>

SOLUCIÓN

>

ECUACIONES DIFERENCIALES
SEGUNDO EXAMEN FINAL

2012-12-07

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1) Obtener la solución

> $Ecuacion := (\exp(y(x)) + \exp(-x)) + (\exp(y(x)) + 2\,y(x)\cdot\exp(-x))\cdot diff(y(x), x) = 0$

$$Ecuacion := e^{y(x)} + e^{-x} + \left(e^{y(x)} + 2\,y(x)\,e^{-x}\right)\left(\frac{d}{dx}\,y(x)\right) = 0 \quad \textbf{(1)}$$

> $Condicion := y(0) = 0$

$$Condicion := y(0) = 0 \quad \textbf{(2)}$$

RESPUESTA 1)

> $with(DEtools):$

> $odeadvisor(Ecuacion)$

$$[y=\_G(x,y')] \quad \textbf{(3)}$$

> $FactInt := intfactor(Ecuacion)$

$$FactInt := e^{x} \quad \textbf{(4)}$$

> $M := \exp(y) + \exp(-x)$

$$M := e^{y} + e^{-x} \quad \textbf{(5)}$$

> $N := e^{y} + 2\,y\,e^{-x}$

$$N := e^{y} + 2\,y\,e^{-x} \quad \textbf{(6)}$$

> $NoExacta := simplify(diff(M, y) - diff(N, x)) \neq 0$

$$NoExacta := e^{y} + 2\,y\,e^{-x} \neq 0 \quad \textbf{(7)}$$

> $MM := expand(FactInt\cdot M);\ NN := expand(FactInt\cdot N);$

$$MM := e^{y}\,e^{x} + 1$$

$$NN := e^{y}\,e^{x} + 2\,y \quad \textbf{(8)}$$

> $Exacta := simplify(diff(MM, y) - diff(NN, x)) = 0$

$$Exacta := 0 = 0 \quad \textbf{(9)}$$

> $IntMMx := int(MM, x)$

$$IntMMx := e^{y}\,e^{x} + x \quad \textbf{(10)}$$

> $SolucionGeneral := IntMMx + int((NN - diff(IntMMx, y)), y) = C_1$

$$SolucionGeneral := e^{y}\,e^{x} + x + y^2 = C_1 \quad \textbf{(11)}$$

> $Parametro := eval(subs(y = 0, x = 0, SolucionGeneral))$

$$Parametro := 1 = C_1 \quad \textbf{(12)}$$

> $SolucionParticular := subs\left(C_1 = lhs(Parametro), SolucionGeneral\right)$

$$SolucionParticular := e^{y}\,e^{x} + x + y^2 = 1 \quad \textbf{(13)}$$

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FIN RESPUESTA 1)

> $restart$

2) Obtenga la solución general

> Ecuacion := diff(y(t), t$2) − diff(y(t), t) − 2 y(t) =−3·exp(−t) + 2

$$Ecuacion := \frac{d^2}{dt^2}\, y(t) - \left(\frac{d}{dt}\, y(t)\right) - 2\, y(t) = -3\, e^{-t} + 2 \qquad \textbf{(14)}$$

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RESPUESTA 2)

> EcuacionHom := lhs(Ecuacion) = 0; Q := rhs(Ecuacion)

$$EcuacionHom := \frac{d^2}{dt^2}\, y(t) - \left(\frac{d}{dt}\, y(t)\right) - 2\, y(t) = 0$$

$$Q := -3\, e^{-t} + 2 \qquad \textbf{(15)}$$

> EcuacionCaract := m··2 − m − 2 = 0

$$EcuacionCaract := m^2 - m - 2 = 0 \qquad \textbf{(16)}$$

> Raiz := solve(EcuacionCaract)

$$Raiz := 2, -1 \qquad \textbf{(17)}$$

> $Sol_1 := y(t) = \exp(Raiz_1 \cdot t);\ Sol_2 := y(t) = \exp(Raiz_2 \cdot t)$

$$Sol_1 := y(t) = e^{2t}$$

$$Sol_2 := y(t) = e^{-t} \qquad \textbf{(18)}$$

> $SolucionHom := y(t) = C_1 \cdot rhs(Sol_1) + C_2 \cdot rhs(Sol_2)$

$$SolucionHom := y(t) = C_1\, e^{2t} + C_2\, e^{-t} \qquad \textbf{(19)}$$

> $SolucionNoHom := y(t) = A \cdot rhs(Sol_1) + B \cdot rhs(Sol_2)$

$$SolucionNoHom := y(t) = A\, e^{2t} + B\, e^{-t} \qquad \textbf{(20)}$$

> with(linalg) :

> $WW := wronskian([rhs(Sol_1), rhs(Sol_2)], t)$

$$WW := \begin{bmatrix} e^{2t} & e^{-t} \\ 2\, e^{2t} & -e^{-t} \end{bmatrix} \qquad \textbf{(21)}$$

> BB := array([0, Q])

$$BB := \begin{bmatrix} 0 & -3\, e^{-t} + 2 \end{bmatrix} \qquad \textbf{(22)}$$

> SOL := linsolve(WW, BB)

$$SOL := \begin{bmatrix} -\frac{1}{3}\, \frac{3\, e^{-t} - 2}{e^{2t}} & \frac{1}{3}\, \frac{3\, e^{-t} - 2}{e^{-t}} \end{bmatrix} \qquad \textbf{(23)}$$

> $A := int(SOL_1, t) + C_1;\ B := int(SOL_2, t) + C_2$

$$A := \frac{1}{3\, (e^{t})^3} - \frac{1}{3\, (e^{t})^2} + C_1$$

$$B := -\frac{2}{3\, e^{-t}} - \ln(e^{-t}) + C_2 \qquad \textbf{(24)}$$

> SolucionGeneral := simplify(expand(SolucionNoHom))

(25)

$$SolucionGeneral := y(t) = \frac{1}{3}\,e^{-t} - 1 + C_1\,e^{2t} - e^{-t}\ln\left(e^{-t}\right) + C_2\,e^{-t} \quad (25)$$

> $comprobacion_1 := simplify(eval(subs(y(t) = rhs(SolucionGeneral), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$comprobacion_1 := 0 = 0 \quad (26)$$

> $SolucionFinal := simplify(dsolve(Ecuacion))$

$$SolucionFinal := y(t) = e^{2t}\,\_C2 + e^{-t}\,\_C1 + \frac{1}{3}\,e^{-t} - 1 + e^{-t}\,t \quad (27)$$

> $comprobacion_2 := simplify(eval(subs(y(t) = rhs(SolucionFinal), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$comprobacion_2 := 0 = 0 \quad (28)$$

FIN RESPUESTA 2)

> *restart*

3) Obtenga la solucion general

> $Ecuacion := y'' + 4\,y = \csc(2\,x)$

$$Ecuacion := \frac{d^2}{dx^2}\,y(x) + 4\,y(x) = \csc(2\,x) \quad (29)$$

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RESPUESTA 3)

> $EcuacionHom := lhs(Ecuacion) = 0$

$$EcuacionHom := \frac{d^2}{dx^2}\,y(x) + 4\,y(x) = 0 \quad (30)$$

> $Q := rhs(Ecuacion)$

$$Q := \csc(2\,x) \quad (31)$$

> $EcuacionCaract := m\cdot\cdot 2 + 4 = 0$

$$EcuacionCaract := m^2 + 4 = 0 \quad (32)$$

> $Raiz := solve(EcuacionCaract)$

$$Raiz := 2\,I,\ -2\,I \quad (33)$$

> $Sol_1 := y(x) = \cos\left(\mathrm{Im}\left(Raiz_1\right)\cdot x\right);\ Sol_2 := y(x) = \sin\left(\mathrm{Im}\left(Raiz_1\right)\cdot x\right)$

$$Sol_1 := y(x) = \cos(2\,x)$$

$$Sol_2 := y(x) = \sin(2\,x) \quad (34)$$

> $SolucionHom := y(x) = C_1\cdot rhs\left(Sol_1\right) + C_2\cdot rhs\left(Sol_2\right)$

$$SolucionHom := y(x) = C_1\cos(2\,x) + C_2\sin(2\,x) \quad (35)$$

> $SolucionNoHom := y(x) = A\cdot rhs\left(Sol_1\right) + B\cdot rhs\left(Sol_2\right)$

$$SolucionNoHom := y(x) = A\cos(2\,x) + B\sin(2\,x) \quad (36)$$

> $with(linalg):$

> $WW := wronskian\left(\left[rhs\left(Sol_1\right), rhs\left(Sol_2\right)\right], x\right)$

$$WW := \begin{bmatrix} \cos(2\,x) & \sin(2\,x) \\ -2\sin(2\,x) & 2\cos(2\,x) \end{bmatrix} \quad (37)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \csc(2x) \end{bmatrix} \quad \textbf{(38)}$$

> $SOL := simplify(linsolve(WW, BB))$

$$SOL := \begin{bmatrix} -\frac{1}{2} & \frac{1}{2}\frac{\cos(2x)}{\sin(2x)} \end{bmatrix} \quad \textbf{(39)}$$

> $A := int(SOL_1, x) + C_1;\ B := int(SOL_2, x) + C_2$

$$A := -\frac{1}{2}x + C_1$$

$$B := \frac{1}{4}\ln(\sin(2x)) + C_2 \quad \textbf{(40)}$$

> $SolucionGeneral := simplify(SolucionNoHom);$

$$SolucionGeneral := y(x) = -\frac{1}{2}\cos(2x)\,x + C_1\cos(2x) + \frac{1}{4}\sin(2x)\ln(\sin(2x)) + C_2\sin(2x) \quad \textbf{(41)}$$

> $comprobacion := simplify(eval(subs(y(x) = rhs(SolucionGeneral), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$comprobacion := 0 = 0 \quad \textbf{(42)}$$

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FIN RESPUESTA 3)

> *restart*

4) Resuleva el sistema de ecuaciones

> $Sistema := 2\cdot diff(x(t), t) + diff(y(t), t) + x(t) + y(t) = t + 1, diff(x(t), t) + 3\cdot x(t) + 2\cdot y(t) = t + 1 : Sistema_1;\ Sistema_2;\ Condiciones := x(0) = -1, y(0) = 3$

$$2\left(\frac{d}{dt}x(t)\right) + \frac{d}{dt}y(t) + x(t) + y(t) = t + 1$$

$$\frac{d}{dt}x(t) + 3x(t) + 2y(t) = t + 1$$

$$Condiciones := x(0) = -1, y(0) = 3 \quad \textbf{(43)}$$

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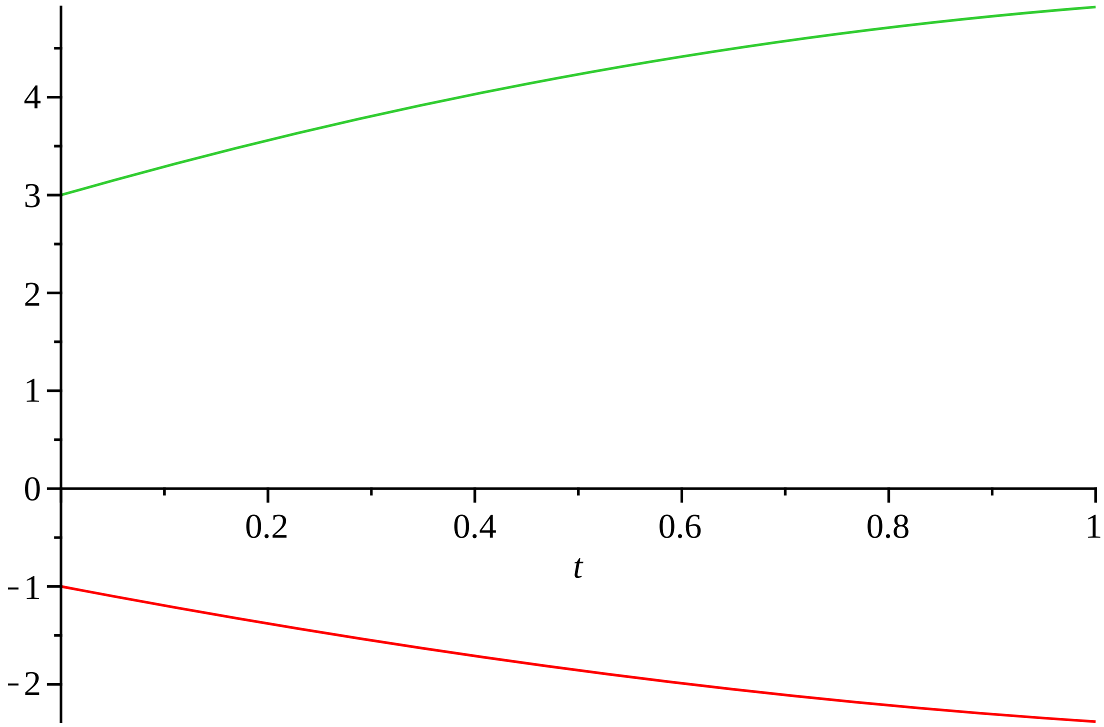
RESPUESTA 4)

> $Solucion := dsolve(\{Sistema, Condiciones\}) : Solucion_1;\ Solucion_2;$

$$x(t) = -\sin(t) - \cos(t) - t$$

$$y(t) = \sin(t) + 2\cos(t) + 1 + 2t \quad \textbf{(44)}$$

> $plot([rhs(Solucion_1), rhs(Solucion_2)], t = 0..1)$



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FIN RESPUESTA 4)

\> *restart*

5) Resuelva la ecuación

\> $Ecuacion := diff(y(t), t) + int(y(v) \cdot \exp(-2 \cdot (t - v)), v = 0..t) = 1$

$$Ecuacion := \frac{d}{dt} y(t) + \int_0^t y(v)\, e^{-2t+2v}\, dv = 1 \quad \textbf{(45)}$$

\> $Condicion := y(0) = 1$

$$Condicion := y(0) = 1 \quad \textbf{(46)}$$

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RESPUESTA 5)

\> $with(inttrans):$

\> $TransLapEcuacion := subs(Condicion, laplace(Ecuacion, t, s))$

$$TransLapEcuacion := s\, laplace(y(t), t, s) - 1 + \frac{1}{2} \frac{laplace(y(t), t, s)}{1 + \frac{1}{2} s} = \frac{1}{s} \quad \textbf{(47)}$$

\> $TransLapSolucion := simplify(isolate(TransLapEcuacion, laplace(y(t), t, s)))$

$$TransLapSolucion := laplace(y(t), t, s) = \frac{2 + s}{(1 + s)\, s} \quad \textbf{(48)}$$

\> $Solucion := invlaplace(TransLapSolucion, s, t)$

$$Solucion := y(t) = 2 - e^{-t} \quad \textbf{(49)}$$

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FIN RESPUESTA 5)

\> *restart*

6a) Separacion Variables para alpha=1

> $Ecuacion := diff(u(x,y), x\$2) + y \cdot diff(u(x,y), y) = 0$

$$Ecuacion := \frac{\partial^2}{\partial x^2} u(x,y) + y \left(\frac{\partial}{\partial y} u(x,y)\right) = 0 \quad \textbf{(50)}$$

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RESPUESTA 6a)

> $EcuacionDos := eval(subs(u(x,y) = F(x) \cdot G(y), Ecuacion))$

$$EcuacionDos := \left(\frac{d^2}{dx^2} F(x)\right) G(y) + y\, F(x) \left(\frac{d}{dy} G(y)\right) = 0 \quad \textbf{(51)}$$

> $EcuacionTres := \frac{\left(lhs(EcuacionDos) - y\, F(x) \left(\frac{d}{dy} G(y)\right)\right)}{F(x) \cdot G(y)} = \left(\frac{rhs(EcuacionDos) - y\, F(x) \left(\frac{d}{dy} G(y)\right)}{F(x) \cdot G(y)}\right)$

$$EcuacionTres := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = -\frac{y \left(\frac{d}{dy} G(y)\right)}{G(y)} \quad \textbf{(52)}$$

> $EcuacionX := lhs(EcuacionTres) = \text{alpha};\ EcuacionY := rhs(EcuacionTres) = \text{alpha}$

$$EcuacionX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \alpha$$

$$EcuacionY := -\frac{y \left(\frac{d}{dy} G(y)\right)}{G(y)} = \alpha \quad \textbf{(53)}$$

> $SolucionX := dsolve(subs(\text{alpha} = 1, EcuacionX))$

$$SolucionX := F(x) = \_C1\, e^{x} + \_C2\, e^{-x} \quad \textbf{(54)}$$

> $SolucionY := dsolve(subs(\text{alpha} = 1, EcuacionY))$

$$SolucionY := G(y) = \frac{\_C1}{y} \quad \textbf{(55)}$$

> $SolucionGeneral := u(x,y) = rhs(SolucionX) \cdot subs(\_C1 = 1, rhs(SolucionY))$

$$SolucionGeneral := u(x,y) = \frac{\_C1\, e^{x} + \_C2\, e^{-x}}{y} \quad \textbf{(56)}$$

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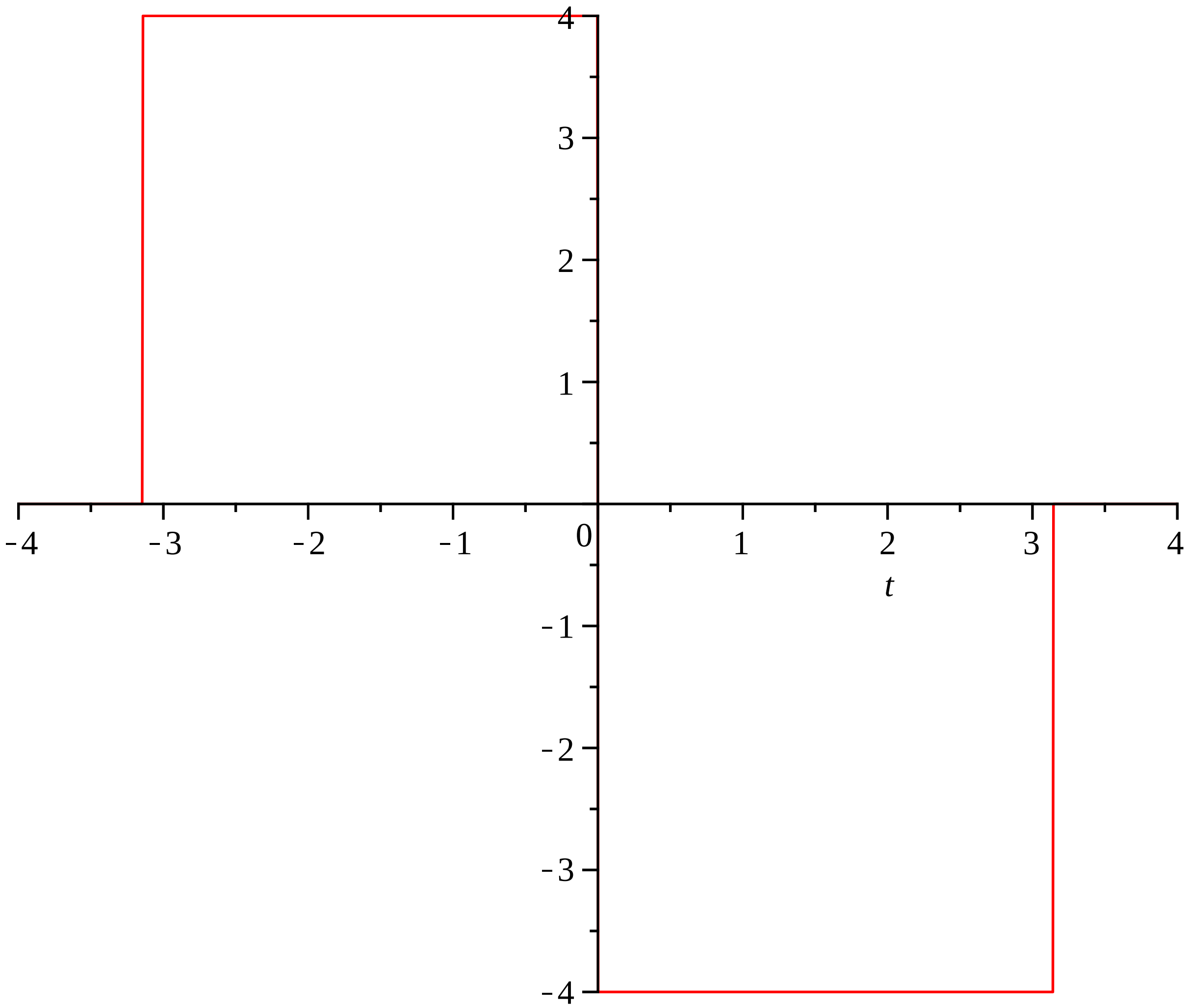
FIN RESPUESTA 6a)

> $restart$

6b) Serie de Fourier

> $f := 4 \cdot \text{Heaviside}(t + \text{Pi}) - 8 \cdot \text{Heaviside}(t) + 4 \cdot \text{Heaviside}(t - \text{Pi}) : plot(f, t = -4..4)$



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RESPUESTA 6b)

> $L := \text{Pi};$

$$L := \pi \qquad (57)$$

> $b_n := subs\left(\cos(n\cdot\text{Pi}) = (-1)^n,\ \left(\frac{1}{L}\right)\cdot int\left(f\cdot\sin\left(\frac{n\cdot\text{Pi}\cdot t}{L}\right), t=-L..L\right)\right)$

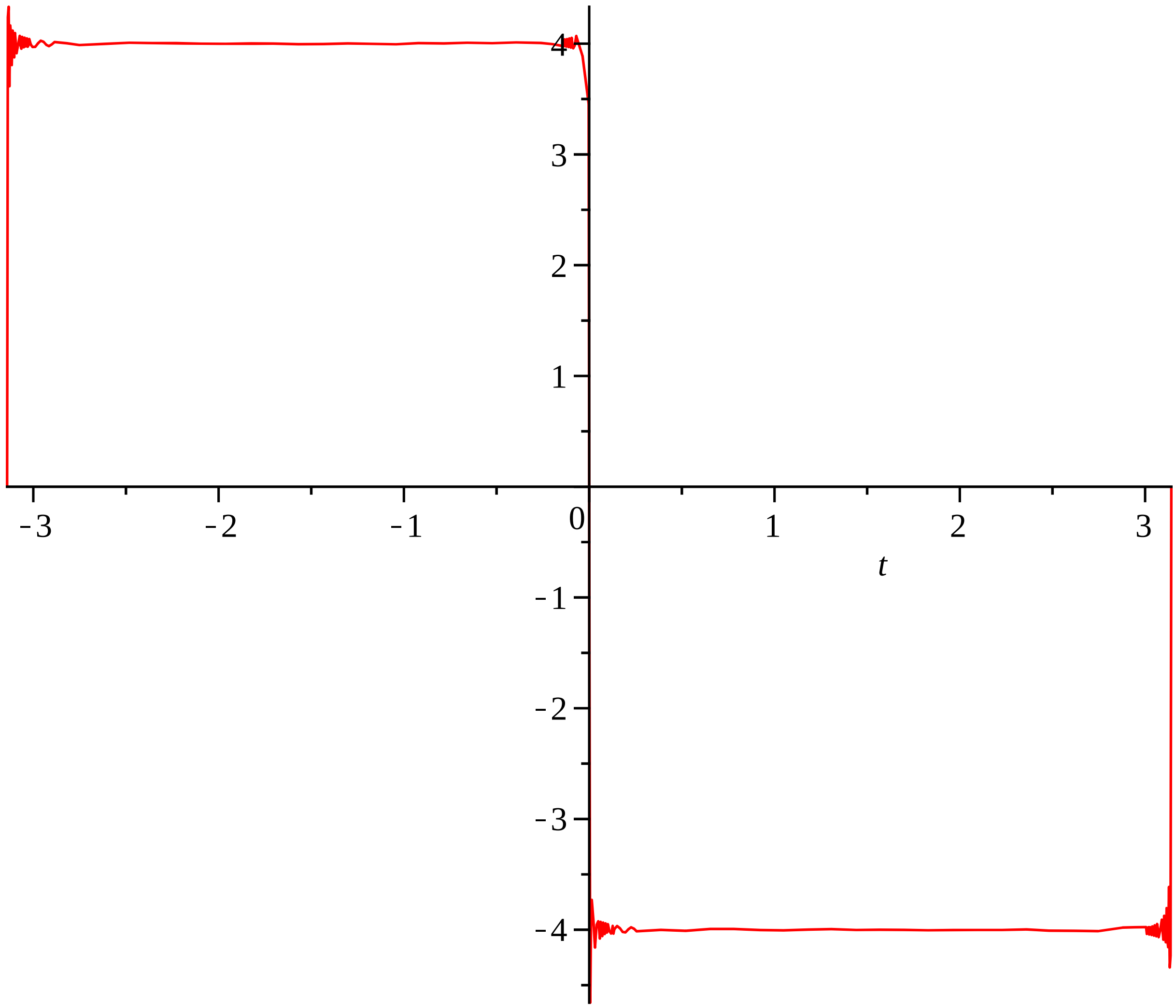
$$b_n := \frac{8\ (-1)^n - 8}{\pi\, n} \qquad (58)$$

> $STF := Sum\left(b_n\cdot\sin\left(\frac{n\cdot\text{Pi}\cdot t}{L}\right), n = 1 ..infinity\right)$

$$STF := \sum_{n=1}^{\infty} \frac{\left(8\ (-1)^n - 8\right)\ \sin(n\, t)}{\pi\, n} \qquad (59)$$

> $STF_{500} := sum\left(b_n\cdot\sin\left(\frac{n\cdot\text{Pi}\cdot t}{L}\right), n = 1 ..500\right)$:

> $plot(STF_{500}, t=-L..L)$



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FIN RESPUESTA 6b)

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FIN RESPUESTAS 6)

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FIN EXAMEN