>

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SERIE 1
DE EJERCICIOS DEL TEMA 1
SEMESTRE 2023-1
SOLUCIÓN

2022-09-12

> *restart*

1) Si conocemos la solución general de una ecuación diferencial ordinaria no lineal desconocida

> $SolucionGeneral := x^2 \cdot \log(y(x)) + \frac{1}{3} \cdot (y(x)^2 + 1)^{\frac{3}{2}} = \_C1$

$$SolucionGeneral := x^2 \ln(y(x)) + \frac{1}{3} \left(y(x)^2 + 1\right)^{3/2} = \_C1 \quad \textbf{(1)}$$

a) obtenga su ecuación diferencial correspondiente: M(x,y)+N(x,y)*y'=0
b) Explique cuál método es el más adecuado para resolverla
b) obtenga la solución particular que satisface la siguiente condición inicial

> $CondicionInicial := y(1) = 2$

$$CondicionInicial := y(1) = 2 \quad \textbf{(2)}$$

RESULTADO
a)

> *SolucionGeneral*

$$x^2 \ln(y(x)) + \frac{1}{3} \left(y(x)^2 + 1\right)^{3/2} = \_C1 \quad \textbf{(3)}$$

> $DerSol := simplify(isolate(diff(SolucionGeneral, x), diff(y(x), x)))$

$$DerSol := \frac{d}{dx} y(x) = -\frac{2\, x \ln(y(x))\, y(x)}{\sqrt{y(x)^2 + 1}\; y(x)^2 + x^2} \quad \textbf{(4)}$$

> $EcuacionDiferencial := 2\, x \ln(y(x))\, y(x) + \left(\sqrt{y(x)^2 + 1}\; y(x)^2 + x^2\right) \cdot diff(y(x), x) = 0$

$$EcuacionDiferencial := 2\, x \ln(y(x))\, y(x) + \left(\sqrt{y(x)^2 + 1}\; y(x)^2 + x^2\right) \left(\frac{d}{dx} y(x)\right) = 0 \quad \textbf{(5)}$$

b)

> *with*(*DEtools*) :

> *intfactor*(*EcuacionDiferencial*)

$$\frac{1}{y(x)} \quad \textbf{(6)}$$

Por el método de Factor Integrante

>

c)

> $Parametro := subs(x = 1, y(1) = 2, SolucionGeneral)$

$$Parametro := \ln(2) + \frac{5}{3} \sqrt{5} = \_C1 \quad \textbf{(7)}$$

> $SolucionParticular := subs(\_C1 = lhs(Parametro), SolucionGeneral)$

$$SolucionParticular := x^2 \ln(y(x)) + \frac{1}{3} \left(y(x)^2 + 1\right)^{3/2} = \ln(2) + \frac{5}{3} \sqrt{5} \quad \textbf{(8)}$$

Fin 1)

>

2) Obtener la solución general de la siguiente ecuación (sin usar dsolve) por ambos métodos posibles:

> $EcuacionDiferencial := 4x^2 + x\,y(x) - 3\,y(x)^2 + \left(-5x^2 + 2x\,y(x) + y(x)^2\right)\left(\frac{d}{dx}y(x)\right) = 0$

$$EcuacionDiferencial := 4x^2 + x\,y(x) - 3\,y(x)^2 + \left(-5x^2 + 2x\,y(x) + y(x)^2\right)\left(\frac{d}{dx}y(x)\right) = 0 \quad (9)$$

> $with(DEtools):$

> $odeadvisor(EcuacionDiferencial)$

$$[[\_homogeneous, class\ A], \_rational, \_dAlembert] \quad (10)$$

Por factor integrante

> $intfactor(EcuacionDiferencial)$

$$\frac{1}{(y(x) - 2x)\,(y(x) + 2x)\,(y(x) - x)} \quad (11)$$

> $FactInt := \frac{1}{(y - 2x)\,(y + 2x)\,(y - x)}$

$$FactInt := \frac{1}{(y - x)\,(y - 2x)\,(y + 2x)} \quad (12)$$

> $M := 4x^2 + x\,y - 3y^2$

$$M := 4x^2 + x\,y - 3y^2 \quad (13)$$

> $N := -5x^2 + 2x\,y + y^2$

$$N := -5x^2 + 2x\,y + y^2 \quad (14)$$

> $diff(M, y) \neq diff(N, x)$

$$x - 6y \neq -10x + 2y \quad (15)$$

> $MM := M \cdot FactInt$

$$MM := \frac{4x^2 + x\,y - 3y^2}{(y - x)\,(y - 2x)\,(y + 2x)} \quad (16)$$

> $NN := N \cdot FactInt$

$$NN := \frac{-5x^2 + 2x\,y + y^2}{(y - x)\,(y - 2x)\,(y + 2x)} \quad (17)$$

> $simplify(diff(MM, y) - diff(NN, x)) = 0$

$$0 = 0 \quad (18)$$

> $IntMMx := int(MM, x)$

$$IntMMx := -\frac{5}{12}\ln(y + 2x) + \frac{3}{4}\ln(-y + 2x) + \frac{2}{3}\ln(-y + x) \quad (19)$$

> $SolGral := IntMMx + int((NN - diff(IntMM, y)), y) = \_C1$

$$SolGral := -\frac{5}{6}\ln(y + 2x) + \frac{3}{4}\ln(-y + 2x) + \frac{2}{3}\ln(-y + x) + \frac{2}{3}\ln(y - x) + \frac{3}{4}\ln(y - 2x) = \_C1 \quad (20)$$

> $SolGralDos := simplify(\exp(lhs(SolGral))) = \_C1$

$$SolGralDos := \frac{(-y + 2x)^{3/4}\,(-y + x)^{2/3}\,(y - x)^{2/3}\,(y - 2x)^{3/4}}{(y + 2x)^{5/6}} = \_C1 \quad (21)$$

> $SolGralTres := \frac{(-y(x)+x)^{2/3}(-y(x)+2x)^{3/4}(y(x)-2x)^{3/4}(y(x)-x)^{2/3}}{(y(x)+2x)^{5/6}} = \_C1$

$$SolGralTres := \frac{(-y(x)+x)^{2/3}(-y(x)+2x)^{3/4}(y(x)-2x)^{3/4}(y(x)-x)^{2/3}}{(y(x)+2x)^{5/6}} = \_C1 \quad \textbf{(22)}$$

> $DerSolGral := simplify(isolate(diff(SolGralTres, x), diff(y(x), x)))$

$$DerSolGral := \frac{d}{dx}\, y(x) = \frac{3\,y(x)^2 - x\,y(x) - 4\,x^2}{-5\,x^2 + 2\,x\,y(x) + y(x)^2} \quad \textbf{(23)}$$

> $DerEcua := isolate(EcuacionDiferencial, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}\, y(x) = \frac{3\,y(x)^2 - x\,y(x) - 4\,x^2}{-5\,x^2 + 2\,x\,y(x) + y(x)^2} \quad \textbf{(24)}$$

Por segundo método: coeficientes homogeneos

> $EcuacionDos := simplify(isolate(eval(subs(y(x) = x\cdot u(x), EcuacionDiferencial)), diff(u(x), x)))$

$$EcuacionDos := \frac{d}{dx}\, u(x) = -\frac{u(x)^3 - u(x)^2 - 4\,u(x) + 4}{x\left(u(x)^2 + 2\,u(x) - 5\right)} \quad \textbf{(25)}$$

> $EcuacionTres := x\cdot diff(u(x), x) = rhs(EcuacionDos)\cdot x$

$$EcuacionTres := x\left(\frac{d}{dx}\, u(x)\right) = -\frac{u(x)^3 - u(x)^2 - 4\,u(x) + 4}{u(x)^2 + 2\,u(x) - 5} \quad \textbf{(26)}$$

> $P := 1;\; Q := \frac{u^3 - u^2 - 4\,u + 4}{u^2 + 2\,u - 5}$

$$P := 1$$

$$Q := \frac{u^3 - u^2 - 4\,u + 4}{u^2 + 2\,u - 5} \quad \textbf{(27)}$$

> $R := x;\; S := 1$

$$R := x$$

$$S := 1 \quad \textbf{(28)}$$

> $SolGralCuatro := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, u\right) = \_C2$

$$SolGralCuatro := \ln(x) + \frac{3}{4}\ln(u-2) - \frac{5}{12}\ln(u+2) + \frac{2}{3}\ln(u-1) = \_C2 \quad \textbf{(29)}$$

> $SolGralCinco := subs\left(u = \frac{y}{x}, SolGralCuatro\right)$

$$SolGralCinco := \ln(x) + \frac{3}{4}\ln\left(\frac{y}{x} - 2\right) - \frac{5}{12}\ln\left(\frac{y}{x} + 2\right) + \frac{2}{3}\ln\left(\frac{y}{x} - 1\right) = \_C2 \quad \textbf{(30)}$$

> $SolGralSeis := expand(simplify(\exp(lhs(SolGralCinco)))) = \_C2$

$$SolGralSeis := \frac{x\left(\frac{y}{x} - 2\right)^{3/4}\left(\frac{y}{x} - 1\right)^{2/3}}{\left(\frac{y}{x} + 2\right)^{5/12}} = \_C2 \quad \textbf{(31)}$$

> $SolGralSiete := \frac{x\left(\frac{y(x)}{x}-2\right)^{3/4}\left(\frac{y(x)}{x}-1\right)^{2/3}}{\left(\frac{y(x)}{x}+2\right)^{5/12}} = \_C2$

$$SolGralSiete := \frac{x\left(\frac{y(x)}{x}-2\right)^{3/4}\left(\frac{y(x)}{x}-1\right)^{2/3}}{\left(\frac{y(x)}{x}+2\right)^{5/12}} = \_C2 \quad (32)$$

> $DerSolGralSiete := simplify(isolate(diff(SolGralSiete, x), diff(y(x), x)))$

$$DerSolGralSiete := \frac{d}{dx}\, y(x) = \frac{3\, y(x)^2 - x\, y(x) - 4\, x^2}{-5\, x^2 + 2\, x\, y(x) + y(x)^2} \quad (33)$$

> $DerEcua$

$$\frac{d}{dx}\, y(x) = \frac{3\, y(x)^2 - x\, y(x) - 4\, x^2}{-5\, x^2 + 2\, x\, y(x) + y(x)^2} \quad (34)$$

Fin 2

> $restart$

3) Dada la siguiente ecuación diferencial con condiciones iniciales:
a) Obtener su solución particular (**sin usar dsolve**)

> $EcuacionDiferencial := \frac{\sin(2\,x)}{y(x)} + x + \left(y(x) - \frac{\sin(x)^2}{y(x)^2}\right)\left(\frac{d}{dx}\, y(x)\right) = 0;$

$CondicionesIniciales := y(\pi) = -2$

$$EcuacionDiferencial := \frac{\sin(2\,x)}{y(x)} + x + \left(y(x) - \frac{\sin(x)^2}{y(x)^2}\right)\left(\frac{d}{dx}\, y(x)\right) = 0$$

$$CondicionesIniciales := y(\pi) = -2 \quad (35)$$

b) Graficar dicha solución particular en en el intervalo $-6 < x < 6$ & $-4 < y < 4$

>

Solución a)

> $with(DEtools):$

> $odeadvisor(EcuacionDiferencial)$

$$[\_exact] \quad (36)$$

> $M := \frac{\sin(2\,x)}{y} + x$

$$M := \frac{\sin(2\,x)}{y} + x \quad (37)$$

> $N := y - \frac{\sin(x)^2}{y^2}$

$$N := y - \frac{\sin(x)^2}{y^2} \quad (38)$$

> $simplify(diff(M, y) - diff(N, x)) = 0$

$$0 = 0 \quad (39)$$

> $IntMx := int(M, x)$

(40)

$$IntMx := -\frac{1}{2}\,\frac{\cos(2\,x)}{y}+\frac{1}{2}\,x^2 \quad \textbf{(40)}$$

> $SolGral := IntMx + int((N - diff(IntMx, y)), y) = \_C1$

$$SolGral := \frac{1}{2}\,x^2+\frac{1}{2}\,y^2+\frac{\sin(x)^2}{y} = \_C1 \quad \textbf{(41)}$$

> $Parametro := eval(subs(x = \mathrm{Pi}, y = -2, SolGral))$

$$Parametro := \frac{1}{2}\,\pi^2+2 = \_C1 \quad \textbf{(42)}$$

> $evalf(\%)$

$$6.934802202 = \_C1 \quad \textbf{(43)}$$

> $SolPart := subs(\_C1 = lhs(Parametro), SolGral)$

$$SolPart := \frac{1}{2}\,x^2+\frac{1}{2}\,y^2+\frac{\sin(x)^2}{y} = \frac{1}{2}\,\pi^2+2 \quad \textbf{(44)}$$

> $SolPartUno := \frac{1}{2}\,x^2+\frac{1}{2}\,y(x)^2+\frac{\sin(x)^2}{y(x)} = \frac{1}{2}\,\pi^2+2$

$$SolPartUno := \frac{1}{2}\,x^2+\frac{1}{2}\,y(x)^2+\frac{\sin(x)^2}{y(x)} = \frac{1}{2}\,\pi^2+2 \quad \textbf{(45)}$$

> $DerSolPart := simplify(isolate(diff(SolPartUno, x), diff(y(x), x)))$

$$DerSolPart := \frac{\mathrm{d}}{\mathrm{d}x}\,y(x) = -\frac{(2\sin(x)\cos(x)+x\,y(x))\,y(x)}{y(x)^3+\cos(x)^2-1} \quad \textbf{(46)}$$

> $DerEcua := simplify(isolate(EcuacionDiferencial, diff(y(x), x)))$

$$DerEcua := \frac{\mathrm{d}}{\mathrm{d}x}\,y(x) = \frac{(\sin(2\,x)+x\,y(x))\,y(x)}{-y(x)^3+\sin(x)^2} \quad \textbf{(47)}$$

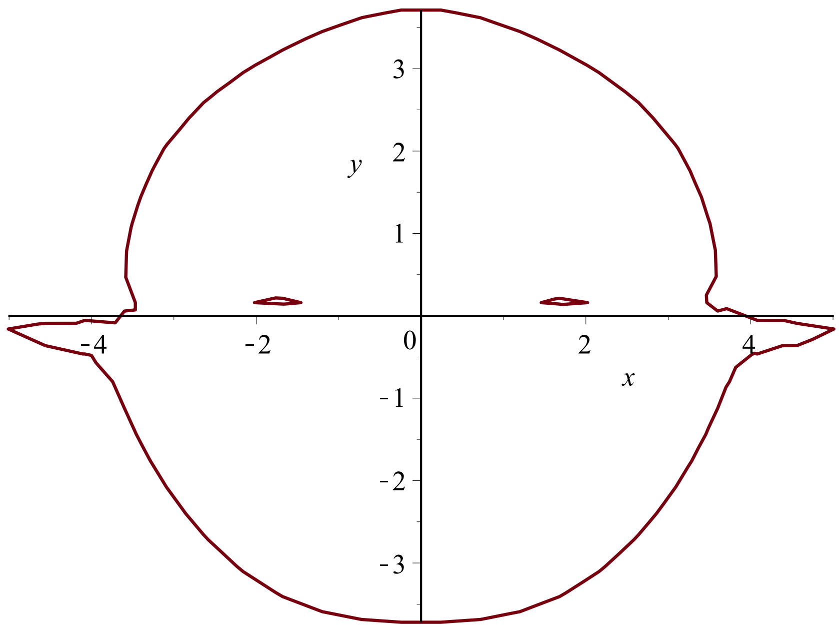
> $simplify(rhs(DerSolPart) - rhs(DerEcua)) = 0$

$$0 = 0 \quad \textbf{(48)}$$

Solucion b)

> $with(plots):$

> $implicitplot(SolPart, x = -6..6, y = -4..4, scaling = CONSTRAINED)$



Fin 3)

> *restart*

4) Obtenga la solución particular de la siguiente ecuación diferencial con la condición inicial dada - utilizando exclusivamente el método del factor integrante (**no utilizar dsolve**)

> $EcuacionDiferencial := x^4 \cdot \log(x) - 2 \cdot x \cdot y(x)^3 + \left(3 \cdot x^2 \cdot y(x)^2\right) \left(\frac{d}{dx} y(x)\right) = 0;$

$CondicionInicial := y(1) = -2$

$$EcuacionDiferencial := x^4 \ln(x) - 2\,x\,y(x)^3 + 3\,x^2\,y(x)^2 \left(\frac{d}{dx} y(x)\right) = 0$$

$$CondicionInicial := y(1) = -2 \quad \mathbf{(49)}$$

Solución 4)

> $with(DEtools):$

> $FactInt := intfactor(EcuacionDiferencial)$

$$FactInt := \frac{1}{x^4} \quad \mathbf{(50)}$$

> $M := x^4 \ln(x) - 2\,x\,y^3$

$$M := x^4 \ln(x) - 2\,x\,y^3 \quad \mathbf{(51)}$$

> $N := 3\,x^2\,y^2$

$$N := 3\,x^2\,y^2 \quad \textbf{(52)}$$

> $MM := expand(FactInt \cdot M)$

$$MM := \ln(x) - \frac{2\,y^3}{x^3} \quad \textbf{(53)}$$

> $NN := FactInt \cdot N$

$$NN := \frac{3\,y^2}{x^2} \quad \textbf{(54)}$$

> $IntMMx := int(MM, x)$

$$IntMMx := x\ln(x) - x + \frac{y^3}{x^2} \quad \textbf{(55)}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = \_C1$

$$SolGral := x\ln(x) - x + \frac{y^3}{x^2} = \_C1 \quad \textbf{(56)}$$

> $Parametro := simplify(subs(x = 1, y = -2, SolGral))$

$$Parametro := -9 = \_C1 \quad \textbf{(57)}$$

> $SolPart := subs(\_C1 = lhs(Parametro), SolGral)$

$$SolPart := x\ln(x) - x + \frac{y^3}{x^2} = -9 \quad \textbf{(58)}$$

> $restart$

5) Dada la siguiente ecuación diferencial, obtenga su solución general (**no se puede utilizar dsolve**)

> $EcuacionDiferencial := 2\,x\,(x^2 + y(x)^2)\left(\frac{d}{dx}\,y(x)\right) = y(x)\,(y(x)^2 + 2\,x^2)$

$$EcuacionDiferencial := 2\,x\,(x^2 + y(x)^2)\left(\frac{d}{dx}\,y(x)\right) = y(x)\,(y(x)^2 + 2\,x^2) \quad \textbf{(59)}$$

Solución

> $with(DEtools):$

> $odeadvisor(EcuacionDiferencial)$

$$[[\_homogeneous,\ class\ A],\ \_rational,\ \_dAlembert] \quad \textbf{(60)}$$

> $EcuacionDos := simplify(isolate(eval(subs(y(x) = x \cdot u(x), EcuacionDiferencial)), diff(u(x), x)))$

$$EcuacionDos := \frac{d}{dx}\,u(x) = -\frac{1}{2}\,\frac{u(x)^3}{x\,(u(x)^2 + 1)} \quad \textbf{(61)}$$

> $P := 1;\ Q := \frac{1}{2}\,\frac{u^3}{(u^2 + 1)};\ R := x;\ S := 1$

$$P := 1$$

$$Q := \frac{1}{2}\,\frac{u^3}{u^2 + 1}$$

$$R := x$$

$$S := 1 \quad \textbf{(62)}$$

> $SolGral := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, u\right) = \_C1$

$$SolGral := \ln(x) - \frac{1}{u^2} + 2\ln(u) = \_C1 \qquad (63)$$

> $SolGralDos := subs\left(u = \frac{y}{x}, SolGral\right)$

$$SolGralDos := \ln(x) - \frac{x^2}{y^2} + 2\ln\left(\frac{y}{x}\right) = \_C1 \qquad (64)$$

> $SolGralTres := \ln(x) - \frac{x^2}{y(x)^2} + 2\ln\left(\frac{y(x)}{x}\right) = \_C1$

$$SolGralTres := \ln(x) - \frac{x^2}{y(x)^2} + 2\ln\left(\frac{y(x)}{x}\right) = \_C1 \qquad (65)$$

> $DerSolGral := simplify(isolate(diff(SolGralTres, x), diff(y(x), x)))$

$$DerSolGral := \frac{d}{dx}\, y(x) = \frac{1}{2}\,\frac{y(x)\left(y(x)^2 + 2x^2\right)}{x\left(x^2 + y(x)^2\right)} \qquad (66)$$

> $DerEcua := isolate(EcuacionDiferencial, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}\, y(x) = \frac{1}{2}\,\frac{y(x)\left(y(x)^2 + 2x^2\right)}{x\left(x^2 + y(x)^2\right)} \qquad (67)$$

> *restart*

FIN SERIE

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