>

SOLUCIÓN

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
TERCER EXAMEN PARCIAL (TEMAS 4 Y 5)
SEMESTRE 2013-1

2012 NOVIEMBRE 21

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1) UTILIZANDO EXCLUSIVAMENTE TRANSFORMADA DE LAPLACE **(sin usar dsolve)**:

a) **(15/100 puntos)** OBTENER LA SOLUCIÓN PARTICULAR DE LA ECUACIÓN DADA CON LAS CONDICIONES INICIALES DADAS

b) **(15/100 puntos)** GRAFICAR - JUNTAS - LA SOLUCIÓN OBTENIDA EN EL INCISO a) Y SU PRIMERA DERIVADA; PARA UN INTERVALO DE $0 < t < 3$

$$\frac{d^2}{dt^2} y(t) + 4\, y(t) = 64\, (t-2)\, \mathrm{Heaviside}(t-2)\, \sin(2\,t-4)$$

$$y(0) = 2$$

$$D(y)(0) = 0 \qquad \textbf{(1)}$$

> *restart*

RESPUESTA 1a)

> $Ecuacion := \frac{d^2}{dt^2} y(t) + 4\, y(t) = 64\, (t-2)\, \mathrm{Heaviside}(t-2)\, \sin(2\,t-4);\ Condiciones := y(0) = 2, D(y)(0) = 0$

$$Ecuacion := \frac{d^2}{dt^2} y(t) + 4\, y(t) = 64\, (t-2)\, \mathrm{Heaviside}(t-2)\, \sin(2\,t-4)$$

$$Condiciones := y(0) = 2, D(y)(0) = 0 \qquad \textbf{(2)}$$

> *with(inttrans)* :

> *TransLapEcuacion := simplify(subs(Condiciones, laplace(Ecuacion, t, s)))*

$$TransLapEcuacion := s^2\, laplace(y(t), t, s) - 2\, s + 4\, laplace(y(t), t, s) = \frac{256\, e^{-2\,s}\, s}{(s^2+4)^2} \qquad \textbf{(3)}$$

> *TransLapSolucion := simplify(isolate(TransLapEcuacion, laplace(y(t), t, s)))*

$$TransLapSolucion := laplace(y(t), t, s) = \frac{2\, s\, (128\, e^{-2\,s} + s^4 + 8\, s^2 + 16)}{(s^2+4)^3} \qquad \textbf{(4)}$$

> *SolucionParticular := invlaplace(TransLapSolucion, s, t)*

$$SolucionParticular := y(t) = 2\cos(2\,t) + 4\,(t-2)\,(\sin(2\,t-4) - 2\cos(2\,t-4)\,(t-2))\,\mathrm{Heaviside}(t-2) \qquad \textbf{(5)}$$

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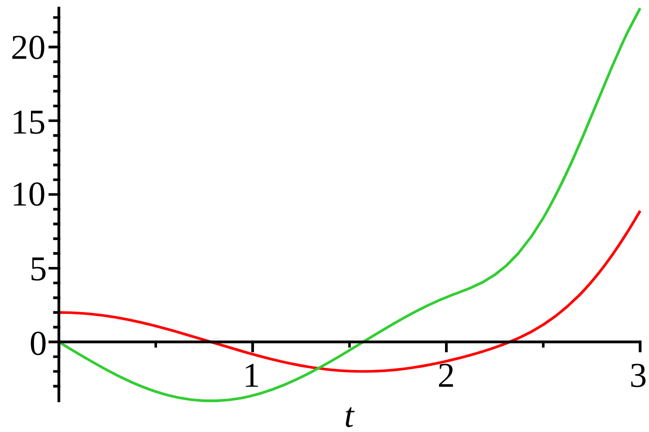
RESPUESTA 1b)

> *DerSolucionParticular := diff(SolucionParticular, t)*

$$DerSolucionParticular := \frac{d}{dt} y(t) = -4\sin(2\,t) + 4\,(\sin(2\,t-4) - 2\cos(2\,t-4)\,(t-2))\,\mathrm{Heaviside}(t-2) + 16\,(t-2)^2 \sin(2\,t-4)\,\mathrm{Heaviside}(t-2) + 4\,(t-2)\,(\sin(2\,t-4) - 2\cos(2\,t-4)\,(t-2))\,\mathrm{Dirac}(t-2) \qquad \textbf{(6)}$$

> $plot([rhs(SolucionParticular), rhs(DerSolucionParticular)], t = 0..3)$



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FIN RESPUESTA 1)

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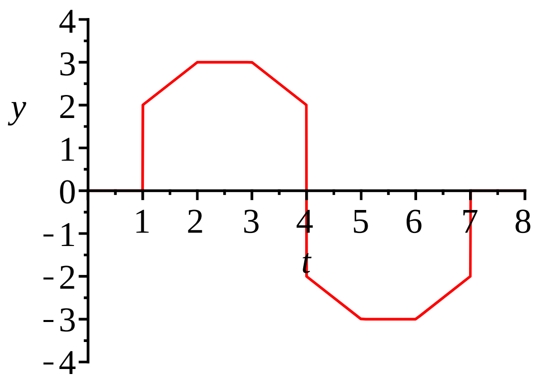
2) DADA LA GRÁFICA DE LA FUNCIÓN SIGUIENTE:



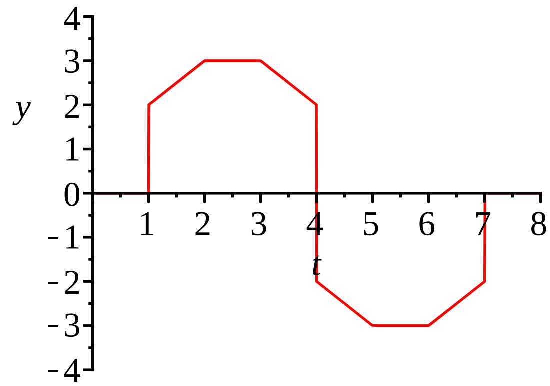
a) **(15/100 puntos)** OBTENER SU TRANSFORMADA DE LAPLACE.
b) **(25/100 puntos)** OBTENER Y GRAFICAR SU SERIE COSENO DE FOURIER PARA 500 TÉRMINOS EN EL MISMO INTERVALO.

> *restart*

RESPUESTA 2a)

> $f := 2\cdot\text{Heaviside}(t-1) + (t-1)\cdot\text{Heaviside}(t-1) - (t-2)\cdot\text{Heaviside}(t-2) - (t-3)\cdot\text{Heaviside}(t-3) + (t-4)\cdot\text{Heaviside}(t-4) - 4\cdot\text{Heaviside}(t-4) - (t-4)\cdot\text{Heaviside}(t-4) + (t-5)\cdot\text{Heaviside}(t-5) + (t-6)\cdot\text{Heaviside}(t-6) - (t-7)\cdot\text{Heaviside}(t-7) + 2\cdot\text{Heaviside}(t-7);\ plot(f, t = 0..8, y = -4..4)$

$$f := 2\,\text{Heaviside}(t-1) + (t-1)\,\text{Heaviside}(t-1) - (t-2)\,\text{Heaviside}(t-2) - (t-3)\,\text{Heaviside}(t-3) - 4\,\text{Heaviside}(t-4) + (t-5)\,\text{Heaviside}(t-5) + (t-6)\,\text{Heaviside}(t-6) - (t-7)\,\text{Heaviside}(t-7) + 2\,\text{Heaviside}(t-7)$$



> $with(inttrans):$

> $F := laplace(f, t, s)$

$$F := \frac{e^{-s} - e^{-7s} + e^{-6s} + e^{-5s} - e^{-3s} - e^{-2s}}{s^2} + \frac{2\left(e^{-s} + e^{-7s} - 2\,e^{-4s}\right)}{s} \qquad \textbf{(7)}$$

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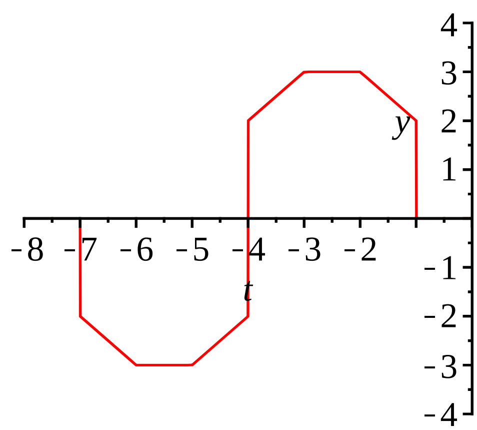
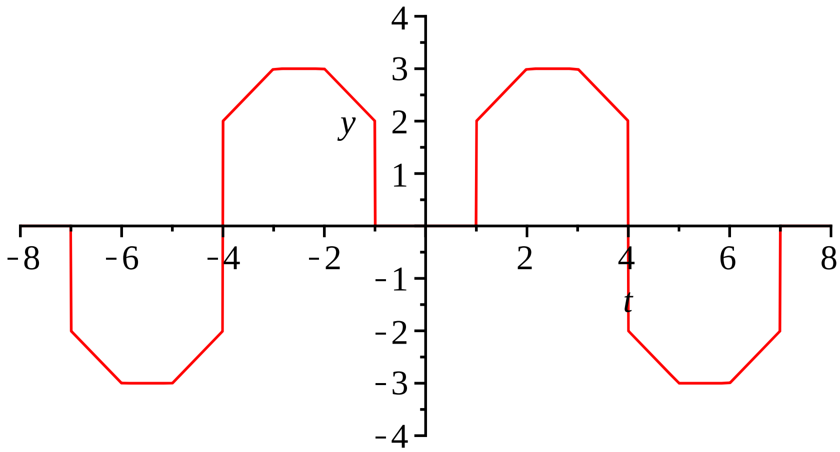
RESPUESTA 2b)

> $g := -2\cdot \text{Heaviside}(t+7) - (t+7)\cdot\text{Heaviside}(t+7) + (t+6)\cdot\text{Heaviside}(t+6) + (t+5)\cdot\text{Heaviside}(t+5) - (t+4)\cdot\text{Heaviside}(t+4) + 4\cdot\text{Heaviside}(t+4) + (t+4)\cdot\text{Heaviside}(t+4) - (t+3)\cdot\text{Heaviside}(t+3) - (t+2)\cdot\text{Heaviside}(t+2) + (t+1)\cdot\text{Heaviside}(t+1) - 2\cdot\text{Heaviside}(t+1) : plot(g, t=-8..0, y=-4..4)$



> $h := f + g; plot(h, t=-8..8, y=-4..4)$

$h := 2\,\text{Heaviside}(t-1) + (t-1)\,\text{Heaviside}(t-1) - (t-2)\,\text{Heaviside}(t-2) - (t-3)\,\text{Heaviside}(t-3) - 4\,\text{Heaviside}(t-4) + (t-5)\,\text{Heaviside}(t-5) + (t-6)\,\text{Heaviside}(t-6) - (t-7)\,\text{Heaviside}(t-7) + 2\,\text{Heaviside}(t-7) - 2\,\text{Heaviside}(t+7) - (t+7)\,\text{Heaviside}(t+7) + (t+6)\,\text{Heaviside}(t+6) + (t+5)\,\text{Heaviside}(t+5) + 4\,\text{Heaviside}(t+4) - (t+3)\,\text{Heaviside}(t+3) - (t+2)\,\text{Heaviside}(t+2) + (t+1)\,\text{Heaviside}(t+1) - 2\,\text{Heaviside}(t+1)$



> $L := 8$

$$L := 8 \tag{8}$$

> $a_0 := \left(\frac{1}{L}\right) \cdot int(h, t = -L..L);\ C := \frac{a_0}{2}$

$$a_0 := 0$$
$$C := 0 \tag{9}$$

> $a_n := simplify\left(\left(\frac{1}{L}\right) \cdot int\left(h \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = -L..L\right)\right)$

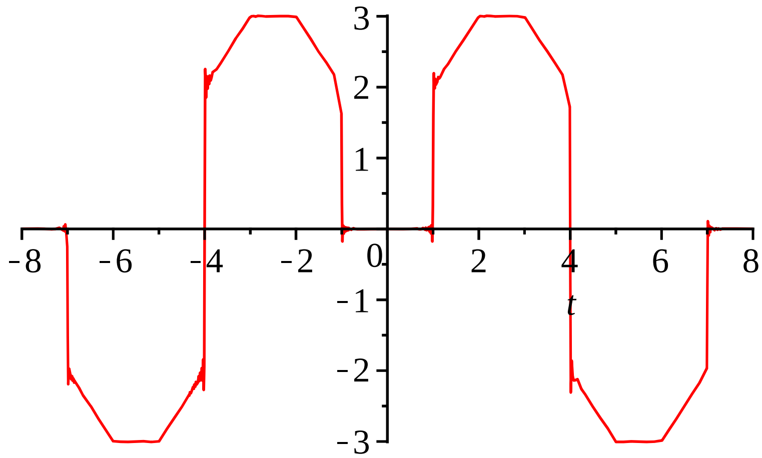
$$a_n := -\frac{1}{n^2 \pi^2} \left(4 \left(-4 \cos\left(\frac{1}{4} n \pi\right) + 4 \cos\left(\frac{1}{8} n \pi\right) + n \pi \sin\left(\frac{1}{8} n \pi\right) + 4 \cos\left(\frac{5}{8} n \pi\right) + 4 \cos\left(\frac{3}{4} n \pi\right) - 4 \cos\left(\frac{7}{8} n \pi\right) + n \pi \sin\left(\frac{7}{8} n \pi\right) - 2 \sin\left(\frac{1}{2} n \pi\right) n \pi - 4 \cos\left(\frac{3}{8} n \pi\right)\right)\right) \tag{10}$$

> $b_n := simplify\left(\left(\frac{1}{L}\right) \cdot int\left(h \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = -L..L\right)\right)$

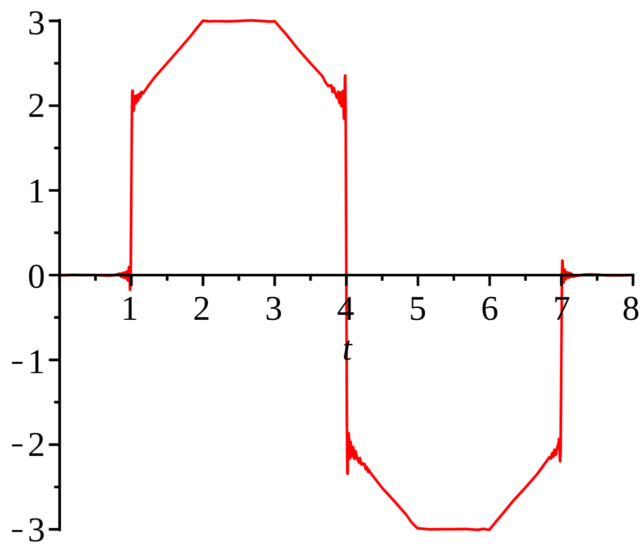
$$b_n := 0 \tag{11}$$

> $STF_{500} := sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), n = 1..500\right):$

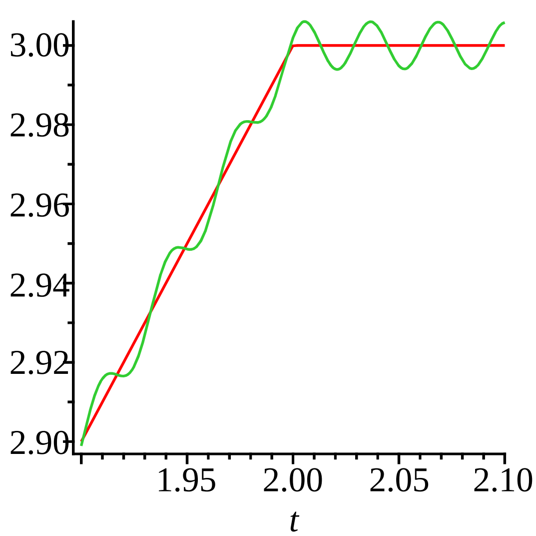
> $plot(STF_{500}, t = -8..8)$



> $plot(STF_{500}, t = 0 ..8)$



> $plot([f, STF_{500}], t = 1.9 ..2.1)$



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FIN RESPUESTA 2)

> *restart*

3) (30/100 puntos) OBTENER LA SOLUCIÓN DE LA SIGUIENTE ECUACIÓN EN DERIVADAS PARCIALES, UTILIZANDO EL MÉTODO DE SEPARACIÓN DE VARIABLES CON UNA **CONSTANTE DE SEPARACIÓN NEGATIVA**:

$$\frac{\partial^2}{\partial x^2} y(x,t) + t^2 \left(\frac{\partial}{\partial t} y(x,t)\right) = \frac{\partial}{\partial x} y(x,t) \tag{12}$$

> *restart*

RESPUESTA 3)

> $Ecuacion := \frac{\partial^2}{\partial x^2} y(x,t) + t^2 \left(\frac{\partial}{\partial t} y(x,t)\right) = \frac{\partial}{\partial x} y(x,t)$

$$Ecuacion := \frac{\partial^2}{\partial x^2} y(x,t) + t^2 \left(\frac{\partial}{\partial t} y(x,t)\right) = \frac{\partial}{\partial x} y(x,t) \tag{13}$$

> $EcuacionDos := eval(subs(y(x,t) = F(x) \cdot G(t), Ecuacion))$

$$EcuacionDos := \left(\frac{d^2}{dx^2} F(x)\right) G(t) + t^2 F(x) \left(\frac{d}{dt} G(t)\right) = \left(\frac{d}{dx} F(x)\right) G(t) \tag{14}$$

> $EcuacionTres := lhs(EcuacionDos) - t^2 F(x) \left(\frac{d}{dt} G(t)\right) - \left(\frac{d}{dx} F(x)\right) G(t) = rhs(EcuacionDos) - t^2 F(x) \left(\frac{d}{dt} G(t)\right) - \left(\frac{d}{dx} F(x)\right) G(t)$

$$EcuacionTres := \left(\frac{d^2}{dx^2} F(x)\right) G(t) - \left(\frac{d}{dx} F(x)\right) G(t) = -t^2 F(x) \left(\frac{d}{dt} G(t)\right) \tag{15}$$

> $EcuacionSeparada := simplify\left(\frac{lhs(EcuacionTres)}{F(x) \cdot G(t)} = \frac{rhs(EcuacionTres)}{F(x) \cdot G(t)}\right)$

$$EcuacionSeparada := \frac{\frac{d^2}{dx^2} F(x) - \left(\frac{d}{dx} F(x)\right)}{F(x)} = -\frac{t^2 \left(\frac{d}{dt} G(t)\right)}{G(t)} \tag{16}$$

> $EcuacionX := lhs(EcuacionSeparada) = -\text{beta} \cdot \cdot 2; EcuacionT := rhs(EcuacionSeparada) = -\text{beta} \cdot \cdot 2$

$$EcuacionX := \frac{\frac{d^2}{dx^2} F(x) - \left(\frac{d}{dx} F(x)\right)}{F(x)} = -\beta^2$$

$$EcuacionT := -\frac{t^2 \left(\frac{d}{dt} G(t)\right)}{G(t)} = -\beta^2 \tag{17}$$

> $SolucionX := dsolve(EcuacionX); SolucionT := dsolve(EcuacionT)$

$$SolucionX := F(x) = \_C1\, e^{\left(\frac{1}{2} + \frac{1}{2}\sqrt{1-4\beta^2}\right)x} + \_C2\, e^{\left(\frac{1}{2} - \frac{1}{2}\sqrt{1-4\beta^2}\right)x}$$

$$SolucionT := G(t) = \_C1\, e^{-\frac{\beta^2}{t}} \tag{18}$$

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> $SolucionNegativa := y(x,t) = rhs(SolucionX) \cdot subs(\_C1 = 1, rhs(SolucionT))$

$$SolucionNegativa := y(x,t) = \left(\_C1\, e^{\left(\frac{1}{2} + \frac{1}{2}\sqrt{1-4\beta^2}\right)x} + \_C2\, e^{\left(\frac{1}{2} - \frac{1}{2}\sqrt{1-4\beta^2}\right)x}\right) e^{-\frac{\beta^2}{t}} \quad \textbf{(19)}$$

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FIN RESPUESTA 3)

> *restart*

FIN DEL SOLUCIÓN

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