SOLUCIÓN

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SEMESTRE 2014-2
SEGUNDO EXAMEN PARCIAL

ABRIL 21 DE 2014

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> restart
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1) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL LINEAL (sin usar dsolve o relativos)

$$2x^2y(x)+2y(x)+5+(2x^3+2x)\left(\frac{d}{dx}y(x)\right)=0 \quad (1)$$

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RESPUESTA 1)

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> Ecuacion := 2·x··2·y(x) + 2·y(x) + 5 + (2·x··3 + 2·x)·diff(y(x), x) = 0
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$$Ecuacion := 2x^2y(x)+2y(x)+5+(2x^3+2x)\left(\frac{d}{dx}y(x)\right)=0 \quad (2)$$

```
> with(DEtools):
> odeadvisor(Ecuacion)
```

$$[\_linear] \quad (3)$$

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RUTA 1) COMO LINEAL

```
> EcuacionLineal := (2·x··3 + 2·x)·diff(y(x), x) + (2·x··2 + 2)·y(x) = -5
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$$EcuacionLineal := (2x^3+2x)\left(\frac{d}{dx}y(x)\right)+(2x^2+2)y(x)=-5 \quad (4)$$

```
> EcuacionLinealNormalizada := expand(simplify(lhs(EcuacionLineal)/(2x^3+2x)))
    = rhs(EcuacionLineal)/(2x^3+2x)
```

$$EcuacionLinealNormalizada := \frac{d}{dx}y(x)+\frac{y(x)}{x}=-\frac{5}{2x^3+2x} \quad (5)$$

```
> p := 1/x; q := rhs(EcuacionLinealNormalizada)
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$$p := \frac{1}{x}$$

$$q := -\frac{5}{2x^3+2x} \quad (6)$$

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> SolucionGeneral := y(x) = C1·exp(-int(p, x)) + exp(-int(p, x))·(int(exp(int(p, x))·q, x))
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$$SolucionGeneral := y(x)=\frac{C_1}{x}-\frac{5}{2}\frac{\arctan(x)}{x} \quad (7)$$

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> Comprobacion1 := simplify(eval(subs(y(x) = rhs(SolucionGeneral), Ecuacion)))
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$$Comprobacion_1 := 0=0 \quad (8)$$

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RUTA 2) POR FACTOR INTEGRANTE

> *Ecuacion*

$$2x^2y(x)+2y(x)+5+\left(2x^3+2x\right)\left(\frac{d}{dx}y(x)\right)=0 \quad (9)$$

> *FactInt* := *intfactor*(*Ecuacion*)

$$FactInt := \frac{1}{x^2+1} \quad (10)$$

> $M := 2x^2y+2y+5;\ N := \left(2x^3+2x\right)$

$$M := 2x^2y+2y+5$$

$$N := 2x^3+2x \quad (11)$$

> *MM* := *simplify*(*FactInt*·*M*); *NN* := *simplify*(*FactInt*·*N*);

$$MM := \frac{2x^2y+2y+5}{x^2+1}$$

$$NN := 2x \quad (12)$$

> *Comprobacion* := *simplify*(*diff*(*MM*, *y*) − *diff*(*NN*, *x*)) = 0

$$Comprobacion := 0=0 \quad (13)$$

> *IntMMx* := *int*(*MM*, *x*)

$$IntMMx := 2yx+5\arctan(x) \quad (14)$$

> *SolucionGeneralDos* := *IntMMx* + *int*((*NN* − *diff*(*IntMMx*, *y*)), *y*) = C_1

$$SolucionGeneralDos := 2yx+5\arctan(x)=C_1 \quad (15)$$

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COMPROBACIÓN

> *dsolve*(*Ecuacion*)

$$y(x)=\frac{-\frac{5}{2}\arctan(x)+\_C1}{x} \quad (16)$$

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FIN RESPUESTA 1)

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> *restart*

2) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)

$$y(x)+\sqrt{y(x)^2-x^2}-x\left(\frac{d}{dx}y(x)\right)=0 \quad (17)$$

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RESPUESTA 2)

> *Ecuacion* := $y(x)+\mathrm{sqrt}(y(x)\cdot\cdot 2-x\cdot\cdot 2)-x\cdot diff(y(x),x)=0$

$$Ecuacion := y(x)+\sqrt{y(x)^2-x^2}-x\left(\frac{d}{dx}y(x)\right)=0 \quad (18)$$

> *with*(*DEtools*) :

> *odeadvisor*(*Ecuacion*)

$$[[\_homogeneous,\ class\ A],\ \_rational,\ \_dAlembert] \quad (19)$$

> $EcuacionSeparable := simplify(isolate(eval(subs(y(x) = u(x)\cdot x, Ecuacion)), diff(u(x), x)))$

$$EcuacionSeparable := \frac{d}{dx}u(x) = \frac{\sqrt{x^2\left(u(x)^2 - 1\right)}}{x^2} \quad (20)$$

> $P := x; R := \sqrt{u^2 - 1}$

$$P := x$$

$$R := \sqrt{u^2 - 1} \quad (21)$$

> $SolucionUno := int\left(\frac{1}{P}, x\right) - int\left(\frac{1}{R}, u\right) = C_1$

$$SolucionUno := \ln(x) - \ln\left(u + \sqrt{u^2 - 1}\right) = C_1 \quad (22)$$

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> $SolucionDos := expand\left(simplify\left(subs\left(u = \frac{y}{x}, SolucionUno\right)\right)\right)$

$$SolucionDos := \ln(x) - \ln\left(\frac{y + \sqrt{-\frac{-y^2 + x^2}{x^2}}\,x}{x}\right) = C_1 \quad (23)$$

> $SolucionTres := isolate\left(simplify(\exp(lhs(SolucionDos))) = \frac{1}{C_1}, C_1\right)$

$$SolucionTres := C_1 = \frac{y + \sqrt{-\frac{-y^2 + x^2}{x^2}}\,x}{x^2} \quad (24)$$

> $SolucionCuatro := \frac{(rhs(SolucionTres)\cdot x\cdot\cdot 2 - y)}{x} = \frac{(lhs(SolucionTres)\cdot x\cdot\cdot 2 - y)}{x}$

$$SolucionCuatro := \sqrt{-\frac{-y^2 + x^2}{x^2}} = \frac{C_1 x^2 - y}{x} \quad (25)$$

> $SolucionCinco := lhs(SolucionCuatro)\cdot\cdot 2 = rhs(SolucionCuatro)\cdot\cdot 2$

$$SolucionCinco := -\frac{-y^2 + x^2}{x^2} = \frac{\left(C_1 x^2 - y\right)^2}{x^2} \quad (26)$$

> $SolucionSeis := lhs(SolucionCinco)\cdot x\cdot\cdot 2 = rhs(SolucionCinco)\cdot x\cdot\cdot 2$

$$SolucionSeis := y^2 - x^2 = \left(C_1 x^2 - y\right)^2 \quad (27)$$

> $Parametro := isolate(SolucionSeis, C_1)$

$$Parametro := C_1 = \frac{y + \sqrt{y^2 - x^2}}{x^2} \quad (28)$$

> $SolucionGeneral := rhs(Parametro) = C_1$

$$SolucionGeneral := \frac{y + \sqrt{y^2 - x^2}}{x^2} = C_1 \quad (29)$$

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COMPROBACIÓN

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> dsolve(Ecuacion)

$$\frac{y(x)}{x^2} + \frac{\sqrt{y(x)^2 - x^2}}{x^2} - \_C1 = 0 \quad (30)$$

FIN RESPUESTA 2)

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> restart

3) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)

$$1 - x^2 y(x) + x^2 (y(x) - x) \left(\frac{d}{dx} y(x) \right) = 0 \quad (31)$$

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RESPUESTA 3)

> $Ecuacion := 1 - x^2 y(x) + x^2 (y(x) - x) \left(\frac{d}{dx} y(x) \right) = 0$

$$Ecuacion := 1 - x^2 y(x) + x^2 (y(x) - x) \left(\frac{d}{dx} y(x) \right) = 0 \quad (32)$$

> with(DEtools) :

> odeadvisor(Ecuacion)

[_rational, [_1st_order, _with_symmetry_[F(x),G(x)]], [_Abel, 2nd type, class B]] (33)

> $FactInt := intfactor(Ecuacion)$

$$FactInt := \frac{1}{x^2} \quad (34)$$

> $M := 1 - x^2 y; N := x^2 (y - x)$

$$M := 1 - x^2 y$$

$$N := x^2 (y - x) \quad (35)$$

> $MM := expand(FactInt \cdot M); NN := simplify(FactInt \cdot N)$

$$MM := \frac{1}{x^2} - y$$

$$NN := y - x \quad (36)$$

> $Comprobacion_1 := simplify(diff(MM, y) - diff(NN, x)) = 0$

$$Comprobacion_1 := 0 = 0 \quad (37)$$

> $IntMMx := int(MM, x)$

$$IntMMx := -\frac{1}{x} - y x \quad (38)$$

> $SolucionGeneral := IntMMx + int((NN - diff(IntMMx, y)), y) = C_1$

$$SolucionGeneral := -\frac{1}{x} - y x + \frac{1}{2} y^2 = C_1 \quad (39)$$

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COMPROBACIÓN

> $EcuacionUno := isolate(Ecuacion, diff(y(x), x))$

$$EcuacionUno := \frac{d}{dx} y(x) = \frac{-1 + x^2 y(x)}{x^2 (y(x) - x)} \quad (40)$$

> $Solucion := -\frac{1}{x} - y(x)\, x + \frac{1}{2} y(x)^2 = C_1$

$$Solucion := -\frac{1}{x} - y(x)\, x + \frac{1}{2} y(x)^2 = C_1 \quad (41)$$

> $DerSolucion := simplify(isolate(diff(Solucion, x), diff(y(x), x)))$

$$DerSolucion := \frac{d}{dx} y(x) = \frac{-1 + x^2 y(x)}{x^2 (y(x) - x)} \quad (42)$$

> $Comprobacion_2 := simplify(rhs(EcuacionUno) - rhs(DerSolucion)) = 0$

$$Comprobacion_2 := 0 = 0 \quad (43)$$

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FIN RESPUESTA 3)

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> *restart*

4) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)

$$(1 + y(x)^2) \left(e^{2x} - e^{y(x)} \left(\frac{d}{dx} y(x)\right)\right) - (1 + y(x)) \left(\frac{d}{dx} y(x)\right) = 0 \quad (44)$$

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RESPUESTA 4)

> $Ecuacion := (1 + y(x)\cdot\cdot 2)\cdot(\exp(2x) - \exp(y(x))\cdot diff(y(x), x)) - (1 + y(x))\cdot diff(y(x), x) = 0$

$$Ecuacion := (1 + y(x)^2) \left(e^{2x} - e^{y(x)} \left(\frac{d}{dx} y(x)\right)\right) - (1 + y(x)) \left(\frac{d}{dx} y(x)\right) = 0 \quad (45)$$

> $with(DEtools)$:

> $odeadvisor(Ecuacion)$

$$[\_separable] \quad (46)$$

> $EcuacionUno := simplify(isolate(Ecuacion, diff(y(x), x)))$

$$EcuacionUno := \frac{d}{dx} y(x) = \frac{(1 + y(x)^2)\, e^{2x}}{e^{y(x)} + y(x)^2 e^{y(x)} + 1 + y(x)} \quad (47)$$

> $M := -((1 + y^2)\, e^{2x})$

$$M := -(1 + y^2)\, e^{2x} \quad (48)$$

> $N := e^y + y^2 e^y + 1 + y$

$$N := e^y + e^y y^2 + 1 + y \quad (49)$$

> $P := -\exp(2x); Q := 1 + y\cdot\cdot 2$

$$P := -e^{2x}$$

$$Q := 1 + y^2 \quad (50)$$

> $R := 1; S := N$

$$R := 1$$

$$(51)$$

$$S := e^{y} + e^{y} y^{2} + 1 + y \tag{51}$$

> $SolucionGeneral := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, y\right) = C_1$

$$SolucionGeneral := -\frac{1}{2} e^{2x} + e^{y} + \arctan(y) + \frac{1}{2} \ln(1 + y^{2}) = C_1 \tag{52}$$

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COMPROBACIÓN

> $Solucion := -\frac{1}{2} e^{2x} + e^{y(x)} + \arctan(y(x)) + \frac{1}{2} \ln(1 + y(x)^{2}) = C_1$

$$Solucion := -\frac{1}{2} e^{2x} + e^{y(x)} + \arctan(y(x)) + \frac{1}{2} \ln(1 + y(x)^{2}) = C_1 \tag{53}$$

> $DerSol := simplify(isolate(diff(Solucion, x), diff(y(x), x)))$

$$DerSol := \frac{d}{dx} y(x) = \frac{(1 + y(x)^{2}) e^{2x}}{e^{y(x)} + y(x)^{2} e^{y(x)} + 1 + y(x)} \tag{54}$$

> $Comprobacion_1 := simplify(rhs(EcuacionUno) - rhs(DerSol)) = 0$

$$Comprobacion_1 := 0 = 0 \tag{55}$$

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FIN RESPUESTA 4)

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> *restart*

5) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)

$$2x + \frac{x^{2} + y(x)^{2}}{x^{2} y(x)} - \frac{(x^{2} + y(x)^{2}) \left(\frac{d}{dx} y(x)\right)}{x y(x)^{2}} = 0 \tag{56}$$

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RESPUESTA 5)

> $Ecuacion := 2 \cdot x + \frac{(x\cdot\cdot 2 + y(x)\cdot\cdot 2)}{x\cdot\cdot 2 \cdot y(x)} - \left(\frac{(x\cdot\cdot 2 + y(x)\cdot\cdot 2)}{x \cdot y(x)\cdot\cdot 2}\right) \cdot diff(y(x), x) = 0$

$$Ecuacion := 2x + \frac{x^{2} + y(x)^{2}}{x^{2} y(x)} - \frac{(x^{2} + y(x)^{2}) \left(\frac{d}{dx} y(x)\right)}{x y(x)^{2}} = 0 \tag{57}$$

> $with(DEtools)$:

> $odeadvisor(Ecuacion)$

$$[[\_homogeneous, class\ D], \_exact, \_rational] \tag{58}$$

> $M := 2 \cdot x + \frac{(x\cdot\cdot 2 + y\cdot\cdot 2)}{x\cdot\cdot 2 \cdot y}; N := -\left(\frac{(x\cdot\cdot 2 + y\cdot\cdot 2)}{x \cdot y\cdot\cdot 2}\right)$

$$M := 2x + \frac{x^{2} + y^{2}}{x^{2} y}$$

$$N := -\frac{x^{2} + y^{2}}{x y^{2}} \tag{59}$$

> $Comprobacion_1 := simplify(diff(M, y) - diff(N, x)) = 0$

$$Comprobacion_1 := 0 = 0 \quad \textbf{(60)}$$

> $IntMx := simplify(int(M, x))$

$$IntMx := \frac{x^3 y + x^2 - y^2}{x y} \quad \textbf{(61)}$$

> $SolucionGeneral := simplify(IntMx + int((N - diff(IntMx, y)), y)) = C_1$

$$SolucionGeneral := \frac{x^3 y + x^2 - y^2}{x y} = C_1 \quad \textbf{(62)}$$

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COMPROBACIONES

> $EcuacionUno := simplify(isolate(Ecuacion, diff(y(x), x)))$

$$EcuacionUno := \frac{d}{dx} y(x) = \frac{y(x)\left(2 x^3 y(x) + x^2 + y(x)^2\right)}{\left(x^2 + y(x)^2\right) x} \quad \textbf{(63)}$$

> $Solucion := \frac{x^3 y(x) + x^2 - y(x)^2}{x y(x)} = C_1$

$$Solucion := \frac{x^3 y(x) + x^2 - y(x)^2}{x y(x)} = C_1 \quad \textbf{(64)}$$

> $DerSolucion := simplify(isolate(diff(Solucion, x), diff(y(x), x)))$

$$DerSolucion := \frac{d}{dx} y(x) = \frac{y(x)\left(2 x^3 y(x) + x^2 + y(x)^2\right)}{\left(x^2 + y(x)^2\right) x} \quad \textbf{(65)}$$

> $Comprobacion_1 := simplify(rhs(EcuacionUno) - rhs(DerSolucion)) = 0$

$$Comprobacion_1 := 0 = 0 \quad \textbf{(66)}$$

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FIN RESPUESTA 5)

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> *restart*

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FIN DEL EXAMEN