Ecuaciones Diferenciales
grupo 15 semestre 2023-1
Segundo Examen Parcial: Temas 3 & 4
SOLUCIÓN

2023-01-05

PREGUNTA 1 (20 puntos) Mediante la Transformada de Laplace obtenga la solución de la ecuación diferencial, sujeta a las condiciones iniciales dadas (***sin usar dsolve***)

> *restart*

> $Ecua := 3\ (diff(y(t), t\$2)) + y(t) = \sin(t)\ \text{Heaviside}(t - 2\,\pi)$

$$Ecua := 3\left(\frac{d^2}{dt^2}\,y(t)\right) + y(t) = \sin(t)\ \text{Heaviside}(t - 2\,\pi) \qquad (1)$$

> $Cond := y(0) = 1, \mathrm{D}(y)(0) = 0$

$$Cond := y(0) = 1, \mathrm{D}(y)(0) = 0 \qquad (2)$$

> $with(inttrans):$

> $EcuaTransLap := subs(Cond, laplace(Ecua, t, s))$

$$EcuaTransLap := 3\,s^2\,laplace(y(t), t, s) - 3\,s + laplace(y(t), t, s) = \frac{e^{-2\,s\,\pi}}{s^2 + 1} \qquad (3)$$

> $SolTransLap := simplify(isolate(EcuaTransLap, laplace(y(t), t, s)))$

$$SolTransLap := laplace(y(t), t, s) = \frac{3\,s^3 + e^{-2\,s\,\pi} + 3\,s}{(s^2 + 1)\,(3\,s^2 + 1)} \qquad (4)$$

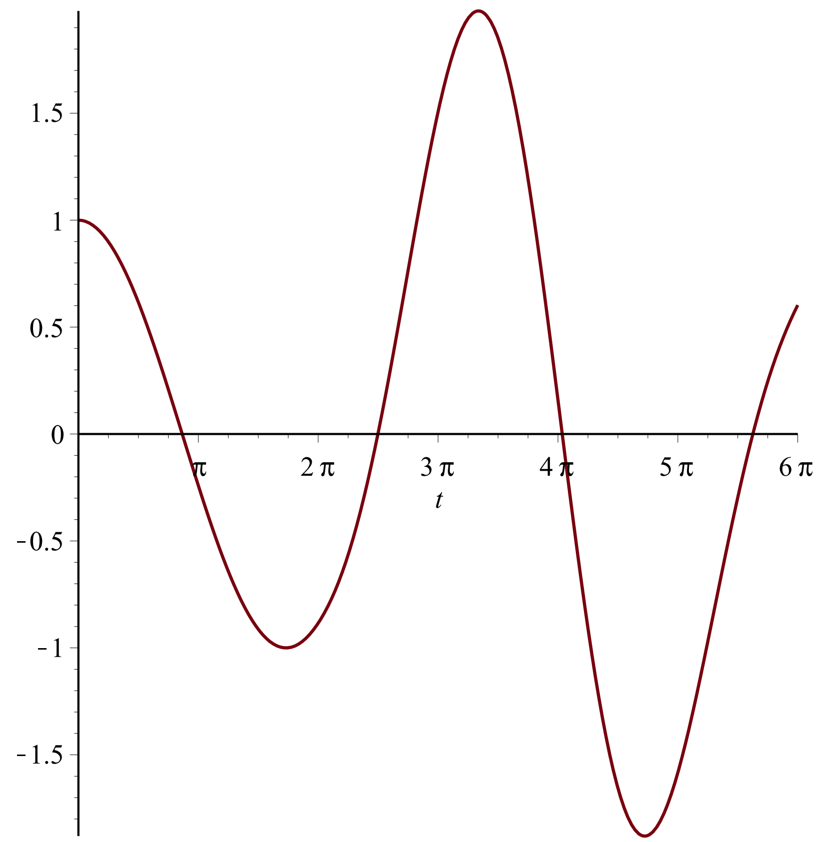
> $SolPart := invlaplace(SolTransLap, s, t)$

$$SolPart := y(t) = \cos\left(\frac{1}{3}\sqrt{3}\,t\right) + \frac{1}{2}\left(-\sin(t) + \sqrt{3}\,\sin\left(\frac{1}{3}\sqrt{3}\,(t - 2\,\pi)\right)\right)\text{Heaviside}(t - 2\,\pi) \qquad (5)$$

> $Comprob := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprob := 0 = 0 \qquad (6)$$

> $plot(rhs(SolPart), t = 0 ..6\cdot \mathrm{Pi})$



> *restart*

PREGUNTA 2 (20 puntos) Obtener la solución particular del sistema de ecuaciones diferenciales con las condiciones iniciales dadas (***sin usar dsolve***)

> $SistEcua := diff(y[1](t), t) = y[2](t), diff(y[2](t), t) = -4 \cdot y[1](t) + 2 \cdot \cos(t) : SistEcua[1];$
> $SistEcua[2];$

$$\frac{d}{dt} y_1(t) = y_2(t)$$

$$\frac{d}{dt} y_2(t) = -4\, y_1(t) + 2\cos(t) \qquad \textbf{(7)}$$

> $Cond := y[1](0) = 0, y[2](0) = 0$

$$Cond := y_1(0) = 0, y_2(0) = 0 \qquad \textbf{(8)}$$

> $AA := array([[0, 1], [-4, 0]])$

$$AA := \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} \qquad \textbf{(9)}$$

> $BB := array([0, 2 \cdot \cos(t)])$

$$BB := \begin{bmatrix} 0 & 2\cos(t) \end{bmatrix} \qquad \textbf{(10)}$$

> $with(linalg) :$

> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} \cos(2\,t) & \frac{1}{2}\sin(2\,t) \\ -2\sin(2\,t) & \cos(2\,t) \end{bmatrix} \quad \textbf{(11)}$$

> $Xcero := array([0, 0])$

$$Xcero := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad \textbf{(12)}$$

> $SolGral := evalm(MatExp \&* Xcero)$

$$SolGral := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad \textbf{(13)}$$

> $MatExpTau := map(rcurry(eval, t='t - \mathrm{tau}'), MatExp)$

$$MatExpTau := \begin{bmatrix} \cos(2\,t-2\,\tau) & \frac{1}{2}\sin(2\,t-2\,\tau) \\ -2\sin(2\,t-2\,\tau) & \cos(2\,t-2\,\tau) \end{bmatrix} \quad \textbf{(14)}$$

> $BBtau := map(rcurry(eval, t='\mathrm{tau}'), BB)$

$$BBtau := \begin{bmatrix} 0 & 2\cos(\tau) \end{bmatrix} \quad \textbf{(15)}$$

> $AAtau := evalm(\ MatExpTau \&* BBtau)$

$$AAtau := \begin{bmatrix} \sin(2\,t-2\,\tau)\cos(\tau) & 2\cos(2\,t-2\,\tau)\cos(\tau) \end{bmatrix} \quad \textbf{(16)}$$

> $SolPart := map(int, AAtau, \mathrm{tau}=0..t) : y[1](t) = SolPart[1]; y[2](t) = SolPart[2]$

$$y_1(t) = -\frac{2}{3}\cos(2\,t) + \frac{2}{3}\cos(t)$$

$$y_2(t) = \frac{4}{3}\sin(2\,t) - \frac{2}{3}\sin(t) \quad \textbf{(17)}$$

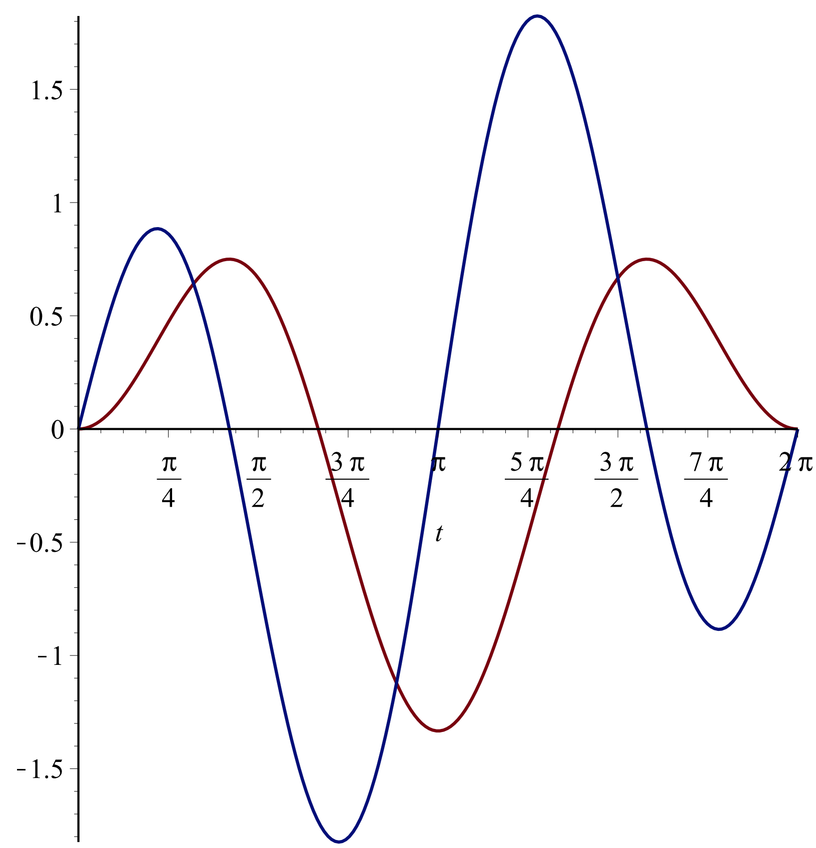
> $CompUno := eval(subs(y[1](t) = SolPart[1], y[2](t) = SolPart[2], lhs(SistEcua[1]) - rhs(SistEcua[1]) = 0)\ )$

$$CompUno := 0 = 0 \quad \textbf{(18)}$$

> $CompDos := eval(subs(y[1](t) = SolPart[1], y[2](t) = SolPart[2], lhs(SistEcua[2]) - rhs(SistEcua[2]) = 0)\ )$

$$CompDos := 0 = 0 \quad \textbf{(19)}$$

> $plot([SolPart[1], SolPart[2]], t = 0..2\cdot\mathrm{Pi})$



> *restart*

PREGUNTA 3 (30 puntos) Detremine una solución completa de la ecuación diferencial utilizando el método de separación de variables para una constante de separación nula (***sin usar pdsolve***)

> *Ecua* := *diff*(*y*(*x*, *t*), *t*$2) + *diff*(*y*(*x*, *t*), *x*, *t*) = 2·*x*³·*diff*(*y*(*x*, *t*), *t*)

$$Ecua := \frac{\partial^2}{\partial t^2}\, y(x,t) + \frac{\partial^2}{\partial x\,\partial t}\, y(x,t) = 2\,x^3 \left(\frac{\partial}{\partial t}\, y(x,t)\right) \quad \textbf{(20)}$$

> *EcuaSeparable* := *eval*(*subs*(*y*(*x*, *t*) = *F*(*x*)·*G*(*t*), *Ecua*))

$$EcuaSeparable := F(x)\left(\frac{d^2}{dt^2}\, G(t)\right) + \left(\frac{d}{dx}\, F(x)\right)\left(\frac{d}{dt}\, G(t)\right) = 2\,x^3\, F(x)\left(\frac{d}{dt}\, G(t)\right) \quad \textbf{(21)}$$

> $$EcuaSeparada := \frac{\left(lhs(EcuaSeparable) - \left(\frac{d}{dx}\, F(x)\right)\left(\frac{d}{dt}\, G(t)\right)\right)}{F(x)\cdot diff(\,G(t), t)} = simplify\left(\frac{\left(rhs(EcuaSeparable) - \left(\frac{d}{dx}\, F(x)\right)\left(\frac{d}{dt}\, G(t)\right)\right)}{F(x)\cdot diff(\,G(t), t)}\right)$$

(22)

$$EcuaSeparada := \frac{\frac{d^2}{dt^2} G(t)}{\frac{d}{dt} G(t)} = \frac{2 F(x) x^3 - \left(\frac{d}{dx} F(x)\right)}{F(x)} \quad (22)$$

> $EcuaX := rhs(EcuaSeparada) = 0$

$$EcuaX := \frac{2 F(x) x^3 - \left(\frac{d}{dx} F(x)\right)}{F(x)} = 0 \quad (23)$$

> $EcuaT := lhs(EcuaSeparada) = 0$

$$EcuaT := \frac{\frac{d^2}{dt^2} G(t)}{\frac{d}{dt} G(t)} = 0 \quad (24)$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = \_C1\, e^{\frac{1}{2} x^4} \quad (25)$$

> $SolT := dsolve(EcuaT)$

$$SolT := G(t) = \_C1\, t + \_C2 \quad (26)$$

> $SolGralCero := y(x, t) = rhs(SolT) \cdot subs(\_C1 = 1, rhs(SolX))$

$$SolGralCero := y(x, t) = (\_C1\, t + \_C2)\, e^{\frac{1}{2} x^4} \quad (27)$$

> $Comprobacion := simplify(eval(subs(y(x, t) = rhs(SolGralCero), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobacion := 0 = 0 \quad (28)$$

> restart

PREGUNTA 4 (30 puntos) Determinar la solución de la ecuación diferencial considerando una constante de separación positiva (***sin usar pdsolve***)

> $Ecua := diff(z(x, y), x\$2, y) = diff(z(x, y), x)$

$$Ecua := \frac{\partial^3}{\partial y\, \partial x^2} z(x, y) = \frac{\partial}{\partial x} z(x, y) \quad (29)$$

> $EcuaSeparable := eval(subs(z(x, y) = F(x) \cdot G(y), Ecua))$

$$EcuaSeparable := \left(\frac{d^2}{dx^2} F(x)\right) \left(\frac{d}{dy} G(y)\right) = \left(\frac{d}{dx} F(x)\right) G(y) \quad (30)$$

> $EcuaSeparada := \frac{lhs(EcuaSeparable)}{\left(\frac{d}{dx} F(x)\right) \cdot \left(\frac{d}{dy} G(y)\right)} = \frac{rhs(EcuaSeparable)}{\left(\frac{d}{dx} F(x)\right) \cdot \left(\frac{d}{dy} G(y)\right)}$

$$EcuaSeparada := \frac{\frac{d^2}{dx^2} F(x)}{\frac{d}{dx} F(x)} = \frac{G(y)}{\frac{d}{dy} G(y)} \quad (31)$$

> $EcuaX := lhs(EcuaSeparada) = \beta^2$

$$EcuaX := \frac{\frac{d^2}{dx^2} F(x)}{\frac{d}{dx} F(x)} = \beta^2 \quad (32)$$

> $EcuaY := rhs(EcuaSeparada) = \beta^2$

$$EcuaY := \frac{G(y)}{\frac{d}{dy} G(y)} = \beta^2 \quad (33)$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = \_C1 + \_C2\, e^{\beta^2 x} \quad (34)$$

> $SolY := dsolve(EcuaY)$

$$SolY := G(y) = \_C1\, e^{\frac{y}{\beta^2}} \quad (35)$$

> $SolGral := z(x, y) = subs(\_C1 = 1, rhs(SolY)) \cdot rhs(SolX)$

$$SolGral := z(x, y) = e^{\frac{y}{\beta^2}} \left(\_C1 + \_C2\, e^{\beta^2 x}\right) \quad (36)$$

> $Comp := eval(subs(z(x, y) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0))$

$$Comp := 0 = 0 \quad (37)$$

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FIN DE LA SOLUCIÓN

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