```
> restart
```

SOLUCIÓN TIPO B

PRIMER EXAMEN FINAL COLEGIADO
ECUACIONES DIFERENCIALES

```
> restart
```

RESPUESTA 1)

```
> CondicionInicial := T(0) = 900; CondicionFinal := T(60) = 50
```

$$CondicionInicial := T(0) = 900$$

$$CondicionFinal := T(60) = 50 \quad \textbf{(1)}$$

```
> Ecuacion := diff(T(t), t) = K·(T(t) − 20)
```

$$Ecuacion := \frac{d}{dt} T(t) = K\,(T(t) - 20) \quad \textbf{(2)}$$

```
> Solucion := dsolve({Ecuacion, CondicionInicial})
```

$$Solucion := T(t) = 20 + 880\, e^{K t} \quad \textbf{(3)}$$

```
> Parametro := isolate(subs(t = 60, rhs(Solucion) = rhs(CondicionFinal)), K); evalf(%)
```

$$Parametro := K = \frac{1}{60} \ln\left(\frac{3}{88}\right)$$

$$K = -0.05631207544 \quad \textbf{(4)}$$

```
> SolucionParticular := subs(K = rhs(Parametro), Solucion)
```

$$SolucionParticular := T(t) = 20 + 880\, e^{\frac{1}{60} \ln\left(\frac{3}{88}\right) t} \quad \textbf{(5)}$$

```
> Tiempo_35 := isolate(rhs(SolucionParticular) = 35, t); evalf(%)
```

$$Tiempo_{35} := t = \frac{60 \ln\left(\frac{3}{176}\right)}{\ln\left(\frac{3}{88}\right)}$$

$$t = 72.30903276 \quad \textbf{(6)}$$

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FIN RESPUESTA 1)

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> restart
```

RESPUESTA 2)

```
> Ecuacion := diff(y(x), x$2) + 3·diff(y(x), x) + 2·y(x) = cos(exp(x))
```

$$Ecuacion := \frac{d^2}{dx^2} y(x) + 3\left(\frac{d}{dx} y(x)\right) + 2\, y(x) = \cos\left(e^x\right) \quad \textbf{(7)}$$

```
> SolucionUno := expand(dsolve(Ecuacion))
```

$$SolucionUno := y(x) = -\frac{2}{\left(e^x\right)^2 \left(1 + \tan\left(\frac{1}{2} e^x\right)^2\right)} - \frac{\_C1}{\left(e^x\right)^2} + \frac{\_C2}{e^x} \quad \textbf{(8)}$$

```
> SolucionDos := simplify(dsolve(Ecuacion))
```

$$SolucionDos := y(x) = \left(-2 \cos\left(\frac{1}{2} e^x\right)^2 - \_C1 + \_C2\, e^x\right) e^{-2x} \quad \textbf{(9)}$$

```
> comprobacion_1 := simplify(eval(subs(y(x) = rhs(SolucionUno), Ecuacion)))
```

(10)

$$comprobacion_1 := -1 + 2\cos\left(\frac{1}{2}\,e^{x}\right)^2 = \cos\left(e^{x}\right) \quad (10)$$

> $comprobacion_2 := simplify(eval(subs(y(x) = rhs(SolucionDos), Ecuacion)))$

$$comprobacion_2 := -1 + 2\cos\left(\frac{1}{2}\,e^{x}\right)^2 = \cos\left(e^{x}\right) \quad (11)$$

> $SolucionTres := y(x) = C_1 \cdot \exp(-2x) + C_2 \cdot \exp(-x) - \exp(-2x) \cdot \cos(\exp(x))$

$$SolucionTres := y(x) = C_1\,e^{-2x} + C_2\,e^{-x} - e^{-2x}\cos\left(e^{x}\right) \quad (12)$$

> $comprobacion_3 := simplify(eval(subs(y(x) = rhs(SolucionTres), Ecuacion)))$

$$comprobacion_3 := \cos\left(e^{x}\right) = \cos\left(e^{x}\right) \quad (13)$$

FIN RESPUESTA 2)

> *restart*

RESPUESTA 3)

> $Ecuacion := diff(x(t), t\$4) - x(t) = 0$

$$Ecuacion := \frac{d^4}{dt^4}\,x(t) - x(t) = 0 \quad (14)$$

> $EcuacionCaracteristica := m\cdot\cdot 4 - 1 = 0$

$$EcuacionCaracteristica := m^4 - 1 = 0 \quad (15)$$

> $Raiz := solve(EcuacionCaracteristica, m)$

$$Raiz := 1,\ -1,\ I,\ -I \quad (16)$$

> $Sol_1 := x(t) = \exp\left(Raiz_1 \cdot t\right); Sol_2 := x(t) = \exp\left(Raiz_2 \cdot t\right); Sol_3 := x(t) = \exp\left(\mathrm{Re}\left(Raiz_3\right) \cdot t\right) \cdot\cos\left(\mathrm{Im}\left(Raiz_3\right) \cdot t\right); Sol_4 := x(t) = \exp\left(\mathrm{Re}\left(Raiz_3\right) \cdot t\right) \cdot \sin\left(\mathrm{Im}\left(Raiz_3\right) \cdot t\right)$

$$Sol_1 := x(t) = e^{t}$$

$$Sol_2 := x(t) = e^{-t}$$

$$Sol_3 := x(t) = \cos(t)$$

$$Sol_4 := x(t) = \sin(t) \quad (17)$$

> $SolucionGeneral := x(t) = C_1 \cdot rhs\left(Sol_1\right) + C_2 \cdot rhs\left(Sol_2\right) + C_3 \cdot rhs\left(Sol_3\right) + C_4 \cdot rhs\left(Sol_4\right)$

$$SolucionGeneral := x(t) = C_1\,e^{t} + C_2\,e^{-t} + C_3\cos(t) + C_4\sin(t) \quad (18)$$

>

FIN RESPUESTA 3)

> *restart*

RESPUESTA 4)

> $Sistema := diff(x(t), t) = x(t) - y(t), diff(y(t), t) = -x(t) + 2\,y(t) + \exp(-t) : Sistema_1;$
$Sistema_2;$

$$\frac{d}{dt}\,x(t) = x(t) - y(t)$$

$$\frac{d}{dt}\,y(t) = -x(t) + 2\,y(t) + e^{-t} \quad (19)$$

> $Variable := isolate\left(Sistema_2, x(t)\right)$

$$Variable := x(t) = -\left(\frac{d}{dt}y(t)\right) + 2\,y(t) + e^{-t} \quad (20)$$

> $Ecuacion := eval(subs(x(t) = rhs(Variable), Sistema_1))$

$$Ecuacion := -\left(\frac{d^2}{dt^2}y(t)\right) + 2\left(\frac{d}{dt}y(t)\right) - e^{-t} = -\left(\frac{d}{dt}y(t)\right) + y(t) + e^{-t} \quad (21)$$

> $NoHom := isolate(Ecuacion, \exp(-t))$

$$NoHom := e^{-t} = -\frac{1}{2}\frac{d^2}{dt^2}y(t) + \frac{3}{2}\frac{d}{dt}y(t) - \frac{1}{2}y(t) \quad (22)$$

> $EcuacionFinal := -2\cdot rhs(NoHom) = -2\cdot \exp(-t)$

$$EcuacionFinal := \frac{d^2}{dt^2}y(t) - 3\left(\frac{d}{dt}y(t)\right) + y(t) = -2\,e^{-t} \quad (23)$$

>

FIN RESPUESTA 4)

> $restart$

RESPUESTA 5)

> $Sistema := diff(y(t), t\$2) + diff(x(t), t\$2) = \text{Heaviside}(t), diff(y(t), t\$2) - diff(x(t), t\$2) = 4\,t : Sistema_1; Sistema_2$

$$\frac{d^2}{dt^2}y(t) + \frac{d^2}{dt^2}x(t) = \text{Heaviside}(t)$$

$$\frac{d^2}{dt^2}y(t) - \left(\frac{d^2}{dt^2}x(t)\right) = 4\,t \quad (24)$$

> $Condiciones := x(0) = 0, D(x)(0) = 0, y(0) = 0, D(y)(0) = 0$

$$Condiciones := x(0) = 0, D(x)(0) = 0, y(0) = 0, D(y)(0) = 0 \quad (25)$$

> $with(inttrans) :$

> $TransLapSisUno := laplace(Sistema_1, t, s)$

$$TransLapSisUno := s^2\,laplace(y(t), t, s) - D(y)(0) - s\,y(0) + s^2\,laplace(x(t), t, s) - D(x)(0) - s\,x(0) = \frac{1}{s} \quad (26)$$

> $TransLapSisDos := laplace(Sistema_2, t, s)$

$$TransLapSisDos := s^2\,laplace(y(t), t, s) - D(y)(0) - s\,y(0) - s^2\,laplace(x(t), t, s) + D(x)(0) + s\,x(0) = \frac{4}{s^2} \quad (27)$$

> $TransLapVariableUno := isolate(TransLapSisUno, laplace(y(t), t, s))$

$$TransLapVariableUno := laplace(y(t), t, s) = \frac{\frac{1}{s} + D(y)(0) + s\,y(0) - s^2\,laplace(x(t), t, s) + D(x)(0) + s\,x(0)}{s^2} \quad (28)$$

> $TransLapEcuacionDos := subs(laplace(y(t), t, s) = rhs(TransLapVariableUno), TransLapSisDos)$

$$TransLapEcuacionDos := \frac{1}{s} - 2\,s^2\,laplace(x(t), t, s) + 2\,D(x)(0) + 2\,s\,x(0) = \frac{4}{s^2} \quad (29)$$

```
> TransLapSolucionDos := isolate(TransLapEcuacionDos, laplace(x(t), t, s))
```

$$TransLapSolucionDos := laplace(x(t), t, s) = -\frac{1}{2}\,\frac{\frac{4}{s^2} - \frac{1}{s} - 2\,D(x)(0) - 2\,s\,x(0)}{s^2} \quad (30)$$

```
> TransLapSolucionUno := subs(laplace(x(t), t, s) = rhs(TransLapSolucionDos),
      TransLapVariableUno)
```

$$TransLapSolucionUno := laplace(y(t), t, s) = \frac{\frac{1}{2\,s} + D(y)(0) + s\,y(0) + \frac{2}{s^2}}{s^2} \quad (31)$$

```
> SolucionUno := expand(subs(Condiciones, invlaplace(TransLapSolucionUno, s, t)))
```

$$SolucionUno := y(t) = \frac{1}{4}\,t^2 + \frac{1}{3}\,t^3 \quad (32)$$

```
> SolucionDos := expand(subs(Condiciones, invlaplace(TransLapSolucionDos, s, t)))
```

$$SolucionDos := x(t) = -\frac{1}{3}\,t^3 + \frac{1}{4}\,t^2 \quad (33)$$

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FIN RESPUESTA 5)

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> restart
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RESPUESTA 6)

```
> Ecuacion := diff(z(x, y), x, y, y) = diff(z(x, y), y)
```

$$Ecuacion := \frac{\partial^3}{\partial y^2\,\partial x}\,z(x,y) = \frac{\partial}{\partial y}\,z(x,y) \quad (34)$$

```
> EcuacionDos := eval(subs(z(x, y) = F(x)·G(y), Ecuacion))
```

$$EcuacionDos := \left(\frac{d}{dx}\,F(x)\right)\left(\frac{d^2}{dy^2}\,G(y)\right) = F(x)\left(\frac{d}{dy}\,G(y)\right) \quad (35)$$

```
> EcuacionSeparada := lhs(EcuacionDos)/(diff(G(y), y)·(diff(F(x), x))) = rhs(EcuacionDos)/(diff(G(y), y)·(diff(F(x), x)))
```

$$EcuacionSeparada := \frac{\frac{d^2}{dy^2}\,G(y)}{\frac{d}{dy}\,G(y)} = \frac{F(x)}{\frac{d}{dx}\,F(x)} \quad (36)$$

```
> EcuacionY := lhs(EcuacionSeparada) = alpha; EcuacionX := rhs(EcuacionSeparada) = alpha
```

$$EcuacionY := \frac{\frac{d^2}{dy^2}\,G(y)}{\frac{d}{dy}\,G(y)} = \alpha$$

$$EcuacionX := \frac{F(x)}{\frac{d}{dx}\,F(x)} = \alpha \quad (37)$$

```
> SolucionX := dsolve(subs(alpha = beta··2, EcuacionX))
```

$$SolucionX := F(x) = \_C1\,e^{\frac{x}{\beta^2}} \quad (38)$$

```
> SolucionY := dsolve( subs( alpha = β··2, EcuacionY) )
```

$$SolucionY := G(y) = \_C1 + \_C2\, e^{\beta^2 y} \tag{39}$$

```
> SolucionGeneral := z(x, y) = rhs(SolucionY)·subs(_C1 = 1, rhs(SolucionX) )
```

$$SolucionGeneral := z(x, y) = \left(\_C1 + \_C2\, e^{\beta^2 y}\right) e^{\frac{x}{\beta^2}} \tag{40}$$

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FIN RESPUESTA 6)
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FIN EXAMEN
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