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SOLUCIÓN

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
PRIMER EXAMEN FINAL
SEMESTRE 2015-2

2015 MAYO 29

\> *restart*

1) Resolver

\> $(6\cdot x+1)\cdot y(x)\cdot\cdot 2\cdot diff(y(x),x)+3\cdot x\cdot\cdot 2+2\cdot y(x)\cdot\cdot 3=0$

$$(6x+1)\,y(x)^2\left(\frac{d}{dx}\,y(x)\right)+3x^2+2\,y(x)^3=0 \qquad \textbf{(1)}$$

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RESPUESTA 1)

\> $Ecuacion := (6\cdot x+1)\cdot y(x)\cdot\cdot 2\cdot diff(y(x),x)+3\cdot x\cdot\cdot 2+2\cdot y(x)\cdot\cdot 3=0$

$$Ecuacion := (6x+1)\,y(x)^2\left(\frac{d}{dx}\,y(x)\right)+3x^2+2\,y(x)^3=0 \qquad \textbf{(2)}$$

\> $Solucion := dsolve(Ecuacion) : Solucion_1; Solucion_2; Solucion_3$

$$y(x)=\frac{\left((-3x^3+\_C1)\,(6x+1)^2\right)^{1/3}}{6x+1}$$

$$y(x)=-\frac{1}{2}\,\frac{\left((-3x^3+\_C1)\,(6x+1)^2\right)^{1/3}}{6x+1}-\frac{\frac{1}{2}\,\mathrm{I}\sqrt{3}\,\left((-3x^3+\_C1)\,(6x+1)^2\right)^{1/3}}{6x+1}$$

$$y(x)=-\frac{1}{2}\,\frac{\left((-3x^3+\_C1)\,(6x+1)^2\right)^{1/3}}{6x+1}+\frac{\frac{1}{2}\,\mathrm{I}\sqrt{3}\,\left((-3x^3+\_C1)\,(6x+1)^2\right)^{1/3}}{6x+1} \qquad \textbf{(3)}$$

\> *with(DEtools) :*

\> *odeadvisor(Ecuacion)*

$$[\_exact,\ \_rational,\ \_Bernoulli] \qquad \textbf{(4)}$$

\> $M := 3\cdot x\cdot\cdot 2+2\cdot y\cdot\cdot 3$

$$M := 3x^2+2y^3 \qquad \textbf{(5)}$$

\> $N := expand((6\cdot x+1)\cdot y\cdot\cdot 2)$

$$N := 6y^2x+y^2 \qquad \textbf{(6)}$$

\> $Comprobacion_1 := diff(M,y)-diff(N,x)=0$

$$Comprobacion_1 := 0=0 \qquad \textbf{(7)}$$

\> $IntMx := int(M,x)$

$$IntMx := x^3+2y^3x \qquad \textbf{(8)}$$

\> $SolucionGeneral := IntMx+int((N-diff(IntMx,y)),y)=C_1$

$$SolucionGeneral := x^3+2y^3x+\frac{1}{3}\,y^3=C_1 \qquad \textbf{(9)}$$

\> $SolucionGeneralDos := lhs(SolucionGeneral)\cdot 3=C_1$

(10)

$$SolucionGeneralDos := 3\,x^3 + 6\,y^3\,x + y^3 = C_1 \quad \textbf{(10)}$$

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FIN RESPUESTA 1)

> *restart*

2) Resolver la ecuación diferencial

> $x \cdot diff(x \cdot y(x), x) + x \cdot \cdot 2 \cdot diff(y(x), x\$2) - x \cdot y(x) = x \cdot \cdot 2 \cdot (\exp(-x) + 2 \cdot \cos(x))$

$$x\left(y(x) + x\left(\frac{d}{dx}\,y(x)\right)\right) + x^2\left(\frac{d^2}{dx^2}\,y(x)\right) - x\,y(x) = x^2\left(e^{-x} + 2\cos(x)\right) \quad \textbf{(11)}$$

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RESPUESTA 2)

> $Ec := x \cdot diff(x \cdot y(x), x) + x \cdot \cdot 2 \cdot diff(y(x), x\$2) - x \cdot y(x) = x \cdot \cdot 2 \cdot (\exp(-x) + 2 \cdot \cos(x))$

$$Ec := x\left(y(x) + x\left(\frac{d}{dx}\,y(x)\right)\right) + x^2\left(\frac{d^2}{dx^2}\,y(x)\right) - x\,y(x) = x^2\left(e^{-x} + 2\cos(x)\right) \quad \textbf{(12)}$$

> $Sol := dsolve(Ec)$

$$Sol := y(x) = -e^{-x}x - e^{-x} + \sin(x) - \cos(x) - e^{-x}\_C1 + \_C2 \quad \textbf{(13)}$$

> $Ecuacion := simplify(x \cdot diff(x \cdot y(x), x) + x \cdot \cdot 2 \cdot diff(y(x), x\$2) - x \cdot y(x)) = x \cdot \cdot 2 \cdot (\exp(-x) + 2 \cdot \cos(x))$

$$Ecuacion := x^2\left(\frac{d}{dx}\,y(x) + \frac{d^2}{dx^2}\,y(x)\right) = x^2\left(e^{-x} + 2\cos(x)\right) \quad \textbf{(14)}$$

> $EcuacionDos := \dfrac{lhs(Ecuacion)}{x \cdot \cdot 2} = \dfrac{rhs(Ecuacion)}{x \cdot \cdot 2}$

$$EcuacionDos := \frac{d}{dx}\,y(x) + \frac{d^2}{dx^2}\,y(x) = e^{-x} + 2\cos(x) \quad \textbf{(15)}$$

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> $Solucion := dsolve(EcuacionDos)$

$$Solucion := y(x) = -e^{-x}x - e^{-x} + \sin(x) - \cos(x) - e^{-x}\_C1 + \_C2 \quad \textbf{(16)}$$

> $SolucionDos := y(x) = C_1 + C_2 \cdot \exp(-x) - x \cdot \exp(-x) + \sin(x) - \cos(x)$

$$SolucionDos := y(x) = C_1 + C_2\,e^{-x} - e^{-x}x + \sin(x) - \cos(x) \quad \textbf{(17)}$$

> $Comprobacion_2 := simplify(eval(subs(y(x) = rhs(SolucionDos), lhs(EcuacionDos) - rhs(EcuacionDos) = 0)))$

$$Comprobacion_2 := 0 = 0 \quad \textbf{(18)}$$

> $EcuacionHom := lhs(EcuacionDos) = 0$

$$EcuacionHom := \frac{d}{dx}\,y(x) + \frac{d^2}{dx^2}\,y(x) = 0 \quad \textbf{(19)}$$

> $Q := rhs(EcuacionDos)$

$$Q := e^{-x} + 2\cos(x) \quad \textbf{(20)}$$

> $EcuacionCarac := m \cdot \cdot 2 + m = 0$

$$EcuacionCarac := m^2 + m = 0 \quad \textbf{(21)}$$

> $Raiz := solve(EcuacionCarac)$

(22)

$$Raiz := 0, -1 \quad (22)$$

> $SolUno := y(x) = \exp(Raiz_1 \cdot x); SolDos := y(x) = \exp(Raiz_2 \cdot x)$

$$SolUno := y(x) = 1$$
$$SolDos := y(x) = e^{-x} \quad (23)$$

> $with(linalg):$

> $WW := wronskian([rhs(SolUno), rhs(SolDos)], x); BB := array([0, Q])$

$$WW := \begin{bmatrix} 1 & e^{-x} \\ 0 & -e^{-x} \end{bmatrix}$$
$$BB := \begin{bmatrix} 0 & e^{-x} + 2\cos(x) \end{bmatrix} \quad (24)$$

> $SOL := linsolve(WW, BB); Aprima := SOL_1; Bprima := expand(SOL_2)$

$$SOL := \begin{bmatrix} e^{-x} + 2\cos(x) & -\frac{e^{-x} + 2\cos(x)}{e^{-x}} \end{bmatrix}$$
$$Aprima := e^{-x} + 2\cos(x)$$
$$Bprima := -1 - 2e^{x}\cos(x) \quad (25)$$

> $A := int(Aprima, x) + C_1; B := int(Bprima, x) + C_2$

$$A := -e^{-x} + 2\sin(x) + C_1$$
$$B := -x - e^{x}\cos(x) - e^{x}\sin(x) + C_2 \quad (26)$$

> $SolucionGeneral := y(x) = simplify(A \cdot rhs(SolUno) + B \cdot rhs(SolDos))$

$$SolucionGeneral := y(x) = -e^{-x} + \sin(x) + C_1 - e^{-x}x - \cos(x) + C_2 e^{-x} \quad (27)$$

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FIN RESPUESTA 2)

> *restart*

3) Sea el sistema de ecuaciones diferenciales

> $b \cdot diff(x(t), t) + a \cdot diff(y(t), t) - 4 \cdot x(t) = 5 \cdot y(t); a \cdot diff(x(t), t) - b \cdot diff(y(t), t) = 3 \cdot x(t)$

$$b\left(\frac{d}{dt}x(t)\right) + a\left(\frac{d}{dt}y(t)\right) - 4x(t) = 5y(t)$$
$$a\left(\frac{d}{dt}x(t)\right) - b\left(\frac{d}{dt}y(t)\right) = 3x(t) \quad (28)$$

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Determinar el valor de las constantes "a" y "b"de manera que

> $x(t) = \exp(t); y(t) = -\exp(t);$

$$x(t) = e^{t}$$
$$y(t) = -e^{t} \quad (29)$$

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sean solución del sistema dado para los condiciones iniciales

> $x(0) = 1; y(0) = -1$

$$x(0) = 1$$
$$y(0) = -1 \quad (30)$$

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RESPUESTA 3)

> $Sistema := b \cdot diff(x(t), t) + a \cdot diff(y(t), t) = 4 \cdot x(t) + 5 \cdot y(t), a \cdot diff(x(t), t) - b \cdot diff(y(t), t) = 3 \cdot x(t) : Sistema_1; Sistema_2$

$$b\left(\frac{d}{dt} x(t)\right) + a\left(\frac{d}{dt} y(t)\right) = 4\, x(t) + 5\, y(t)$$

$$a\left(\frac{d}{dt} x(t)\right) - b\left(\frac{d}{dt} y(t)\right) = 3\, x(t) \quad \textbf{(31)}$$

> $SolUno := x(t) = e^t; SolDos := y(t) = -e^t$

$$SolUno := x(t) = e^t$$

$$SolDos := y(t) = -e^t \quad \textbf{(32)}$$

> $SistDos := eval(subs(x(t) = rhs(SolUno), y(t) = rhs(SolDos), Sistema_1)), eval(subs(x(t) = rhs(SolUno), y(t) = rhs(SolDos), Sistema_2)) : SistDos_1; SistDos_2$

$$b\, e^t - a\, e^t = -e^t$$

$$a\, e^t + b\, e^t = 3\, e^t \quad \textbf{(33)}$$

> $Coef := solve(\{SistDos\}, \{a, b\})$

$$Coef := \{a = 2, b = 1\} \quad \textbf{(34)}$$

> $SistTres := subs(a = rhs(Coef_1), b = rhs(Coef_2), Sistema_1), subs(a = rhs(Coef_1), b = rhs(Coef_2), Sistema_2) : SistTres_1; SistTres_2$

$$\frac{d}{dt} x(t) + 2\left(\frac{d}{dt} y(t)\right) = 4\, x(t) + 5\, y(t)$$

$$2\left(\frac{d}{dt} x(t)\right) - \left(\frac{d}{dt} y(t)\right) = 3\, x(t) \quad \textbf{(35)}$$

> $SolucionGeneral := dsolve(\{SistTres\}) : SolucionGeneral_1; SolucionGeneral_2$

$$x(t) = \_C1\, e^{3t} + \_C2\, e^t$$

$$y(t) = \_C1\, e^{3t} - \_C2\, e^t \quad \textbf{(36)}$$

> $Condiciones := x(0) = 1, y(0) = -1$

$$Condiciones := x(0) = 1, y(0) = -1 \quad \textbf{(37)}$$

> $SolucionParticular := dsolve(\{SistTres, Condiciones\}) : SolucionParticular_1; SolucionParticular_2$

$$x(t) = e^t$$

$$y(t) = -e^t \quad \textbf{(38)}$$

> *restart*

FIN RESPUESTA 3)

> *restart*

4) Determinar la solución del siguiente sistema de ecuaciones diferenciales haciendo uso de la transformada de Laplace

> $diff(x(t), t) + diff(y(t), t) = 1; diff(x(t), t) = x(t) - 6 \cdot y(t)$

$$\frac{d}{dt} x(t) + \frac{d}{dt} y(t) = 1$$

(39)

$$\frac{d}{dt}\, x(t) = x(t) - 6\, y(t) \quad \textbf{(39)}$$

sujeto a

> $x(0) = -1;\, y(0) = -1$

$$x(0) = -1$$
$$y(0) = -1 \quad \textbf{(40)}$$

obtener sólo x(t)

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RESPUESTA 4)

> $Sistema := \frac{d}{dt}\, x(t) + \frac{d}{dt}\, y(t) = 1,\ \frac{d}{dt}\, x(t) = x(t) - 6\, y(t) : Sistema_1;\ Sistema_2$

$$\frac{d}{dt}\, x(t) + \frac{d}{dt}\, y(t) = 1$$
$$\frac{d}{dt}\, x(t) = x(t) - 6\, y(t) \quad \textbf{(41)}$$

> $Condiciones := x(0) = -1,\, y(0) = -1$

$$Condiciones := x(0) = -1,\, y(0) = -1 \quad \textbf{(42)}$$

> $with(inttrans) :$

> $TLSist := subs\big(Condiciones,\, laplace\big(Sistema_1,\, t,\, s\big)\big),\, subs\big(Condiciones,\, laplace\big(Sistema_2,\, t,\, s\big)\big) : TLSist_1;\ TLSist_2$

$$s\, laplace(x(t), t, s) + 2 + s\, laplace(y(t), t, s) = \frac{1}{s}$$
$$s\, laplace(x(t), t, s) + 1 = laplace(x(t), t, s) - 6\, laplace(y(t), t, s) \quad \textbf{(43)}$$

> $TLSol := solve(\{TLSist\},\, \{laplace(x(t), t, s),\, laplace(y(t), t, s)\}) : TLSol_1;\ TLSol_2$

$$laplace(x(t), t, s) = -\frac{s^2 - 12\, s + 6}{s^2\, (-7 + s)}$$
$$laplace(y(t), t, s) = -\frac{s^2 - 3\, s + 1}{s^2\, (-7 + s)} \quad \textbf{(44)}$$

> $SolucionX := invlaplace\big(TLSol_1,\, s,\, t\big)$

$$SolucionX := x(t) = \frac{6}{7}\, t + \frac{29}{49}\, e^{7\, t} - \frac{78}{49} \quad \textbf{(45)}$$

> $SolucionY := invlaplace\big(TLSol_2,\, s,\, t\big)$

$$SolucionY := y(t) = \frac{1}{7}\, t - \frac{29}{49}\, e^{7\, t} - \frac{20}{49} \quad \textbf{(46)}$$

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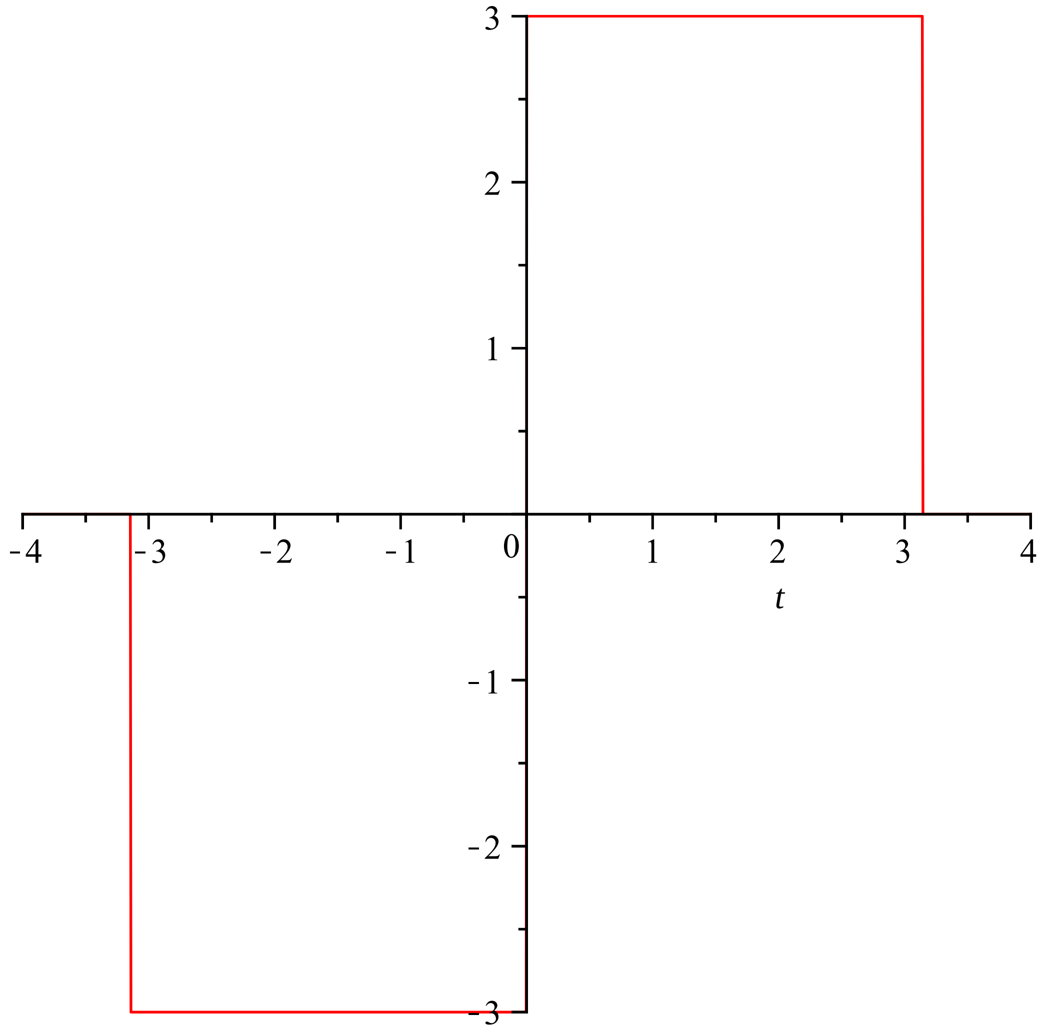
FIN RESPUESTA 4)

> *restart*

5) Desarrollar la función

> $f := -3 \cdot \mathrm{Heaviside}(t + \mathrm{Pi}) + 6 \cdot \mathrm{Heaviside}(t) - 3 \cdot \mathrm{Heaviside}(t - \mathrm{Pi});\ plot(f,\, t = -4..4)$

$$f := -3\, \mathrm{Heaviside}(t + \pi) + 6\, \mathrm{Heaviside}(t) - 3\, \mathrm{Heaviside}(t - \pi)$$



en una serie seno

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RESPUESTA 5)

> $L := \mathrm{Pi}$

$$L := \pi \quad \textbf{(47)}$$

> $a_0 := \frac{1}{L} \cdot int(f, t = -L..L)$

$$a_0 := 0 \quad \textbf{(48)}$$

> $a_n := \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)$

$$a_n := 0 \quad \textbf{(49)}$$

> $b_n := subs\left(\cos(n \cdot \mathrm{Pi}) = (-1)^{\cdot} n, \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)\right)$

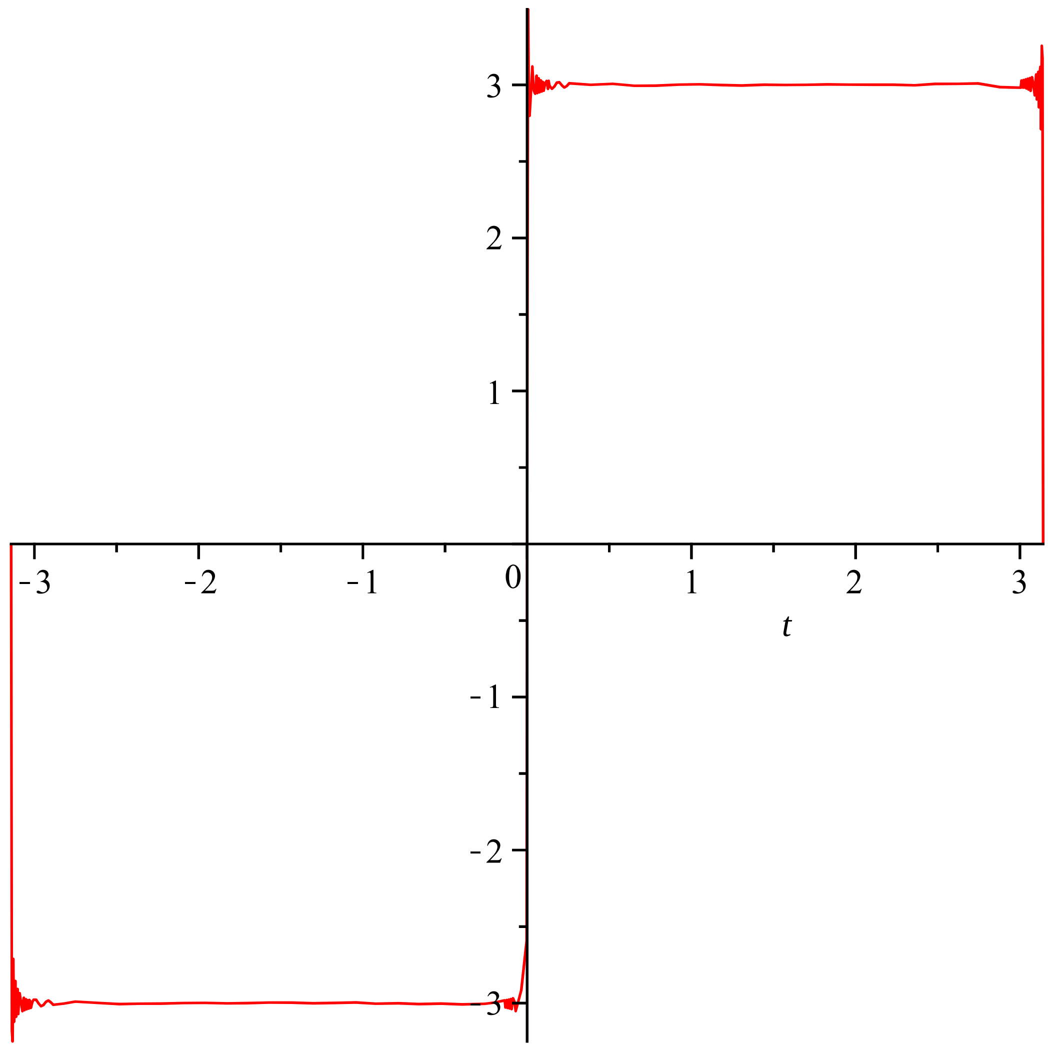
$$b_n := \frac{-6\ (-1)^n + 6}{\pi\ n} \quad \textbf{(50)}$$

> $STF := Sum\left(b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), n = 1..infinity\right)$

$$STF := \sum_{n=1}^{\infty} \frac{(-6(-1)^n + 6)\sin(n t)}{\pi n} \quad \mathbf{(51)}$$

> $STF_{500} := sum\left(b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), n = 1..500\right):$

> $plot(STF_{500}, t = -\mathrm{Pi}..\mathrm{Pi})$



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FIN RESPUESTA 5)

> *restart*

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FIN DEL EXAMEN