>

SOLUCIÓN

FACULTAD DE INGENIERIA
DIVISIÓN DE CIENCIAS BÁSICAS
ECUACIONES DIFERENCIALES
SEGUNDO EXAMEN FINAL

SEMESTRE 2014-2
JUNIO 4 DE 2014
TIPO "A"

> *restart*

1) Resolver la ecuación diferencial

> $Ecuacion := (\sin(x)\cdot\cos(x))\cdot y' + y = \tan(x)\cdot\cdot 2$

$$Ecuacion := \sin(x)\,\cos(x)\left(\frac{d}{dx}\,y(x)\right) + y(x) = \tan(x)^2 \quad (1)$$

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RESPUESTA 1)

Método 1 = Como ecuación diferencial lineal de coeficentes variables no homogénea

> $EcuacionDos := expand\left(\frac{lhs(Ecuacion)}{\sin(x)\cdot\cos(x)}\right) = simplify\left(\frac{rhs(Ecuacion)}{\sin(x)\cdot\cos(x)}\right)$

$$EcuacionDos := \frac{d}{dx}\,y(x) + \frac{y(x)}{\sin(x)\,\cos(x)} = \frac{\sin(x)}{\cos(x)^3} \quad (2)$$

> $p := \frac{1}{\sin(x)\,\cos(x)};\ q := rhs(EcuacionDos)$

$$p := \frac{1}{\sin(x)\,\cos(x)}$$

$$q := \frac{\sin(x)}{\cos(x)^3} \quad (3)$$

> $ExpPos := \exp(int(p, x))$

$$ExpPos := \tan(x) \quad (4)$$

> $ExpNeg := \exp(-int(p, x))$

$$ExpNeg := \frac{1}{\tan(x)} \quad (5)$$

> $SolucionCero := C_1\cdot ExpNeg + simplify(ExpNeg\cdot int(ExpPos\cdot q, x))$

$$SolucionCero := \frac{C_1}{\tan(x)} + \frac{1}{3}\,\frac{\sin(x)^2}{\cos(x)^2} \quad (6)$$

> $SolucionUno := C_1\cdot ExpNeg + \frac{1}{3}\cdot\tan(x)\cdot\cdot 2$

$$SolucionUno := \frac{C_1}{\tan(x)} + \frac{1}{3}\,\tan(x)^2 \quad (7)$$

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Método 2 = Como no lineal

> *with*(*DEtools*) :

> *odeadvisor*(*Ecuacion*)

[_linear] **(8)**

> $FactInt := simplify(intfactor(Ecuacion))$

$$FactInt := \frac{2}{\cos(x)^2} \qquad (9)$$

> $M := y - \tan(x)^2;\ N := \sin(x)\cdot\cos(x)$

$$M := y - \tan(x)^2$$

$$N := \sin(x)\cos(x) \qquad (10)$$

> $MM := simplify(FactInt \cdot M);\ NN := simplify(FactInt \cdot N)$

$$MM := \frac{2\left(y\cos(x)^2 - 1 + \cos(x)^2\right)}{\cos(x)^4}$$

$$NN := \frac{2\sin(x)}{\cos(x)} \qquad (11)$$

> $comprobacion := \mathrm{simplify}(\mathrm{diff}(MM, y) - diff(NN, x)) = 0$

$$comprobacion := 0 = 0 \qquad (12)$$

> $SolucionDos := \frac{(int(MM, x) + simplify(int((NN - diff(int(MM, x), y)), y)))}{2} = C_1$

$$SolucionDos := \frac{y\sin(x)}{\cos(x)} - \frac{1}{3}\frac{\sin(x)}{\cos(x)^3} + \frac{1}{3}\frac{\sin(x)}{\cos(x)} = C_1 \qquad (13)$$

> $SolucionDosMedio := y(x)\cdot\tan(x) + simplify\left(-\frac{1}{3}\frac{\sin(x)}{\cos(x)^3} + \frac{1}{3}\frac{\sin(x)}{\cos(x)}\right) = C_1$

$$SolucionDosMedio := y(x)\tan(x) - \frac{1}{3}\frac{\sin(x)^3}{\cos(x)^3} = C_1 \qquad (14)$$

> $SolucionDosCuartos := y(x)\cdot\tan(x) - \frac{1}{3}\cdot\tan(x)^3 = C_1$

$$SolucionDosCuartos := y(x)\tan(x) - \frac{1}{3}\tan(x)^3 = C_1 \qquad (15)$$

> $SolucionTres := expand(isolate(SolucionDos, y))$

$$SolucionTres := y = \frac{1}{2}\frac{C_1\cos(x)}{\sin(x)} + \frac{1}{3\cos(x)^2} - \frac{1}{3} \qquad (16)$$

> $SolucionCuatro := y(x) = \frac{C_1}{\tan(x)} + simplify\left(\frac{1}{3}\cdot\left(\frac{1}{\cos(x)^2} - 1\right)\right)$

$$SolucionCuatro := y(x) = \frac{C_1}{\tan(x)} + \frac{1}{3}\frac{\sin(x)^2}{\cos(x)^2} \qquad (17)$$

> $SolucionCinco := y(x) = \frac{C_1}{\tan(x)} + \frac{1}{3}\cdot\tan(x)^2$

$$SolucionCinco := y(x) = \frac{C_1}{\tan(x)} + \frac{1}{3}\tan(x)^2 \qquad (18)$$

Método 3 = directo

> $SolucionSeis := expand(simplify(dsolve(Ecuacion)))$

$$SolucionSeis := y(x) = \frac{\cos(x)\ \_C1}{\sin(x)} + \frac{1}{3\cos(x)^2} - \frac{1}{3} \qquad (19)$$

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FIN RESPUESTA 1)

> *restart*

2) Resolver la ecuación diferencial

> $Ecuacion := y''' - 4\,y' = 4 + 32\,\sin(2\,x)$

$$Ecuacion := \frac{d^3}{dx^3}\,y(x) - 4\left(\frac{d}{dx}\,y(x)\right) = 4 + 32\,\sin(2\,x) \quad \textbf{(20)}$$

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RESPUESTA 2)

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Método 1 = Parámetros Variables

> $EcuacionHom := lhs(Ecuacion) = 0$

$$EcuacionHom := \frac{d^3}{dx^3}\,y(x) - 4\left(\frac{d}{dx}\,y(x)\right) = 0 \quad \textbf{(21)}$$

> $Q := rhs(Ecuacion)$

$$Q := 4 + 32\,\sin(2\,x) \quad \textbf{(22)}$$

> $EcuacionCarac := m\cdot\cdot 3 - 4\cdot m = 0$

$$EcuacionCarac := m^3 - 4\,m = 0 \quad \textbf{(23)}$$

> $Raiz := solve(EcuacionCarac)$

$$Raiz := 0, 2, -2 \quad \textbf{(24)}$$

> $SolUno := y(x) = \exp(Raiz_1\cdot x);\ SolDos := y(x) = \exp(Raiz_2\cdot x);\ SolTres := y(x) = \exp(Raiz_3\cdot x)$

$$SolUno := y(x) = 1$$
$$SolDos := y(x) = e^{2\,x}$$
$$SolTres := y(x) = e^{-2\,x} \quad \textbf{(25)}$$

> $SolucionHom := y(x) = C_1\cdot rhs(SolUno) + C_2\cdot rhs(SolDos) + C_3\cdot rhs(SolTres)$

$$SolucionHom := y(x) = C_1 + C_2\,e^{2\,x} + C_3\,e^{-2\,x} \quad \textbf{(26)}$$

> $SolucionNoHom := y(x) = A\cdot rhs(SolUno) + B\cdot rhs(SolDos) + E\cdot rhs(SolTres)$

$$SolucionNoHom := y(x) = A + B\,e^{2\,x} + E\,e^{-2\,x} \quad \textbf{(27)}$$

> $with(linalg):$

> $WW := wronskian([rhs(SolUno), rhs(SolDos), rhs(SolTres)], x)$

$$WW := \begin{bmatrix} 1 & e^{2\,x} & e^{-2\,x} \\ 0 & 2\,e^{2\,x} & -2\,e^{-2\,x} \\ 0 & 4\,e^{2\,x} & 4\,e^{-2\,x} \end{bmatrix} \quad \textbf{(28)}$$

> $BB := array([0, 0, Q])$

$$BB := \begin{bmatrix} 0 & 0 & 4 + 32\,\sin(2\,x) \end{bmatrix} \quad \textbf{(29)}$$

> $SOL := simplify(linsolve(WW, BB))$

$$SOL := \begin{bmatrix} -1 - 8\,\sin(2\,x) & \frac{1}{2}\,(1 + 8\,\sin(2\,x))\,e^{-2\,x} & \frac{1}{2}\,(1 + 8\,\sin(2\,x))\,e^{2\,x} \end{bmatrix} \quad \textbf{(30)}$$

> $Aprima := SOL_1; Bprima := SOL_2; Eprima := SOL_3$

$$Aprima := -1 - 8\sin(2x)$$

$$Bprima := \frac{1}{2}(1 + 8\sin(2x))\,e^{-2x}$$

$$Eprima := \frac{1}{2}(1 + 8\sin(2x))\,e^{2x} \quad \mathbf{(31)}$$

> $A := int(Aprima, x) + C_1; B := int(Bprima, x) + C_2; E := int(Eprima, x) + C_3$

$$A := -x + 4\cos(2x) + C_1$$

$$B := -\frac{1}{4(e^x)^2} + \frac{1}{2}e^{-2x}(-2\sin(2x) - 2\cos(2x)) + C_2$$

$$E := \frac{1}{4}e^{2x} - \cos(2x)\,e^{2x} + e^{2x}\sin(2x) + C_3 \quad \mathbf{(32)}$$

> $SolucionFinal := expand(simplify(SolucionNoHom))$

$$SolucionFinal := y(x) = -x + 4\cos(x)^2 - 2 + C_1 + (e^x)^2 C_2 + \frac{C_3}{(e^x)^2} \quad \mathbf{(33)}$$

Método 2 = directo

> $SolucionDiez := dsolve(Ecuacion)$

$$SolucionDiez := y(x) = \frac{1}{2}e^{2x}\_C2 - \frac{1}{2}e^{-2x}\_C1 + 2\cos(2x) - x + \_C3 \quad \mathbf{(34)}$$

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FIN RESPUESTA 2)

> *restart*

3) Resolver la ecuación diferencial

> $Ecuacion := y''' + y' = \tan(x)$

$$Ecuacion := \frac{d^3}{dx^3}y(x) + \frac{d}{dx}y(x) = \tan(x) \quad \mathbf{(35)}$$

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RESPUESTA 3)

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Método 1 = Parámetros Variables

> $EcuacionHom := lhs(Ecuacion) = 0$

$$EcuacionHom := \frac{d^3}{dx^3}y(x) + \frac{d}{dx}y(x) = 0 \quad \mathbf{(36)}$$

> $Q := rhs(Ecuacion)$

$$Q := \tan(x) \quad \mathbf{(37)}$$

> $EcuacionCarac := m \cdot \cdot 3 + m = 0$

$$EcuacionCarac := m^3 + m = 0 \quad \mathbf{(38)}$$

> $Raiz := solve(EcuacionCarac)$

$$Raiz := 0, I, -I \quad \mathbf{(39)}$$

> $SolUno := y(x) = \exp(Raiz_1 \cdot x); SolDos := y(x) = \exp(\mathrm{Re}(Raiz_2)\cdot x)\cdot\cos(\mathrm{Im}(Raiz_2)\cdot x);$
$SolTres := y(x) = \exp(\mathrm{Re}(Raiz_2)\cdot x)\cdot\sin(\mathrm{Im}(Raiz_2)\cdot x)$

$$SolUno := y(x) = 1$$
$$SolDos := y(x) = \cos(x)$$
$$SolTres := y(x) = \sin(x) \quad \textbf{(40)}$$

> $SolucionHom := y(x) = C_1 \cdot rhs(SolUno) + C_2 \cdot rhs(SolDos) + C_3 \cdot rhs(SolTres)$

$$SolucionHom := y(x) = C_1 + C_2 \cos(x) + C_3 \sin(x) \quad \textbf{(41)}$$

> $SolucionNoHom := y(x) = A \cdot rhs(SolUno) + B \cdot rhs(SolDos) + E \cdot rhs(SolTres)$

$$SolucionNoHom := y(x) = A + B\cos(x) + E\sin(x) \quad \textbf{(42)}$$

> $with(linalg):$

> $WW := wronskian([rhs(SolUno), rhs(SolDos), rhs(SolTres)], x)$

$$WW := \begin{bmatrix} 1 & \cos(x) & \sin(x) \\ 0 & -\sin(x) & \cos(x) \\ 0 & -\cos(x) & -\sin(x) \end{bmatrix} \quad \textbf{(43)}$$

> $BB := array([0, 0, Q])$

$$BB := \begin{bmatrix} 0 & 0 & \tan(x) \end{bmatrix} \quad \textbf{(44)}$$

> $SOL := simplify(linsolve(WW, BB))$

$$SOL := \begin{bmatrix} \tan(x) & -\sin(x) & -\frac{\sin(x)^2}{\cos(x)} \end{bmatrix} \quad \textbf{(45)}$$

> $Aprima := SOL_1; Bprima := SOL_2; Eprima := SOL_3$

$$Aprima := \tan(x)$$
$$Bprima := -\sin(x)$$
$$Eprima := -\frac{\sin(x)^2}{\cos(x)} \quad \textbf{(46)}$$

> $A := int(Aprima, x) + C_1; B := int(Bprima, x) + C_2; E := int(Eprima, x) + C_3$

$$A := -\ln(\cos(x)) + C_1$$
$$B := \cos(x) + C_2$$
$$E := \sin(x) - \ln(\sec(x) + \tan(x)) + C_3 \quad \textbf{(47)}$$

> $SolucionFinal := simplify(SolucionNoHom)$

$$SolucionFinal := y(x) = -\ln(\cos(x)) + C_1 + 1 + C_2\cos(x) - \sin(x)\ln\left(\frac{1+\sin(x)}{\cos(x)}\right) + C_3\sin(x) \quad \textbf{(48)}$$

> $SolucionFinalDos := y(x) = C_1 + C_2 \cdot \cos(x) + C_3 \cdot \sin(x) + \log(\sec(x)) - \sin(x) \cdot \log(\sec(x) + \tan(x))$

$$SolucionFinalDos := y(x) = C_1 + C_2\cos(x) + C_3\sin(x) + \ln(\sec(x)) - \sin(x)\ln(\sec(x) + \tan(x)) \quad \textbf{(49)}$$

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FIN RESPUESTA 3)

> *restart*

4) Obtener Transformada de Laplace

> $f := (2 \cdot t - 5) \cdot \text{Heaviside}(t - 1)$

$$f := (2\,t - 5)\ \mathrm{Heaviside}(t - 1) \qquad \textbf{(50)}$$

> $g := \mathit{Diff}(t \cdot \sin(3 \cdot t), t)$

$$g := \frac{\mathrm{d}}{\mathrm{d}t}\,(t \sin(3\,t)) \qquad \textbf{(51)}$$

> $h := \exp(3\,t) \cdot \mathit{int}(2 \cdot \exp(\mathrm{tau}), \mathrm{tau} = 0\,..t)$

$$h := \mathrm{e}^{3\,t}\,(-2 + 2\,\mathrm{e}^{t}) \qquad \textbf{(52)}$$

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RESPUESTA 4)

> $\mathit{with}(\mathit{inttrans})$:

> $F := \mathit{laplace}(f, t, s)$

$$F := \mathrm{e}^{-s}\left(-\frac{3}{s} + \frac{2}{s^2}\right) \qquad \textbf{(53)}$$

> $G := \mathit{laplace}(g, t, s)$

$$G := \frac{6\,s^2}{(s^2 + 9)^2} \qquad \textbf{(54)}$$

> $H := \mathit{laplace}(\mathrm{Dirac}(t - 1), t, s) \cdot \mathit{laplace}(h, t, s)$

$$H := \frac{2\,\mathrm{e}^{-s}}{(-3 + s)\,(-4 + s)} \qquad \textbf{(55)}$$

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FIN RESPUESTA 4)

> *restart*

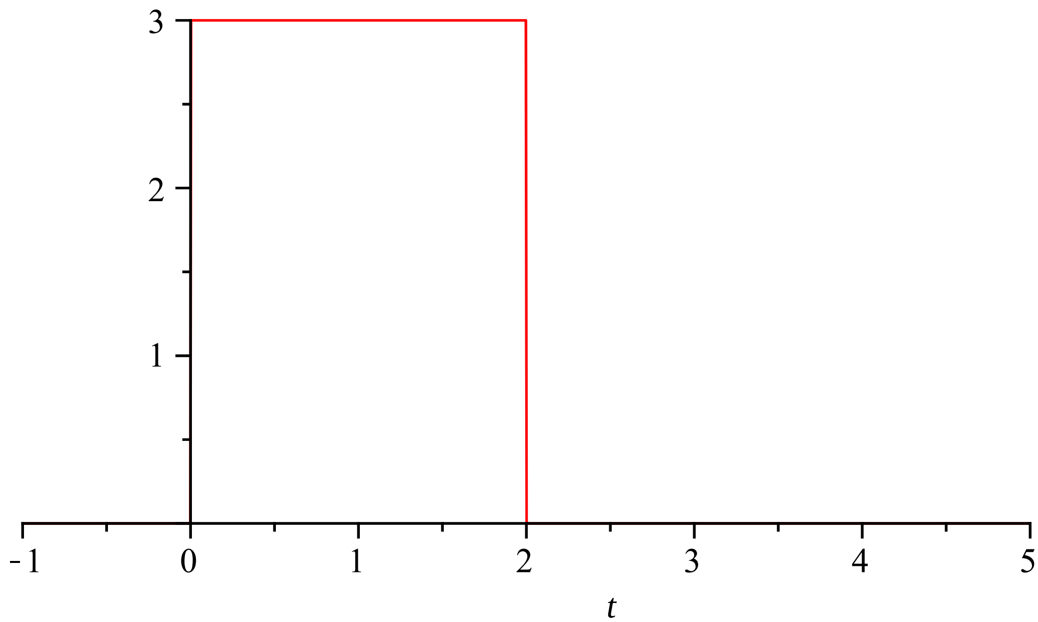
5) Resuelva el circuito RLC

> $R := 4;\ L := 1;\ C := \frac{2}{10}$

$$R := 4$$
$$L := 1$$
$$C := \frac{1}{5} \qquad \textbf{(56)}$$

> $E := 3\ \mathrm{Heaviside}(t) - 3 \cdot \mathrm{Heaviside}(t - 2);\ \mathit{plot}(E, t = -1\,..5, \mathit{scaling} = \mathit{CONSTRAINED})$

$$E := 3\ \mathrm{Heaviside}(t) - 3\ \mathrm{Heaviside}(t - 2)$$



> $Ecuacion := L \cdot diff(i(t), t) + R \cdot i(t) + \frac{1}{C} \cdot int(i(\mathrm{tau}), \mathrm{tau} = 0..t) = E$

$$Ecuacion := \frac{d}{dt}\, i(t) + 4\, i(t) + 5 \left(\int_0^t i(\tau)\, d\tau \right) = 3\, \mathrm{Heaviside}(t) - 3\, \mathrm{Heaviside}(t-2) \quad \textbf{(57)}$$

> $Condicion := i(0) = 0$

$$Condicion := i(0) = 0 \quad \textbf{(58)}$$

RESPUESTA 5)

> $with(inttrans):$

> $TransLapEcuacion := subs(Condicion, laplace(Ecuacion, t, s))$

$$TransLapEcuacion := s\, laplace(i(t), t, s) + 4\, laplace(i(t), t, s) + \frac{5\, laplace(i(t), t, s)}{s} = \frac{3\,(1 - e^{-2s})}{s} \quad \textbf{(59)}$$

> $TransLapSolucion := isolate(TransLapEcuacion, laplace(i(t), t, s))$

$$TransLapSolucion := laplace(i(t), t, s) = \frac{3\,(1 - e^{-2s})}{s\left(s + 4 + \frac{5}{s}\right)} \quad \textbf{(60)}$$

> *SolucionParticular* := *invlaplace*(*TransLapSolucion*, *s*, *t*)

$$SolucionParticular := i(t) = 3\,e^{-2t}\sin(t) - 3\,\mathrm{Heaviside}(t-2)\,e^{-2t+4}\sin(t-2) \quad \textbf{(61)}$$

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FIN RESPUESTA 5)

> *restart*

6) Obtener la solución completa de la ecuación considerando alpha = 3

> $Ecuacion := \frac{1}{t}\cdot diff(u(x,t), x\$2) + 3\cdot diff(u(x,t), t, x) - 3\cdot diff(u(x,t), x) = 0$

$$Ecuacion := \frac{\frac{\partial^2}{\partial x^2} u(x,t)}{t} + 3\left(\frac{\partial^2}{\partial x\,\partial t} u(x,t)\right) - 3\left(\frac{\partial}{\partial x} u(x,t)\right) = 0 \quad \textbf{(62)}$$

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RESPUESTA 6)

> $EcuacionDos := eval(subs(u(x,t) = F(x)\cdot G(t), Ecuacion))$

$$EcuacionDos := \frac{\left(\frac{d^2}{dx^2}F(x)\right)G(t)}{t} + 3\left(\frac{d}{dx}F(x)\right)\left(\frac{d}{dt}G(t)\right) - 3\left(\frac{d}{dx}F(x)\right)G(t) = 0 \quad \textbf{(63)}$$

> $EcuacionTres := lhs(EcuacionDos) - 3\left(\frac{d}{dx}F(x)\right)\left(\frac{d}{dt}G(t)\right) + 3\left(\frac{d}{dx}F(x)\right)G(t) = rhs(EcuacionDos) - 3\left(\frac{d}{dx}F(x)\right)\left(\frac{d}{dt}G(t)\right) + 3\left(\frac{d}{dx}F(x)\right)G(t)$

$$EcuacionTres := \frac{\left(\frac{d^2}{dx^2}F(x)\right)G(t)}{t} = -3\left(\frac{d}{dx}F(x)\right)\left(\frac{d}{dt}G(t)\right) + 3\left(\frac{d}{dx}F(x)\right)G(t) \quad \textbf{(64)}$$

> $EcuacionCuatro := simplify\left(\frac{lhs(EcuacionTres)\cdot t}{G(t)\cdot diff(F(x), x)}\right) = simplify\left(\frac{rhs(EcuacionTres)\cdot t}{G(t)\cdot diff(F(x), x)}\right)$

$$EcuacionCuatro := \frac{\frac{d^2}{dx^2}F(x)}{\frac{d}{dx}F(x)} = -\frac{3\left(\frac{d}{dt}G(t) - G(t)\right)t}{G(t)} \quad \textbf{(65)}$$

> $EcuacionX := lhs(EcuacionCuatro) = \mathrm{alpha};\ EcuacionT := rhs(EcuacionCuatro) = \mathrm{alpha}$

$$EcuacionX := \frac{\frac{d^2}{dx^2}F(x)}{\frac{d}{dx}F(x)} = \alpha$$

$$EcuacionT := -\frac{3\left(\frac{d}{dt}G(t) - G(t)\right)t}{G(t)} = \alpha \quad \textbf{(66)}$$

> $SolucionX := subs(\mathrm{alpha} = 3, dsolve(EcuacionX))$

$$SolucionX := F(x) = \_C1 + \_C2\,e^{3x} \quad \textbf{(67)}$$

> $SolucionT := subs(\mathrm{alpha} = 3, dsolve(EcuacionT))$

(68)

$$SolucionT := G(t) = \frac{\_C1\, e^{t}}{t} \quad \textbf{(68)}$$

> $SolucionCompleta := u(x,t) = rhs(SolucionX) \cdot subs(\_C1 = 1, rhs(SolucionT))$

$$SolucionCompleta := u(x,t) = \frac{(\_C1 + \_C2\, e^{3x})\, e^{t}}{t} \quad \textbf{(69)}$$

> $EcuacionCinco := simplify\left(\frac{lhs(EcuacionTres) \cdot t}{-3 \cdot G(t) \cdot diff(F(x), x)}\right) = simplify\left(\frac{rhs(EcuacionTres) \cdot t}{-3 \cdot G(t) \cdot diff(F(x), x)}\right)$

$$EcuacionCinco := -\frac{1}{3}\, \frac{\frac{d^2}{dx^2} F(x)}{\frac{d}{dx} F(x)} = \frac{\left(\frac{d}{dt} G(t) - G(t)\right) t}{G(t)} \quad \textbf{(70)}$$

> $EcuacionXX := lhs(EcuacionCinco) = \text{alpha};\ EcuacionTT := rhs(EcuacionCinco) = \text{alpha}$

$$EcuacionXX := -\frac{1}{3}\, \frac{\frac{d^2}{dx^2} F(x)}{\frac{d}{dx} F(x)} = \alpha$$

$$EcuacionTT := \frac{\left(\frac{d}{dt} G(t) - G(t)\right) t}{G(t)} = \alpha \quad \textbf{(71)}$$

> $SolucionXX := subs(\text{alpha} = 3, dsolve(EcuacionXX))$

$$SolucionXX := F(x) = \_C1 + \_C2\, e^{-9x} \quad \textbf{(72)}$$

> $SolucionTT := subs(\text{alpha} = 3, dsolve(EcuacionTT))$

$$SolucionTT := G(t) = \_C1\, e^{t}\, t^{3} \quad \textbf{(73)}$$

> $SolucionCompletaDos := u(x,t) = rhs(SolucionXX) \cdot subs(\_C1 = 1, rhs(SolucionTT))$

$$SolucionCompletaDos := u(x,t) = (\_C1 + \_C2\, e^{-9x})\, e^{t}\, t^{3} \quad \textbf{(74)}$$

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FIN RESPUESTA 6)

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FIN EXAMEN

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