x)\left(\frac{d}{dt} H(t)\right) + R\,G\,F(x)\,H(t)\right)}{F(x)\cdot H(t)}\right)$

$$EcuacionSeparada := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \frac{R\left(C\left(\frac{d}{dt} H(t)\right) + G\,H(t)\right)}{H(t)} \quad \textbf{(78)}$$

> $Ecuacion_x := lhs(EcuacionSeparada) = \text{alpha}; Ecuacion_t := rhs(EcuacionSeparada) = \text{alpha}$

$$Ecuacion_x := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \alpha$$

$$Ecuacion_t := \frac{R\left(C\left(\frac{d}{dt} H(t)\right) + G\,H(t)\right)}{H(t)} = \alpha \quad \textbf{(79)}$$

> $Solucion1 := dsolve(subs(\text{alpha} = 1, Ecuacion_x))$

$$Solucion1 := F(x) = \_C1\,e^{x} + \_C2\,e^{-x} \quad \textbf{(80)}$$

> $Solucion2 := dsolve(subs(\text{alpha} = 1, Ecuacion_t))$

$$Solucion2 := H(t) = \_C1\,e^{-\frac{(R\,G-1)\,t}{R\,C}} \quad \textbf{(81)}$$

> $SolucionGeneral := u(x,t) = rhs(Solucion1)\cdot subs(\_C1 = 1, rhs(Solucion2))$

$$SolucionGeneral := u(x,t) = \left(\_C1\,e^{x} + \_C2\,e^{-x}\right) e^{-\frac{(R\,G-1)\,t}{R\,C}} \quad \textbf{(82)}$$

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FIN RESPUESTA 7)

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FIN EXAMEN