>

SOLUCIÓN

>

FACULTAD DE INGENIERÍA
DIVISIÓN DE CIENCIAS BÁSICAS
ECUACIONES DIFERENCIALES
SEMESTRE 2019-1
SEGUNDO EXAMEN PARCIAL

2018-10-11

>

>

DADO EL PROBLEMA DE CONDICIONES INICIALES CON UNA ECUACIÓN DIFERENCIAL ORDINARIA LINEAL DE SEGUNDO ORDEN, COEFICIENTES CONSTANTES - NO HOMOGÉNEA

>

1) **(20/100 puntos)** OBTENER SU SOLUCIÓN GENERAL DE LA ECUACIÓN DIFERENCIAL HOMOGENEA ASOCIADA **(sin utilizar dsolve)**

> *restart*

> $EDO := \frac{d^2}{dt^2}\,x(t) - 9\,x(t) = 14\,e^{3t}$

$$EDO := \frac{d^2}{dt^2}\,x(t) - 9\,x(t) = 14\,e^{3t} \quad \mathbf{(1)}$$

> $Cond := x(0) = 9, D(x)(0) = 12$

$$Cond := x(0) = 9, D(x)(0) = 12 \quad \mathbf{(2)}$$

> $EDOH := lhs(EDO) = 0$

$$EDOH := \frac{d^2}{dt^2}\,x(t) - 9\,x(t) = 0 \quad \mathbf{(3)}$$

> $ParteNoHom := Q(t) = rhs(EDO)$

$$ParteNoHom := Q(t) = 14\,e^{3t} \quad \mathbf{(4)}$$

> $EcuaCarac := m{\cdot\cdot}2 - 9 = 0$

$$EcuaCarac := m^2 - 9 = 0 \quad \mathbf{(5)}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 3, -3 \quad \mathbf{(6)}$$

> $SolUno := x(t) = \exp(Raiz[1]\cdot t); SolDos := x(t) = \exp(Raiz[2]\cdot t)$

$$SolUno := x(t) = e^{3t}$$
$$SolDos := x(t) = e^{-3t} \quad \mathbf{(7)}$$

> $SolucionHomogeneaAsociada := x(t) = C1\cdot rhs(SolUno) + C2\cdot rhs(SolDos)$

$$SolucionHomogeneaAsociada := x(t) = C1\,e^{3t} + C2\,e^{-3t} \quad \mathbf{(8)}$$

> *restart*

2) **(20/100 puntos)** OBTENER SU SOLUCIÓN GENERAL UTILIZANDO EL MÉTODO DE COEFICIENTES INDETERMINADOS **(sin utilizar dsolve)**

> $EDO := \frac{d^2}{dt^2}\,x(t) - 9\,x(t) = 14\,e^{3t}$

$$EDO := \frac{d^2}{dt^2}\, x(t) - 9\, x(t) = 14\, e^{3t} \tag{9}$$

> $SolucionHomogeneaAsociada := x(t) = C1\, e^{3t} + C2\, e^{-3t}$

$$SolucionHomogeneaAsociada := x(t) = C1\, e^{3t} + C2\, e^{-3t} \tag{10}$$

> $ParteNoHom := Q(t) = rhs(EDO)$

$$ParteNoHom := Q(t) = 14\, e^{3t} \tag{11}$$

> $EDONH := (D-3)\cdot(D+3)x(t) = rhs(ParteNoHom)$

$$EDONH := (D-3)\ (D+3)\ x(t) = 14\, e^{3t} \tag{12}$$

> $EDOHuno := (D-3)\cdot(D+3)\cdot(D-3)[A]\, x(t) = 0$

$$EDOHuno := (D-3)\ (D+3)\ (D-3)_A\, x(t) = 0 \tag{13}$$

> $EDOHdos := (D-3)\cdot\cdot 2\cdot(D+3)x(t) = 0$

$$EDOHdos := (D-3)^2\ (D+3)\ x(t) = 0 \tag{14}$$

> $SolucionHomogeneaExtendida := x(t) = C1\cdot\exp(3\cdot t) + C2\cdot t\cdot\exp(3\cdot t) + C3\cdot\exp(-3\cdot t)$

$$SolucionHomogeneaExtendida := x(t) = C1\, e^{3t} + C2\, t\, e^{3t} + C3\, e^{-3t} \tag{15}$$

> $SolucionHomogeneaAsociada$

$$x(t) = C1\, e^{3t} + C2\, e^{-3t} \tag{16}$$

> $SolucionParticularQ := x(t) = A\cdot t\cdot\exp(3\cdot t)$

$$SolucionParticularQ := x(t) = A\, t\, e^{3t} \tag{17}$$

> $SolucionNoHomogenea := x(t) = rhs(SolucionHomogeneaAsociada)$
> $+ rhs(SolucionParticularQ)$

$$SolucionNoHomogenea := x(t) = C1\, e^{3t} + C2\, e^{-3t} + A\, t\, e^{3t} \tag{18}$$

> $CoeficienteIndeterminado := isolate(eval(subs(x(t) = rhs(SolucionParticularQ), EDO)), A)$

$$CoeficienteIndeterminado := A = \frac{7}{3} \tag{19}$$

> $SolucionNoHomogeneaFinal := subs(A = rhs(CoeficienteIndeterminado),$
> $SolucionNoHomogenea)$

$$SolucionNoHomogeneaFinal := x(t) = C1\, e^{3t} + C2\, e^{-3t} + \frac{7}{3}\, t\, e^{3t} \tag{20}$$

> $Comprobacion := eval(subs(x(t) = rhs(SolucionNoHomogeneaFinal), lhs(EDO) - rhs(EDO)$
> $= 0))$

$$Comprobacion := 0 = 0 \tag{21}$$

> $restart$

3) **(20/100 puntos)** OBTENER SU SOLUCIÓN GENERAL UTILIZANDO EL <u>MÉTODO DE PARÁMETROS VARIABLES</u> **(sin utilizar dsolve)**

> $EDO := \frac{d^2}{dt^2}\, x(t) - 9\, x(t) = 14\, e^{3t}$

$$EDO := \frac{d^2}{dt^2}\, x(t) - 9\, x(t) = 14\, e^{3t} \tag{22}$$

> $ParteNoHom := Q(t) = rhs(EDO)$

$$ParteNoHom := Q(t) = 14\, e^{3t} \tag{23}$$

> $SolUno := x(t) = e^{3t}$

$$SolUno := x(t) = e^{3t} \tag{24}$$

> $SolDos := x(t) = e^{-3t}$

$$SolDos := x(t) = e^{-3t} \tag{25}$$

> $SolucionHomogeneaAsociada := x(t) = C1 \cdot rhs(SolUno) + C2 \cdot rhs(SolDos)$

$$SolucionHomogeneaAsociada := x(t) = C1\, e^{3t} + C2\, e^{-3t} \tag{26}$$

> $SolucionNoHom := x(t) = A(t) \cdot rhs(SolUno) + B(t) \cdot rhs(SolDos)$

$$SolucionNoHom := x(t) = A(t)\, e^{3t} + B(t)\, e^{-3t} \tag{27}$$

> $with(linalg):$

> $MM := wronskian([rhs(SolUno), rhs(SolDos)], t)$

$$MM := \begin{bmatrix} e^{3t} & e^{-3t} \\ 3\, e^{3t} & -3\, e^{-3t} \end{bmatrix} \tag{28}$$

> $BB := array([0, rhs(ParteNoHom)])$

$$BB := \begin{bmatrix} 0 & 14\, e^{3t} \end{bmatrix} \tag{29}$$

> $ParaDer := linsolve(MM, BB)$

$$ParaDer := \begin{bmatrix} \frac{7}{3} & -\frac{7}{3}\, \frac{e^{3t}}{e^{-3t}} \end{bmatrix} \tag{30}$$

> $Aprima := ParaDer[1]; Bprima := simplify(ParaDer[2])$

$$Aprima := \frac{7}{3}$$

$$Bprima := -\frac{7}{3}\, e^{6t} \tag{31}$$

> $ParaUno := A(t) = int(Aprima, t) + C1$

$$ParaUno := A(t) = \frac{7}{3}\, t + C1 \tag{32}$$

> $ParaDos := B(t) = int(Bprima, t) + C2$

$$ParaDos := B(t) = -\frac{7}{18}\, e^{6t} + C2 \tag{33}$$

> $SolucionNoHomogeneaFinal := simplify\left(subs\left(C1 = C1 + \frac{7}{18}, simplify(subs(A(t) = rhs(ParaUno), B(t) = rhs(ParaDos), SolucionNoHom))\right)\right)$

$$SolucionNoHomogeneaFinal := x(t) = \frac{7}{3}\, e^{3t}\, t + C1\, e^{3t} + C2\, e^{-3t} \tag{34}$$

> $restart$

4) **(20/100 puntos)** OBTENER LA SOLUCIÓN PARTICULAR PARA LAS CONDICIONES INICIALES DADAS **(sin utilizar dsolve)**

> $SolucionNoHomogeneaFinal := x(t) = \frac{7}{3}\, e^{3t}\, t + C1\, e^{3t} + C2\, e^{-3t}$

$$SolucionNoHomogeneaFinal := x(t) = \frac{7}{3}\, e^{3t}\, t + C1\, e^{3t} + C2\, e^{-3t} \tag{35}$$

> $Cond := x(0) = 9, D(x)(0) = 12$

$$Cond := x(0) = 9, D(x)(0) = 12 \tag{36}$$

> $ParaUno := eval(subs(t = 0, rhs(SolucionNoHomogeneaFinal) = rhs(Cond[1])))$

$$ParaUno := C1 + C2 = 9 \tag{37}$$

> $ParaDos := eval(subs(t = 0, rhs(diff(SolucionNoHomogeneaFinal, t)) = rhs(Cond[2])))$

$$ParaDos := \frac{7}{3} + 3\, C1 - 3\, C2 = 12 \tag{38}$$

> $Parametros := solve(\{ParaUno, ParaDos\}, \{C1, C2\})$

$$Parametros := \left\{C1 = \frac{55}{9}, C2 = \frac{26}{9}\right\} \tag{39}$$

> $SolucionParticular := subs(C1 = rhs(Parametros[1]), C2 = rhs(Parametros[2]),$
$SolucionNoHomogeneaFinal)$

$$SolucionParticular := x(t) = \frac{7}{3}\, e^{3t}\, t + \frac{55}{9}\, e^{3t} + \frac{26}{9}\, e^{-3t} \tag{40}$$

> $restart$

5) **(20/100 puntos)** COMPROBAR LAS CONDICIONES INICIALES MEDIANTE LAS GRÁFICAS DE LA SOLUCIÓN PARTICULAR OBTENIDA Y DE SU PRIMERA DERIVADA, PARA UN INTERVALO DE $0 < t < 0.1$

> $SolucionParticular := x(t) = \frac{7}{3}\, e^{3t}\, t + \frac{55}{9}\, e^{3t} + \frac{26}{9}\, e^{-3t}$

$$SolucionParticular := x(t) = \frac{7}{3}\, e^{3t}\, t + \frac{55}{9}\, e^{3t} + \frac{26}{9}\, e^{-3t} \tag{41}$$

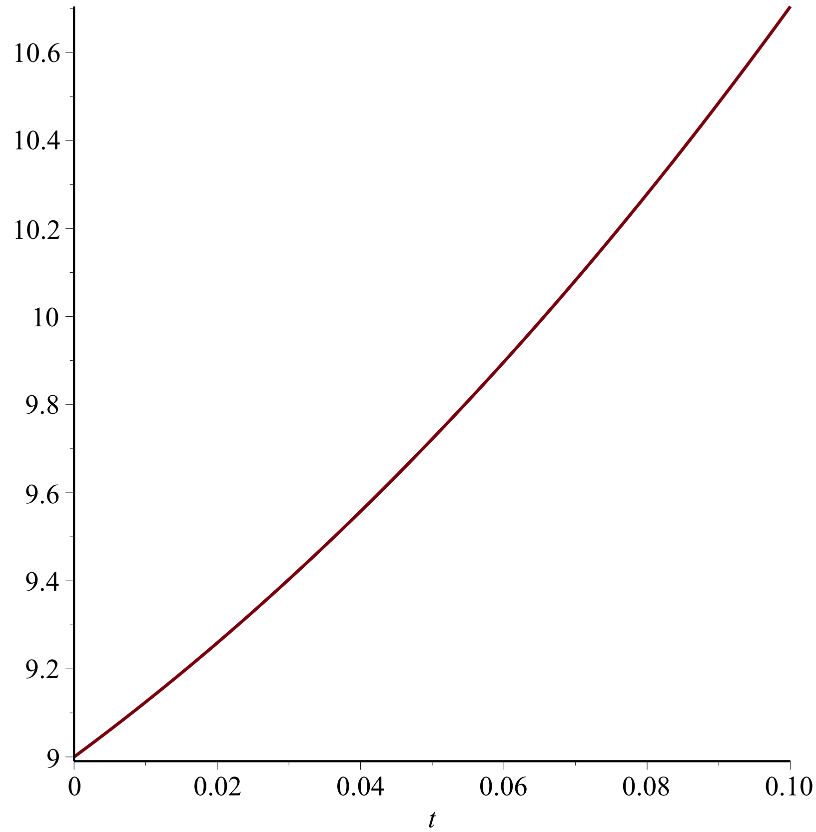
> $CondicionUno := eval(subs(t = 0, SolucionParticular))$

$$CondicionUno := x(0) = 9 \tag{42}$$

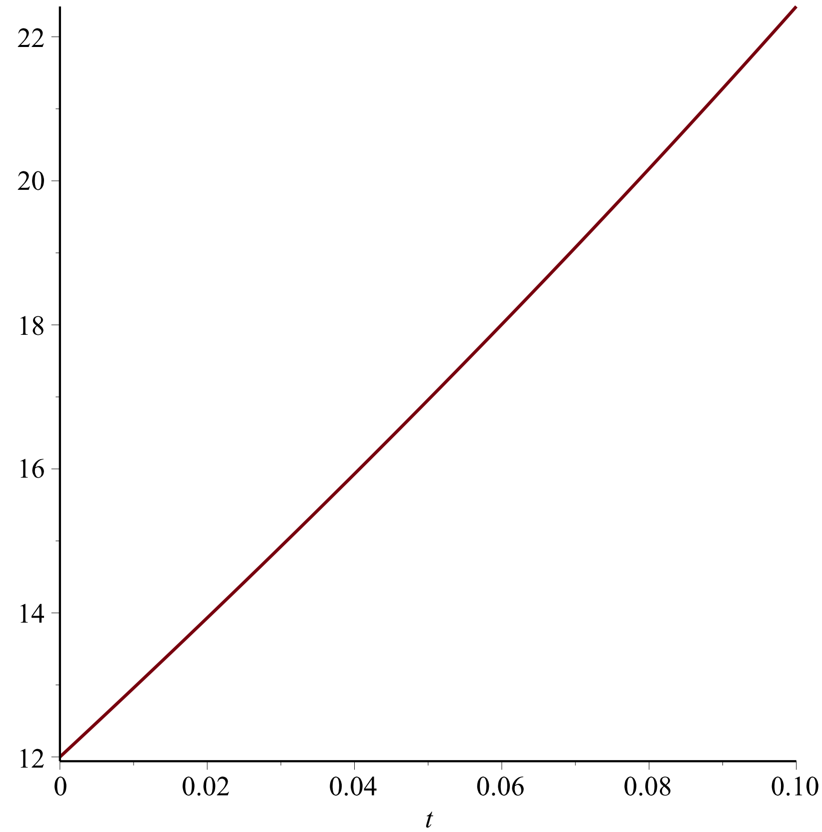
> $CondicionDos := D(x)(0) = eval(subs(t = 0, rhs(diff(SolucionParticular, t))))$

$$CondicionDos := D(x)(0) = 12 \tag{43}$$

> $plot(rhs(SolucionParticular), t = 0 ..0.1)$



> $plot(rhs(diff(SolucionParticular, t)), t = 0\,..0.1)$



> *restart*

FIN DEL EXAMEN

>

>