UNAM
FACULTAD DE INGENIERÍA
DIVISIÓN DE CIENCIAS BÁSICAS
ECUACIONES DIFERENCIALES
GRUPO 15 SEMESTRE 2023-1
PRIMER EXAMEN PARCIAL Temas 1 & 2

2022-11-03

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PREGUNTA 1 (30 puntos) Obtener la solución general de la siguiente ecuación diferencial ordinaria no lineal, utilizando el método de coeficientes homogeneos (***sin usar dsolve***)

> $Ecua := 4x - 3y + (2y - 3x)\cdot y' = 0$

$$Ecua := 4x - 3y(x) + (2y(x) - 3x)\left(\frac{d}{dx}y(x)\right) = 0 \quad (1)$$

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[[\_homogeneous, class\ A], \_exact, \_rational, [\_Abel, 2nd\ type, class\ A]] \quad (2)$$

> $EcuaDos := simplify(isolate(eval(subs(y(x) = u(x)\cdot x, Ecua)), diff(u(x), x)))$

$$EcuaDos := \frac{d}{dx}u(x) = -\frac{2(u(x)^2 - 3u(x) + 2)}{x(2u(x) - 3)} \quad (3)$$

> $odeadvisor(EcuaDos)$

$$[\_separable] \quad (4)$$

> $P := -\frac{2(u^2 - 3u + 2)}{(2u - 3)}$

$$P := -\frac{2(u^2 - 3u + 2)}{2u - 3} \quad (5)$$

> $R := x$

$$R := x \quad (6)$$

> $SolUno := int\left(\frac{1}{P}, u\right) = int\left(\frac{1}{R}, x\right) + \_C1$

$$SolUno := -\frac{1}{2}\ln(u^2 - 3u + 2) = \ln(x) + \_C1 \quad (7)$$

> $SolDos := isolate\left(simplify\left(subs\left(u = \frac{y(x)}{x}, SolUno\right)\right), \_C1\right)$

$$SolDos := \_C1 = -\frac{1}{2}\ln\left(\frac{(y(x) - x)(y(x) - 2x)}{x^2}\right) - \ln(x) \quad (8)$$

> $SolGral := simplify(\exp(rhs(SolDos))) = \_C1$

$$SolGral := \frac{1}{\sqrt{\frac{(y(x) - x)(y(x) - 2x)}{x^2}}\, x} = \_C1 \quad (9)$$

> $SolFinal := \frac{1}{lhs(SolGral)^2} = \_C1$

$$SolFinal := (y(x) - x)(y(x) - 2x) = \_C1 \quad (10)$$

> $DerSolGral := simplify(isolate(diff(SolFinal, x), diff(y(x), x)))$

$$DerSolGral := \frac{d}{dx}\, y(x) = \frac{-4\,x + 3\,y(x)}{2\,y(x) - 3\,x} \qquad (11)$$

> $DerEcua := isolate(Ecua, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}\, y(x) = \frac{-4\,x + 3\,y(x)}{2\,y(x) - 3\,x} \qquad (12)$$

> $Comprobacion := simplify(rhs(DerEcua) - rhs(DerSolGral)) = 0$

$$Comprobacion := 0 = 0 \qquad (13)$$

> $restart$

PREGUNTA 2 (20 puntos) Obtener la solución general de la siguiente ecuación diferencial ordinaria de coeficientes variables no homogenea (***sin usar dsolve***)

> $Ecua := \left(\frac{d}{dx}\, y(x)\right) + 2\cdot x\cdot y(x) = 2\cdot x\cdot \exp(-x^2)$

$$Ecua := \frac{d}{dx}\, y(x) + 2\,x\,y(x) = 2\,x\,e^{-x^2} \qquad (14)$$

>

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d}{dx}\, y(x) + 2\,x\,y(x) = 0 \qquad (15)$$

> $p := 2\cdot x;\ q := rhs(Ecua)$

$$p := 2\,x$$

$$q := 2\,x\,e^{-x^2} \qquad (16)$$

> $SolHom := y(x) = \_C1\cdot \exp(-int(p, x))$

$$SolHom := y(x) = \_C1\,e^{-x^2} \qquad (17)$$

> $SolGral := y(x) = \_C1\cdot \exp(-int(p, x)) + \exp(-int(p, x))\cdot int(\exp(int(p, x))\cdot q, x)$

$$SolGral := y(x) = \_C1\,e^{-x^2} + e^{-x^2}\,x^2 \qquad (18)$$

> $Comprobacion := simplify(eval(subs(y(x) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobacion := 0 = 0 \qquad (19)$$

> $restart$

PREGUNTA 3 (20 puntos) Obtener la solución particular del siguiente problema de ecuaciones diferenciales ordinarias lineales homogeneas con condiciones iniciales (***sin usar dsolve***)

> $Ecua := y'' - 5\cdot y' + 6\cdot y = 0$

$$Ecua := \frac{d^2}{dx^2}\, y(x) - 5\left(\frac{d}{dx}\, y(x)\right) + 6\,y(x) = 0 \qquad (20)$$

> $CondIni := y(0) = 6, D(y)(0) = 10$

$$CondIni := y(0) = 6, D(y)(0) = 10 \qquad (21)$$

> $EcuaCarac := m^2 - 5\,m + 6 = 0$

$$EcuaCarac := m^2 - 5\,m + 6 = 0 \qquad (22)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 3, 2 \qquad (23)$$

> $SolGralHom := y(x) = \_C1\cdot \exp(Raiz[1]\cdot x) + \_C2\cdot \exp(Raiz[2]\cdot x)$

$$SolGralHom := y(x) = \_C1\,e^{3\,x} + \_C2\,e^{2\,x} \qquad (24)$$

> $EcuaUno := eval(subs(x=0, rhs(SolGralHom)=6))$

$$EcuaUno := \_C1 + \_C2 = 6 \quad (25)$$

> $EcuaDos := eval(subs(x=0, rhs(diff(SolGralHom, x))=10))$

$$EcuaDos := 3\ \_C1 + 2\ \_C2 = 10 \quad (26)$$

> $Para := solve([EcuaUno, EcuaDos])$

$$Para := \{\_C1 = -2, \_C2 = 8\} \quad (27)$$

> $SolPart := subs(Para, SolGralHom)$

$$SolPart := y(x) = -2\ e^{3x} + 8\ e^{2x} \quad (28)$$

> $Comprobacion := eval(subs(y(x)=rhs(SolPart), Ecua))$

$$Comprobacion := 0 = 0 \quad (29)$$

> $CondIni$

$$y(0) = 6, D(y)(0) = 10 \quad (30)$$

> $CompUno := simplify(subs(x=0, SolPart))$

$$CompUno := y(0) = 6 \quad (31)$$

> $CompDos := D(y)(0) = simplify(subs(x=0, rhs(diff(SolPart, x))))$

$$CompDos := D(y)(0) = 10 \quad (32)$$

> $restart$

PREGUNTA 4 (30 puntos) Obtener la solución particular del siguiente problema de ecuaciones diferenciales ordinarias no homogeneas con condiciones iniciales (***sin usar dsolve***)

> $Ecua := \frac{d^2}{dx^2}\ y(x) - y(x) = 5\ e^x$

$$Ecua := \frac{d^2}{dx^2}\ y(x) - y(x) = 5\ e^x \quad (33)$$

> $CondIni := y(0) = -1, D(y)(0) = 1$

$$CondIni := y(0) = -1, D(y)(0) = 1 \quad (34)$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\ y(x) - y(x) = 0 \quad (35)$$

> $Q := rhs(Ecua)$

$$Q := 5\ e^x \quad (36)$$

> $EcuaCarac := m^2 - 1 = 0$

$$EcuaCarac := m^2 - 1 = 0 \quad (37)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 1, -1 \quad (38)$$

> $yy[1] := \exp(Raiz[1]\cdot x); yy[2] := \exp(Raiz[2]\cdot x)$

$$yy_1 := e^x$$

$$yy_2 := e^{-x} \quad (39)$$

> $SolHom := y(x) = \_C1\cdot yy[1] + \_C2\cdot yy[2]$

$$SolHom := y(x) = \_C1\ e^x + \_C2\ e^{-x} \quad (40)$$

> $SolNoHom := y(x) = A\cdot yy[1] + B\cdot yy[2]$

$$SolNoHom := y(x) = A\ e^x + B\ e^{-x} \quad (41)$$

> with(linalg) :

> WW := wronskian([yy[1], yy[2]], x)

$$WW := \begin{bmatrix} e^{x} & e^{-x} \\ e^{x} & -e^{-x} \end{bmatrix} \quad (42)$$

> BB := array([0, Q])

$$BB := \begin{bmatrix} 0 & 5\,e^{x} \end{bmatrix} \quad (43)$$

> Para := simplify(linsolve(WW, BB))

$$Para := \begin{bmatrix} \frac{5}{2} & -\frac{5}{2}\,e^{2x} \end{bmatrix} \quad (44)$$

> Aprima := Para[1]; Bprima := Para[2]

$$Aprima := \frac{5}{2}$$

$$Bprima := -\frac{5}{2}\,e^{2x} \quad (45)$$

> A := int(Aprima, x) + _C10; B := int(Bprima, x) + _C20

$$A := \frac{5}{2}\,x + \_C10$$

$$B := -\frac{5}{4}\,e^{2x} + \_C20 \quad (46)$$

> SolGral := simplify(subs(_C10 = _C1 + 1, _C20 = _C2, simplify(SolNoHom)))

$$SolGral := y(x) = \frac{5}{2}\,e^{x}\,x + \_C1\,e^{x} - \frac{1}{4}\,e^{x} + \_C2\,e^{-x} \quad (47)$$

> CondIni

$$y(0) = -1, D(y)(0) = 1 \quad (48)$$

> ParaUno := simplify(subs(x = 0, rhs(SolGral) =-1))

$$ParaUno := \_C1 - \frac{1}{4} + \_C2 = -1 \quad (49)$$

> ParaDos := simplify(subs(x = 0, rhs(diff(SolGral, x)) = 1))

$$ParaDos := \frac{9}{4} + \_C1 - \_C2 = 1 \quad (50)$$

> Parametros := solve([ParaUno, ParaDos])

$$Parametros := \left\{\_C1 = -1, \_C2 = \frac{1}{4}\right\} \quad (51)$$

> SolPart := subs(Parametros, SolGral)

$$SolPart := y(x) = \frac{5}{2}\,e^{x}\,x - \frac{5}{4}\,e^{x} + \frac{1}{4}\,e^{-x} \quad (52)$$

> Comprobacion := simplify(eval(subs(y(x) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))

$$Comprobacion := 0 = 0 \quad (53)$$

> CondIni

$$y(0) = -1, D(y)(0) = 1 \quad (54)$$

> CompUno := simplify(subs(x = 0, SolPart))

$$CompUno := y(0) = -1 \quad (55)$$

> $CompDos := D(y)(0) = simplify(subs(x=0, rhs(diff(SolPart, x))))$

$$CompDos := D(y)(0) = 1 \quad \textbf{(56)}$$

> $restart$

FIN DEL EXAMEN

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