>

SOLUCIÓN

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
TERCER EXAMEN PARCIAL (TEMAS 4 Y 5)
SEMESTRE 2014-2

2014 MAYO 19

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1) UTILIZANDO EXCLUSIVAMENTE TRANSFORMADA DE LAPLACE **(sin usar dsolve)**:

a) **(15/100 puntos)** OBTENER LA SOLUCIÓN PARTICULAR DE LA ECUACIÓN DADA CON LAS CONDICIONES INICIALES DADAS

b) **(15/100 puntos)** GRAFICAR - JUNTAS - LA SOLUCIÓN OBTENIDA EN EL INCISO a) Y SU PRIMERA DERIVADA; PARA UN INTERVALO DE 0 < t < 3

$$\frac{d^2}{dt^2} y(t) + 4\,y(t) = 64\,(t-1)\,\mathrm{Heaviside}(t-1)\cos(2t-2)$$

$$y(0) = 1$$

$$D(y)(0) = 2 \qquad (1)$$

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RESULTADO 1a)

> $Ecuacion := \frac{d^2}{dt^2} y(t) + 4\,y(t) = 64\,(t-1)\,\mathrm{Heaviside}(t-1)\cos(2t-2)$

$$Ecuacion := \frac{d^2}{dt^2} y(t) + 4\,y(t) = 64\,(t-1)\,\mathrm{Heaviside}(t-1)\cos(2t-2) \qquad (2)$$

> $Condiciones := y(0) = 1, D(y)(0) = 2$

$$Condiciones := y(0) = 1, D(y)(0) = 2 \qquad (3)$$

> $with(inttrans):$

> $TransLapEcua := subs(Condiciones, laplace(Ecuacion, t, s))$

$$TransLapEcua := s^2\,laplace(y(t), t, s) - 2 - s + 4\,laplace(y(t), t, s) = \frac{64\,e^{-s}\,(s^2-4)}{(s^2+4)^2} \qquad (4)$$

> $TransLapSol := isolate(TransLapEcua, laplace(y(t), t, s))$

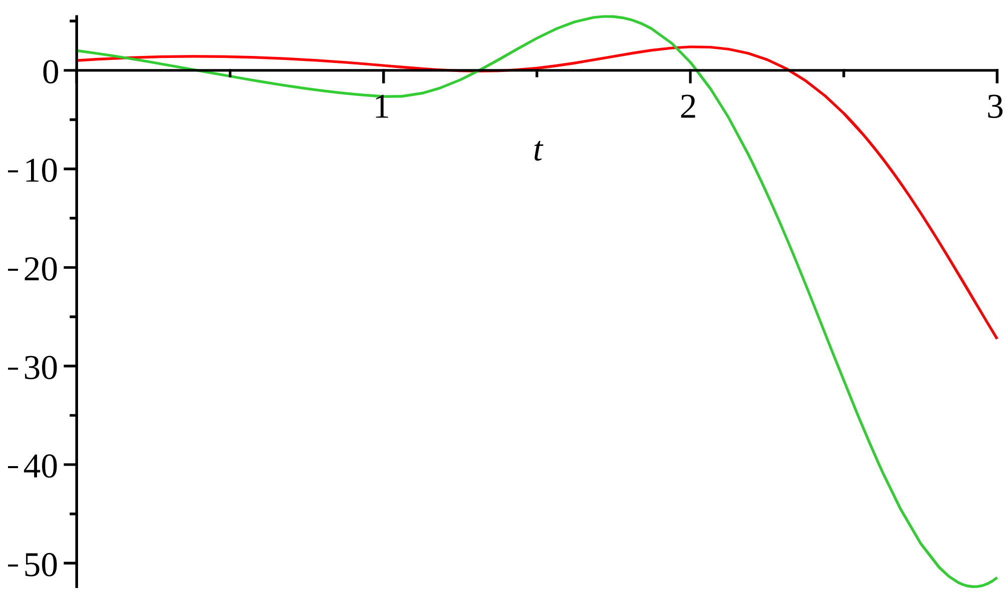
$$TransLapSol := laplace(y(t), t, s) = \frac{\frac{64\,e^{-s}\,(s^2-4)}{(s^2+4)^2} + 2 + s}{s^2+4} \qquad (5)$$

> $SolucionParticular := invlaplace(TransLapSol, s, t)$

$$SolucionParticular := y(t) = \cos(2t) + \sin(2t) + 2\,(2\,(t-1)\cos(2t-2) + \sin(2t-2)\,(2t-1)\,(2t-3))\,\mathrm{Heaviside}(t-1) \qquad (6)$$

RESULTADO 1b)

> $plot([rhs(SolucionParticular), rhs(diff(SolucionParticular, t))], t = 0..3)$



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FIN PREGUNTA 1)

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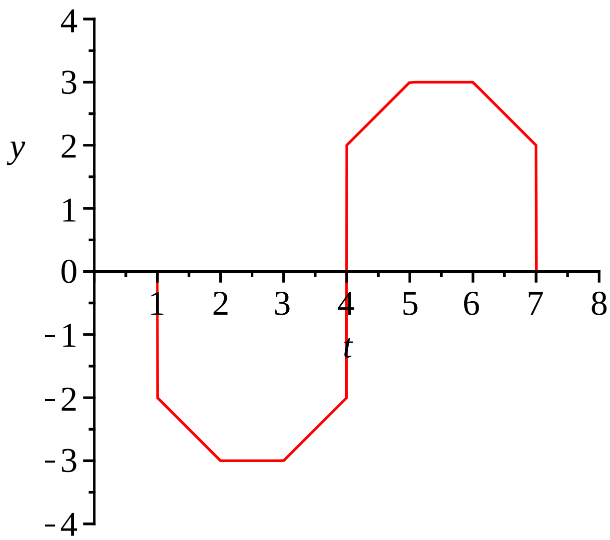
2) DADA LA GRÁFICA DE LA FUNCIÓN SIGUIENTE:



a) **(15/100 puntos)** OBTENER SU TRANSFORMADA DE LAPLACE.
b) **(25/100 puntos)** OBTENER Y GRAFICAR SU SERIE COSENO DE FOURIER PARA 500 TÉRMINOS EN EL MISMO INTERVALO.

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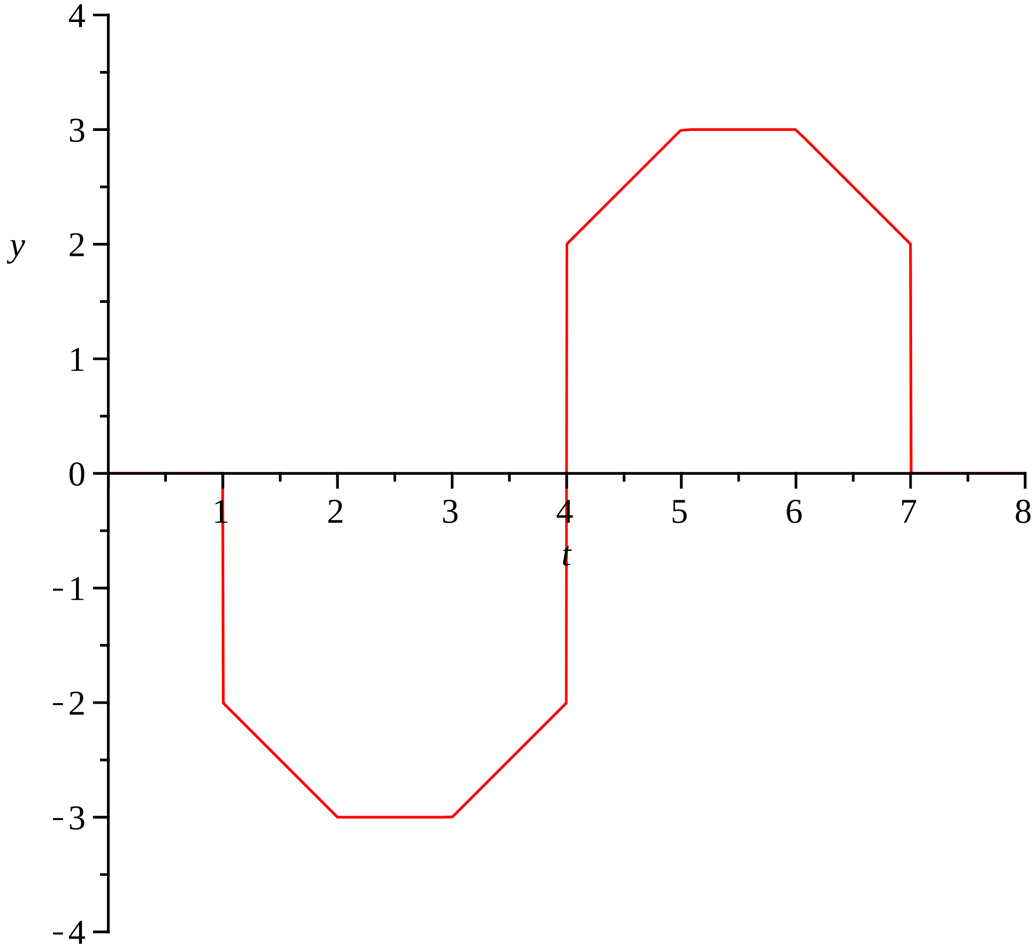
RESULTADO 2a)

> $f := -2\,\text{Heaviside}(t-1) - (t-1)\cdot\text{Heaviside}(t-1) + (t-2)\cdot\text{Heaviside}(t-2) + (t-3)\cdot\text{Heaviside}(t-3) - (t-4)\cdot\text{Heaviside}(t-4) + 4\cdot\text{Heaviside}(t-4) + (t-4)\cdot\text{Heaviside}(t-4) - (t-5)\cdot\text{Heaviside}(t-5) - (t-6)\cdot\text{Heaviside}(t-6) + (t-7)\cdot\text{Heaviside}(t-7) - 2\cdot\text{Heaviside}(t-7)$: $plot(f, t=0..8, y=-4..4, scaling = CONSTRAINED)$



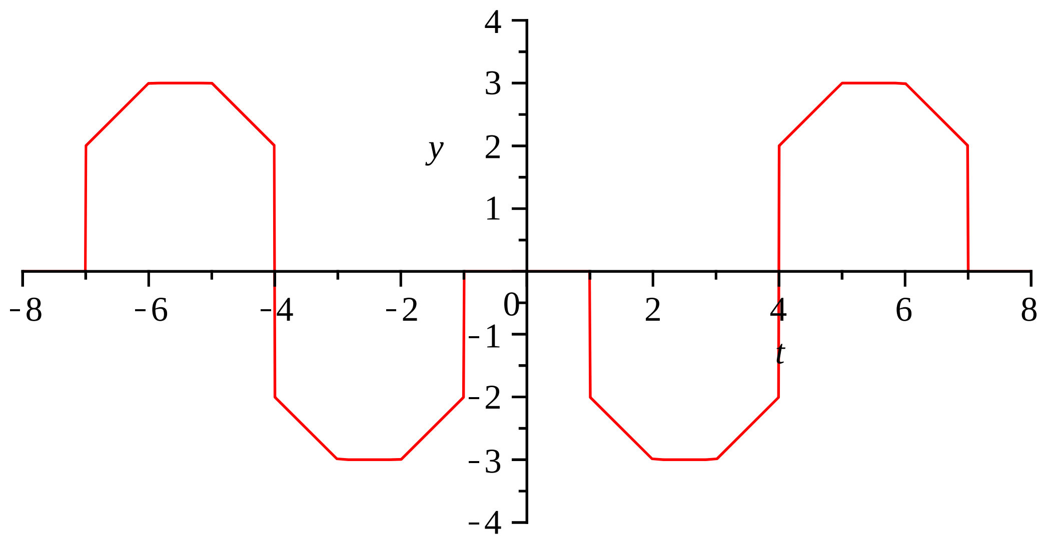
> $with(inttrans):$

> $F := laplace(f, t, s)$

$$F := -\frac{e^{-s} - e^{-7s} + e^{-6s} + e^{-5s} - e^{-3s} - e^{-2s}}{s^2} - \frac{2\left(e^{-s} + e^{-7s} - 2\,e^{-4s}\right)}{s} \qquad \textbf{(7)}$$

RESULTADO 2b)

> $g := 2\,\text{Heaviside}(t+1) - (t+1)\cdot\text{Heaviside}(t+1) + (t+2)\cdot\text{Heaviside}(t+2) + (t+3)\cdot\text{Heaviside}(t+3) - (t+4)\cdot\text{Heaviside}(t+4) - 4\cdot\text{Heaviside}(t+4) + (t+4)\cdot\text{Heaviside}(t+4) - (t+5)\cdot\text{Heaviside}(t+5) - (t+6)\cdot\text{Heaviside}(t+6) + (t+7)\cdot\text{Heaviside}(t+7) + 2\cdot\text{Heaviside}(t+7) : plot(f+g, t=-8..8, y=-4..4, scaling = CONSTRAINED)$



> $L := 8$

$$L := 8 \tag{8}$$

> $a_0 := \left(\frac{1}{L}\right) \cdot int(\,(f+g),\, t=-L..L)$

$$a_0 := 0 \tag{9}$$

> $a_n := \left(\frac{1}{L}\right) \cdot int\left(\,(f+g) \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right),\, t=-L..L\right);$

$$a_n := -\frac{12 \sin\left(\frac{3}{4} n \pi\right)}{n \pi} - \frac{8 \sin\left(\frac{1}{2} n \pi\right)}{n \pi} + \frac{6 \sin\left(\frac{3}{8} n \pi\right)}{n \pi} + \frac{4 \sin\left(\frac{1}{4} n \pi\right)}{n \pi} + \frac{2 \sin\left(\frac{1}{8} n \pi\right)}{n \pi} - \frac{10 \sin\left(\frac{5}{8} n \pi\right)}{n \pi} + \frac{18 \sin\left(\frac{7}{8} n \pi\right)}{n \pi} - \frac{16 \left(\cos\left(\frac{3}{8} n \pi\right) + \frac{3}{8} n \pi \sin\left(\frac{3}{8} n \pi\right)\right)}{n^2 \pi^2} \tag{10}$$

$$+\frac{16\left(\cos\left(\frac{1}{8}n\pi\right)+\frac{1}{8}n\pi\sin\left(\frac{1}{8}n\pi\right)\right)}{n^2\pi^2}$$

$$-\frac{16\left(\cos\left(\frac{1}{4}n\pi\right)+\frac{1}{4}n\pi\sin\left(\frac{1}{4}n\pi\right)\right)}{n^2\pi^2}$$

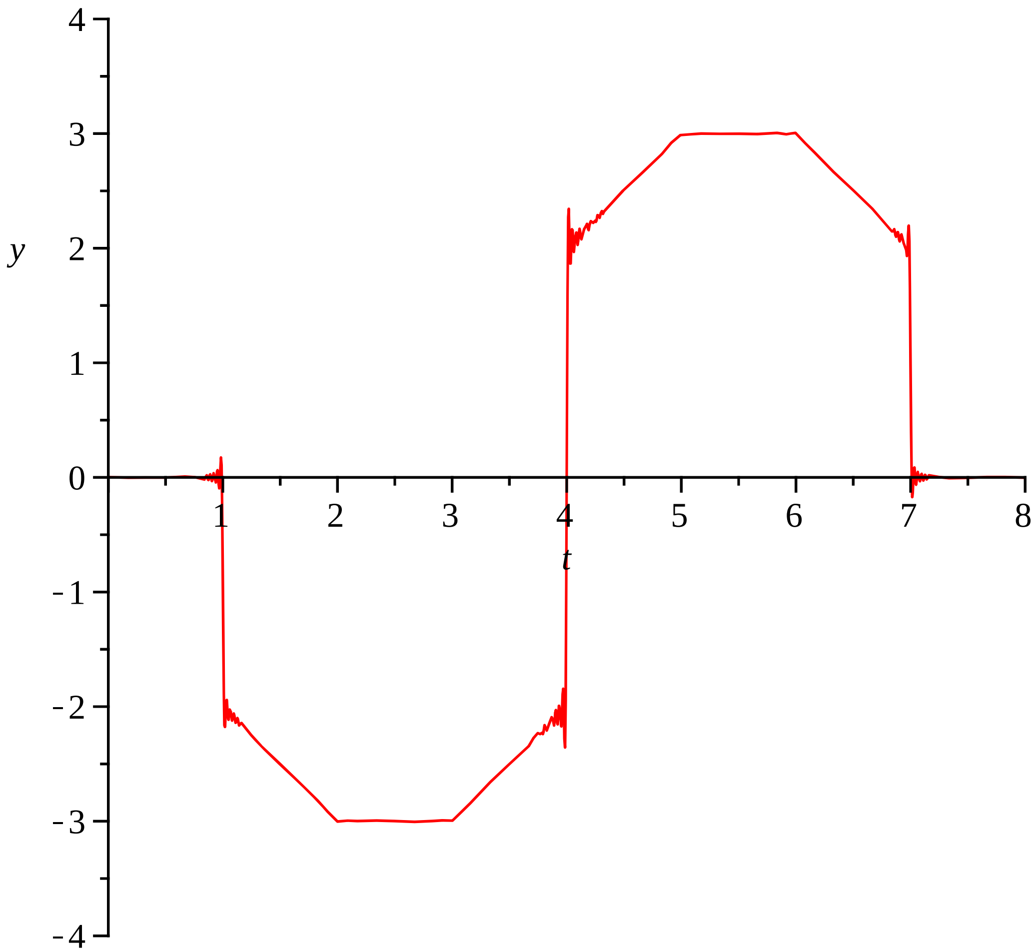
$$+\frac{16\left(\cos\left(\frac{3}{4}n\pi\right)+\frac{3}{4}n\pi\sin\left(\frac{3}{4}n\pi\right)\right)}{n^2\pi^2}$$

$$-\frac{16\left(\cos\left(\frac{7}{8}n\pi\right)+\frac{7}{8}n\pi\sin\left(\frac{7}{8}n\pi\right)\right)}{n^2\pi^2}$$

$$+\frac{16\left(\cos\left(\frac{5}{8}n\pi\right)+\frac{5}{8}n\pi\sin\left(\frac{5}{8}n\pi\right)\right)}{n^2\pi^2}$$

> $STF_{500} := sum\left(a_n\cdot\cos\left(\frac{n\cdot \mathrm{Pi}\cdot t}{L}\right), n = 1..500\right):$

> $plot\left(STF_{500}, t = 0..8, y = -4..4, scaling = CONSTRAINED\right)$



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FIN PREGUNTA 2)

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3) (30/100 puntos) OBTENER LA SOLUCIÓN DE LA SIGUIENTE ECUACIÓN EN DERIVADAS PARCIALES, UTILIZANDO EL MÉTODO DE SEPARACIÓN DE VARIABLES CON UNA **CONSTANTE DE SEPARACIÓN POSITIVA**:

$$\frac{\partial^2}{\partial t^2} y(x,t) + \frac{\partial}{\partial t} y(x,t) = x^2 \left(\frac{\partial}{\partial x} y(x,t)\right) \quad \textbf{(11)}$$

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RESULTADO 3)

> $Ecuacion := \frac{\partial^2}{\partial t^2} y(x,t) + \frac{\partial}{\partial t} y(x,t) = x^2 \left(\frac{\partial}{\partial x} y(x,t)\right)$

$$Ecuacion := \frac{\partial^2}{\partial t^2} y(x,t) + \frac{\partial}{\partial t} y(x,t) = x^2 \left(\frac{\partial}{\partial x} y(x,t)\right) \quad \textbf{(12)}$$

> $EcuacionDos := eval(subs(y(x,t) = F(x) \cdot G(t), Ecuacion))$

$$EcuacionDos := F(x) \left(\frac{d^2}{dt^2} G(t)\right) + F(x) \left(\frac{d}{dt} G(t)\right) = x^2 \left(\frac{d}{dx} F(x)\right) G(t) \quad \textbf{(13)}$$

> $EcuacionSeparada := simplify\left(\frac{lhs(EcuacionDos)}{F(x) \cdot G(t)}\right) = \frac{rhs(EcuacionDos)}{F(x) \cdot G(t)}$

$$EcuacionSeparada := \frac{\frac{d^2}{dt^2} G(t) + \frac{d}{dt} G(t)}{G(t)} = \frac{x^2 \left(\frac{d}{dx} F(x)\right)}{F(x)} \quad \textbf{(14)}$$

> $EcuacionXpos := rhs(EcuacionSeparada) = \text{beta} \cdot \cdot 2; EcuacionTpos := lhs(EcuacionSeparada) = \text{beta} \cdot \cdot 2$

$$EcuacionXpos := \frac{x^2 \left(\frac{d}{dx} F(x)\right)}{F(x)} = \beta^2$$

$$EcuacionTpos := \frac{\frac{d^2}{dt^2} G(t) + \frac{d}{dt} G(t)}{G(t)} = \beta^2 \quad \textbf{(15)}$$

> $SolucionXpos := dsolve(EcuacionXpos); SolucionTpos := dsolve(EcuacionTpos)$

$$SolucionXpos := F(x) = \_C1\, e^{-\frac{\beta^2}{x}}$$

$$SolucionTpos := G(t) = \_C1\, e^{\left(-\frac{1}{2} + \frac{1}{2}\sqrt{1+4\beta^2}\right) t} + \_C2\, e^{\left(-\frac{1}{2} - \frac{1}{2}\sqrt{1+4\beta^2}\right) t} \quad \textbf{(16)}$$

> $SolucionGeneralPos := y(x, t) = subs(\_C1 = 1, rhs(SolucionXpos)) \cdot rhs(SolucionTpos)$

$$SolucionGeneralPos := y(x, t) = e^{-\frac{\beta^2}{x}} \left(\_C1\, e^{\left(-\frac{1}{2} + \frac{1}{2}\sqrt{1+4\beta^2}\right) t} + \_C2\, e^{\left(-\frac{1}{2} - \frac{1}{2}\sqrt{1+4\beta^2}\right) t}\right) \quad \textbf{(17)}$$

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FIN PREGUNTA 3)

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FIN DEL EXAMEN

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