

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SERIE 2
DE EJERCICIOS DEL TEMA 2
SEMESTRE 2023-1
SOLUCIÓN

2022-09-29

[>

> restart

1) OBTENER LA SOLUCIÓN GENERAL DE LA ECUACIÓN DIFERENCIAL SIGUIENTE (sin utilizar dsolve)

$$\begin{aligned} > x \ln(x) \left(\frac{d}{dx} y(x) \right) - (1 + \ln(x)) y(x) + \frac{1}{2} \sqrt{x} (2 + \ln(x)) = 0 \\ & \quad x \ln(x) \left(\frac{d}{dx} y(x) \right) - (1 + \ln(x)) y(x) + \frac{1}{2} \sqrt{x} (2 + \ln(x)) = 0 \end{aligned} \quad (1)$$

>

SOLUCIÓN 1

$$\begin{aligned} > Ecuacion := x \ln(x) \left(\frac{d}{dx} y(x) \right) - (1 + \ln(x)) y(x) + \frac{1}{2} \sqrt{x} (2 + \ln(x)) = 0 \\ & \quad Ecuacion := x \ln(x) \left(\frac{d}{dx} y(x) \right) - (1 + \ln(x)) y(x) + \frac{1}{2} \sqrt{x} (2 + \ln(x)) = 0 \end{aligned} \quad (2)$$

$$\begin{aligned} > EcuaDos := \text{expand} \left(\frac{\text{lhs}(Ecuacion)}{x \ln(x)} \right) = 0 \\ & \quad EcuaDos := \frac{d}{dx} y(x) - \frac{y(x)}{x \ln(x)} - \frac{y(x)}{x} + \frac{1}{\sqrt{x} \ln(x)} + \frac{1}{2 \sqrt{x}} = 0 \end{aligned} \quad (3)$$

$$\begin{aligned} > EcuaTres := \text{lhs}(EcuaDos) - \left(\frac{1}{\sqrt{x} \ln(x)} + \frac{1}{2 \sqrt{x}} \right) = \text{rhs}(EcuaDos) - \left(\frac{1}{\sqrt{x} \ln(x)} + \frac{1}{2 \sqrt{x}} \right) \\ & \quad EcuaTres := \frac{d}{dx} y(x) - \frac{y(x)}{x \ln(x)} - \frac{y(x)}{x} = -\frac{1}{\sqrt{x} \ln(x)} - \frac{1}{2 \sqrt{x}} \end{aligned} \quad (4)$$

$$\begin{aligned} > p := \text{factor} \left(-\frac{1}{x \ln(x)} - \frac{1}{x} \right); q := \text{factor} \left(-\frac{1}{\sqrt{x} \ln(x)} - \frac{1}{2 \sqrt{x}} \right) \\ & \quad p := -\frac{1 + \ln(x)}{x \ln(x)} \\ & \quad q := -\frac{1}{2} \frac{2 + \ln(x)}{\sqrt{x} \ln(x)} \end{aligned} \quad (5)$$

$$\begin{aligned} > IntMasP := \text{simplify}(\exp(\text{int}(p, x))) \\ & \quad IntMasP := \frac{1}{x \ln(x)} \end{aligned} \quad (6)$$

$$\begin{aligned} > IntMenosP := \text{simplify}(\exp(-\text{int}(p, x))) \\ & \quad IntMenosP := x \ln(x) \end{aligned} \quad (7)$$

$$> SolGral := y(x) = _C1 \cdot IntMenosP + IntMenosP \cdot \text{int}(IntMasP \cdot q, x)$$

(18)

$$y\left(\frac{3}{2}\pi\right) = 9 \quad (19)$$

SOLUCIÓN 3a)

$$> \text{Ecuacion} := \frac{d^3}{dx^3} y(x) + \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) + y(x) = 0$$

$$\text{Ecuacion} := \frac{d^3}{dx^3} y(x) + \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) + y(x) = 0 \quad (20)$$

$$> \text{EcuaCarac} := m^3 + m^2 + m + 1 = 0$$

$$\text{EcuaCarac} := m^3 + m^2 + m + 1 = 0 \quad (21)$$

$$> \text{Raiz} := \text{solve}(\text{EcuaCarac})$$

$$\text{Raiz} := -1, I, -I \quad (22)$$

$$> yy[1] := \exp(\text{Raiz}[1] \cdot x); yy[2] := \cos(\text{Im}(\text{Raiz}[2]) \cdot x); yy[3] := \sin(\text{Im}(\text{Raiz}[2]) \cdot x)$$

$$yy_1 := e^{-x}$$

$$yy_2 := \cos(x)$$

$$yy_3 := \sin(x) \quad (23)$$

$$> \text{SolGral} := y(x) = _C1 \cdot yy[1] + _C2 \cdot yy[2] + _C3 \cdot yy[3]$$

$$\text{SolGral} := y(x) = _C1 e^{-x} + _C2 \cos(x) + _C3 \sin(x) \quad (24)$$

$$> \text{EcuaUno} := \text{simplify}(\text{subs}(x=0, \text{rhs}(\text{SolGral}) = -3))$$

$$\text{EcuaUno} := _C1 + _C2 = -3 \quad (25)$$

$$> \text{EcuaDos} := \text{simplify}\left(\text{subs}\left(x = \frac{\text{Pi}}{2}, \text{rhs}(\text{SolGral}) = 3\right)\right)$$

$$\text{EcuaDos} := _C1 e^{-\frac{1}{2}\pi} + _C3 = 3 \quad (26)$$

$$> \text{EcuaTres} := \text{simplify}\left(\text{subs}\left(x = \frac{3 \cdot \text{Pi}}{2}, \text{rhs}(\text{SolGral}) = 9\right)\right)$$

$$\text{EcuaTres} := _C1 e^{-\frac{3}{2}\pi} - _C3 = 9 \quad (27)$$

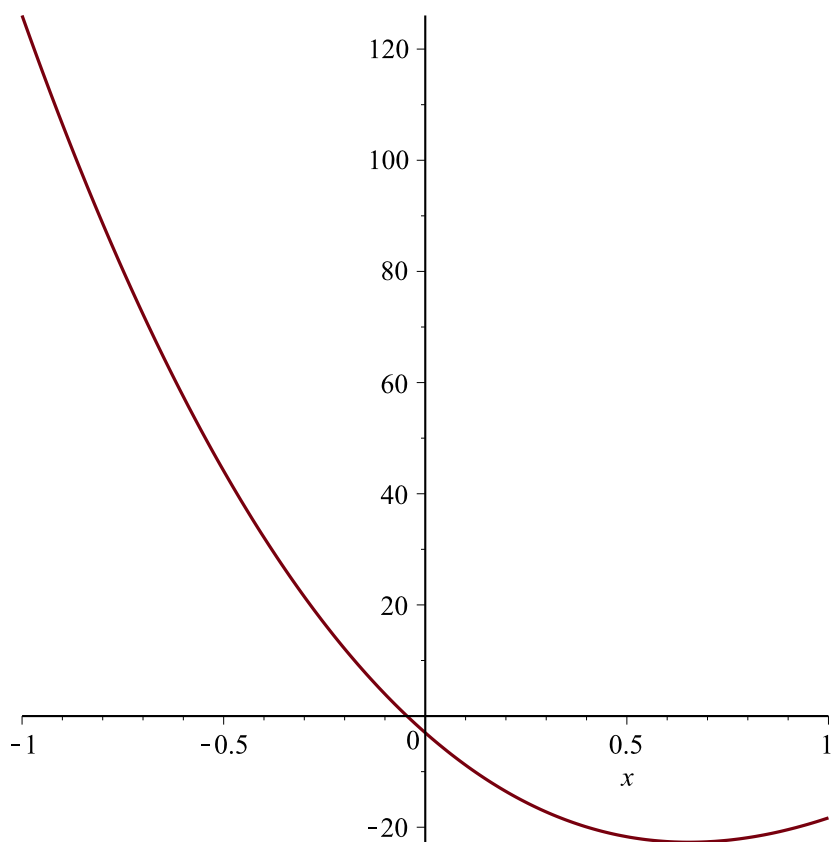
$$> \text{Para} := \text{solve}(\{\text{EcuaUno}, \text{EcuaDos}, \text{EcuaTres}\})$$

$$\text{Para} := \left\{ \begin{aligned} _C1 &= \frac{12}{e^{-\frac{3}{2}\pi} + e^{-\frac{1}{2}\pi}}, _C2 = -\frac{3 \left(e^{-\frac{3}{2}\pi} + e^{-\frac{1}{2}\pi} + 4 \right)}{e^{-\frac{3}{2}\pi} + e^{-\frac{1}{2}\pi}}, _C3 \\ &= \frac{3 \left(e^{-\frac{3}{2}\pi} - 3 e^{-\frac{1}{2}\pi} \right)}{e^{-\frac{3}{2}\pi} + e^{-\frac{1}{2}\pi}} \end{aligned} \right\} \quad (28)$$

$$> \text{SolPart} := \text{subs}(_C1 = \text{rhs}(\text{Para}[1]), _C2 = \text{rhs}(\text{Para}[2]), _C3 = \text{rhs}(\text{Para}[3]), \text{SolGral}) : \text{evalf}(\%, 3)$$

$$y(x) = 55.3 e^{-1 \cdot x} - 58.2 \cos(x) - 8.49 \sin(x) \quad (29)$$

$$> \text{plot}(\text{rhs}(\text{SolPart}), x = -1 .. 1)$$



>

> restart :

b) CON CONDICIONES INICIALES

> $\frac{d^2}{dt^2} x(t) - 7 \left(\frac{d}{dt} x(t) \right) + 12 x(t) = \cos(3 t) + t^2; x(1) = 2; D(x)(1) = -2$

$$\frac{d^2}{dt^2} x(t) - 7 \left(\frac{d}{dt} x(t) \right) + 12 x(t) = \cos(3 t) + t^2$$

$$x(1) = 2$$

$$D(x)(1) = -2$$

(30)

>

SOLUCIÓN 3b)

> $Ecua := \frac{d^2}{dt^2} x(t) - 7 \left(\frac{d}{dt} x(t) \right) + 12 x(t) = \cos(3 t) + t^2$

$$Ecua := \frac{d^2}{dt^2} x(t) - 7 \left(\frac{d}{dt} x(t) \right) + 12 x(t) = \cos(3 t) + t^2$$

(31)

> $EcuaHom := lhs(Ecua) = 0; Q := rhs(Ecua)$

$$EcuaHom := \frac{d^2}{dt^2} x(t) - 7 \left(\frac{d}{dt} x(t) \right) + 12 x(t) = 0$$

$$Q := \cos(3 t) + t^2 \quad (32)$$

$$> EcuaCarac := m^2 - 7 \cdot m + 12 = 0$$

$$EcuaCarac := m^2 - 7 m + 12 = 0 \quad (33)$$

$$> Raiz := solve(EcuaCarac)$$

$$Raiz := 4, 3 \quad (34)$$

$$> xx[1] := \exp(Raiz[1] \cdot t); xx[2] := \exp(Raiz[2] \cdot t)$$

$$xx_1 := e^{4t}$$

$$xx_2 := e^{3t} \quad (35)$$

$$> SolGralHom := x(t) = _C1 \cdot xx[1] + _C2 \cdot xx[2]$$

$$SolGralHom := x(t) = _C1 e^{4t} + _C2 e^{3t} \quad (36)$$

$$> SolGralNoHom := x(t) = AA \cdot xx[1] + BB \cdot xx[2]$$

$$SolGralNoHom := x(t) = AA e^{4t} + BB e^{3t} \quad (37)$$

> with(linalg) :

$$> WW := wronskian([xx[1], xx[2]], t)$$

$$WW := \begin{bmatrix} e^{4t} & e^{3t} \\ 4 e^{4t} & 3 e^{3t} \end{bmatrix} \quad (38)$$

$$> BB := array([0, Q])$$

$$BB := \begin{bmatrix} 0 & \cos(3 t) + t^2 \end{bmatrix} \quad (39)$$

$$> Para := simplify(linsolve(WW, BB))$$

$$Para := \begin{bmatrix} e^{-4t} (\cos(3 t) + t^2) & -e^{-3t} (\cos(3 t) + t^2) \end{bmatrix} \quad (40)$$

$$> Aprima := Para[1]; Bprima := Para[2]$$

$$Aprima := e^{-4t} (\cos(3 t) + t^2)$$

$$Bprima := -e^{-3t} (\cos(3 t) + t^2) \quad (41)$$

$$> AA := simplify(int(Aprima, t) + _C10)$$

$$AA := \frac{1}{800} (384 \cos(t)^2 \sin(t) - 512 \cos(t)^3 - 96 \sin(t) + 384 \cos(t) + 800 _C10 e^{4t} - 200 t^2 - 100 t - 25) e^{-4t} \quad (42)$$

$$> BB := simplify(int(Bprima, t) + _C20)$$

$$BB := -\frac{1}{54} (36 \cos(t)^2 \sin(t) - 36 \cos(t)^3 - 9 \sin(t) + 27 \cos(t) - 54 _C20 e^{3t} - 18 t^2 - 12 t - 4) e^{-3t} \quad (43)$$

$$> SolGralNoHom$$

$$x(t) = \frac{1}{800} (384 \cos(t)^2 \sin(t) - 512 \cos(t)^3 - 96 \sin(t) + 384 \cos(t) + 800 _C10 e^{4t} \quad (44)$$

$$-200 t^2 - 100 t - 25) e^{-4t} e^{4t} - \frac{1}{54} (36 \cos(t)^2 \sin(t) - 36 \cos(t)^3 - 9 \sin(t) \\ + 27 \cos(t) - 54 _C20 e^{3t} - 18 t^2 - 12 t - 4) e^{-3t} e^{3t}$$

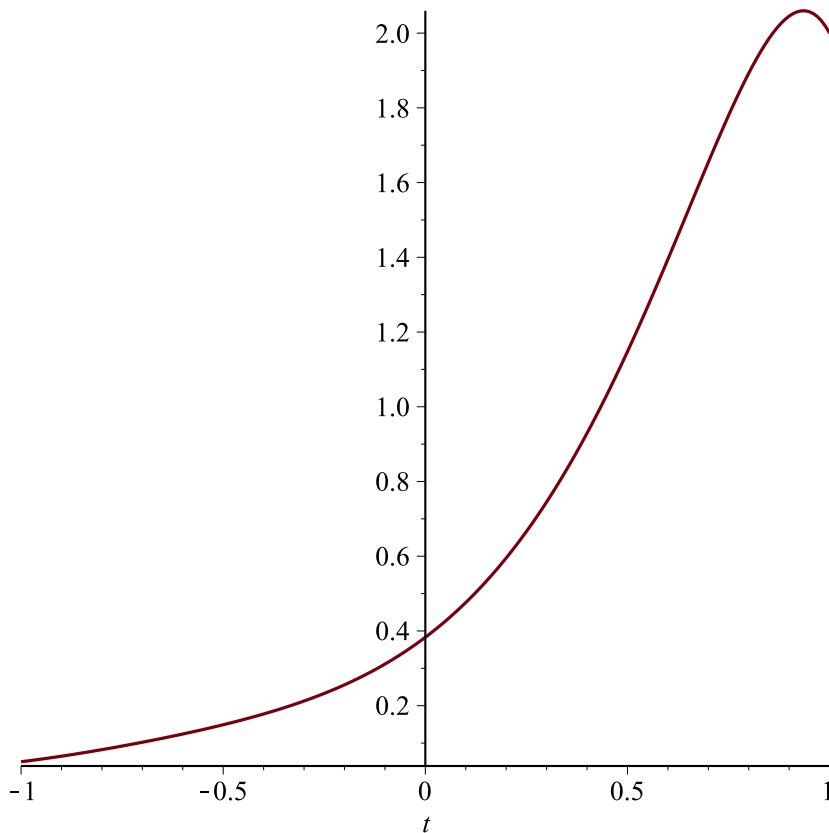
```
> EcuaUno := subs(t=1, rhs(SolGralNoHom) = 2) : evalf(%, 2)
0.24 + 53. _C10 + 21. _C20 = 2. (45)
```

```
> EcuaDos := subs(t=1, rhs(diff(SolGralNoHom, t)) = -2) : evalf(%, 2)
0.42 + 220. _C10 + 61. _C20 = -2. (46)
```

```
> Param := solve({EcuaUno, EcuaDos}) : evalf(%, 2)
{ _C10 = -0.15, _C20 = 0.52 } (47)
```

```
> SolPart := simplify(subs(_C10 = rhs(Param[1]), _C20 = rhs(Param[2]), SolGralNoHom)) :
evalf(%, 2)
x(t) = -0.0069 e^{3. + 4. t} + 0.0087 e^{4. + 3. t} - 0.18 cos(t)^2 sin(t) + 0.027 cos(t)^3 + 0.083 t^2
+ 0.046 sin(t) - 0.020 cos(t) + 0.097 t + 0.042 (48)
```

```
> plot(rhs(SolPart), t=-1..1)
```



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>
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> restart :
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c) CON CONDICIONES INICIALES

$$> \frac{d^2}{dx^2} y(x) + 2 \left(\frac{d}{dx} y(x) \right) + 2 y(x) = 3 e^{2x}; y(0) = -5; D(y)(0) = 8$$

$$\frac{d^2}{dx^2} y(x) + 2 \left(\frac{d}{dx} y(x) \right) + 2 y(x) = 3 e^{2x}$$

$$y(0) = -5$$

$$D(y)(0) = 8$$

(49)

SOLUCIÓN 3c)

$$> Ecua := \frac{d^2}{dx^2} y(x) + 2 \left(\frac{d}{dx} y(x) \right) + 2 y(x) = 3 e^{2x}$$

$$Ecua := \frac{d^2}{dx^2} y(x) + 2 \left(\frac{d}{dx} y(x) \right) + 2 y(x) = 3 e^{2x}$$

(50)

$$> EcuaHom := lhs(Ecua) = 0$$

$$EcuaHom := \frac{d^2}{dx^2} y(x) + 2 \left(\frac{d}{dx} y(x) \right) + 2 y(x) = 0$$

(51)

$$> Q := rhs(Ecua)$$

$$Q := 3 e^{2x}$$

(52)

$$> EcuaCarac := m^2 + 2 m + 2 = 0$$

$$EcuaCarac := m^2 + 2 m + 2 = 0$$

(53)

$$> Raiz := solve(EcuaCarac)$$

$$Raiz := -1 + I, -1 - I$$

(54)

$$> yy[1] := \exp(\operatorname{Re}(Raiz[1]) \cdot x) \cdot \cos(\operatorname{Im}(Raiz[1]) \cdot x); yy[2] := \exp(\operatorname{Re}(Raiz[1]) \cdot x) \cdot \sin(\operatorname{Im}(Raiz[1]) \cdot x)$$

$$yy_1 := e^{-x} \cos(x)$$

$$yy_2 := e^{-x} \sin(x)$$

(55)

$$> SolGralHom := y(x) = _C1 \cdot yy[1] + _C2 \cdot yy[2]$$

$$SolGralHom := y(x) = _C1 e^{-x} \cos(x) + _C2 e^{-x} \sin(x)$$

(56)

$$> SolGralNoHom := y(x) = AA \cdot yy[1] + BB \cdot yy[2]$$

$$SolGralNoHom := y(x) = AA e^{-x} \cos(x) + BB e^{-x} \sin(x)$$

(57)

$$> with(linalg) :$$

$$> WW := wronskian([yy[1], yy[2]], x)$$

$$WW := \begin{bmatrix} e^{-x} \cos(x) & e^{-x} \sin(x) \\ -e^{-x} \cos(x) - e^{-x} \sin(x) & -e^{-x} \sin(x) + e^{-x} \cos(x) \end{bmatrix}$$

(58)

$$> BB := array([0, Q])$$

$$BB := \begin{bmatrix} 0 & 3 e^{2x} \end{bmatrix}$$

(59)

$$\begin{aligned} &> \text{Para} := \text{simplify}(\text{linsolve}(\text{WW}, \text{BB})) \\ &\quad \text{Para} := \begin{bmatrix} -3 e^{3x} \sin(x) & 3 e^{3x} \cos(x) \end{bmatrix} \end{aligned} \quad (60)$$

$$\begin{aligned} &> \text{Aprima} := \text{Para}[1]; \text{Bprima} := \text{Para}[2] \\ &\quad \text{Aprima} := -3 e^{3x} \sin(x) \\ &\quad \text{Bprima} := 3 e^{3x} \cos(x) \end{aligned} \quad (61)$$

$$\begin{aligned} &> \text{AA} := \text{simplify}(\text{int}(\text{Aprima}, x) + _C10) : \text{evalf}(\%, 2) \\ &\quad 0.30 e^{3x} \cos(x) - 0.90 e^{3x} \sin(x) + _C10 \end{aligned} \quad (62)$$

$$\begin{aligned} &> \text{BB} := \text{simplify}(\text{int}(\text{Bprima}, x) + _C20) : \text{evalf}(\%, 2) \\ &\quad 0.90 e^{3x} \cos(x) + 0.30 e^{3x} \sin(x) + _C20 \end{aligned} \quad (63)$$

$$\begin{aligned} &> \text{SolGralNoHom} \\ &\quad y(x) = \left(\frac{3}{10} e^{3x} \cos(x) - \frac{9}{10} e^{3x} \sin(x) + _C10 \right) e^{-x} \cos(x) + \left(\frac{9}{10} e^{3x} \cos(x) \right. \\ &\quad \left. + \frac{3}{10} e^{3x} \sin(x) + _C20 \right) e^{-x} \sin(x) \end{aligned} \quad (64)$$

$$\begin{aligned} &> \text{EcuaUno} := \text{subs}(x=0, \text{rhs}(\text{SolGralNoHom}) = -5) : \text{evalf}(\%, 2) \\ &\quad 0.30 + _C10 = -5. \end{aligned} \quad (65)$$

$$\begin{aligned} &> \text{EcuaDos} := \text{subs}(x=0, \text{rhs}(\text{diff}(\text{SolGralNoHom}, x)) = 8) : \text{evalf}(\%, 2) \\ &\quad 0.60 - 1. _C10 + _C20 = 8. \end{aligned} \quad (66)$$

$$\begin{aligned} &> \text{Param} := \text{solve}(\{\text{EcuaUno}, \text{EcuaDos}\}) : \text{evalf}(\%, 2) \\ &\quad \{ _C10 = -5.3, _C20 = 2.1 \} \end{aligned} \quad (67)$$

$$\begin{aligned} &> \text{SolPart} := \text{simplify}(\text{subs}(_C10 = \text{rhs}(\text{Param}[1]), _C20 = \text{rhs}(\text{Param}[2]), \text{SolGralNoHom})) : \\ &\quad \text{evalf}(\%, 2) \\ &\quad y(x) = 0.10 e^{-1x} (3. e^{3x} - 53. \cos(x) + 21. \sin(x)) \end{aligned} \quad (68)$$

$$> \text{plot}(\text{rhs}(\text{SolPart}), x = -1 .. 1)$$


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>
[=
SOLUCION 4a)
> Ecua :=  $\frac{d^4}{dt^4} y(t) + 5 \left( \frac{d^2}{dt^2} y(t) \right) - 4 y(t) = 5 e^{-3t} \cos(2t)$ 
      Ecua :=  $\frac{d^4}{dt^4} y(t) + 5 \left( \frac{d^2}{dt^2} y(t) \right) - 4 y(t) = 5 e^{-3t} \cos(2t)$  (70)
[=
> EcuaHom := lhs(Ecua) = 0
      EcuaHom :=  $\frac{d^4}{dt^4} y(t) + 5 \left( \frac{d^2}{dt^2} y(t) \right) - 4 y(t) = 0$  (71)
[=
> Q := rhs(Ecua)
      Q :=  $5 e^{-3t} \cos(2t)$  (72)
[=
> EcuaCarac :=  $m^4 + 5 \cdot m^2 - 4 = 0$ 
      EcuaCarac :=  $m^4 + 5 m^2 - 4 = 0$  (73)
[=
> Raiz := solve(EcuaCarac) : evalf(%, 2)
      2.4 I, -2.4 I, 0.85, -0.85 (74)
[=
> yy[1] := cos(Im(Raiz[1])·t) : evalf(%, 2); yy[2] := sin(Im(Raiz[1])·t) : evalf(%, 2); yy[3]
  := exp(Raiz[3]·t) : evalf(%, 2); yy[4] := exp(Raiz[4]·t) : evalf(%, 2)
      cos(2.4 t)
      sin(2.4 t)
      e0.85 t
      e-0.85 t (75)
[=
> SolGralHom := y(t) = _C1·yy[1] + _C2·yy[2] + _C3·yy[3] + _C4·yy[4] : evalf(%, 2)
      y(t) = _C1 cos(2.4 t) + _C2 sin(2.4 t) + _C3 e0.85 t + _C4 e-0.85 t (76)
[=
> SolGralNoHom := y(t) = AA·yy[1] + BB·yy[2] + DD·yy[3] + EE·yy[4] : evalf(%, 2)
      y(t) = AA cos(2.4 t) + BB sin(2.4 t) + DD e0.85 t + EE e-0.85 t (77)
[=
> with(linalg) :
> WW := wronskian([yy[1], yy[2], yy[3], yy[4]], t) : evalf(%, 2)
      
$$\begin{bmatrix} \cos(2.4 t) & \sin(2.4 t) & e^{0.85 t} & e^{-0.85 t} \\ -2.4 \sin(2.4 t) & 2.4 \cos(2.4 t) & 0.85 e^{0.85 t} & -0.85 e^{-0.85 t} \\ -5.8 \cos(2.4 t) & -5.8 \sin(2.4 t) & 0.75 e^{0.85 t} & 0.75 e^{-0.85 t} \\ 13. \sin(2.4 t) & -13. \cos(2.4 t) & 0.59 e^{0.85 t} & -0.59 e^{-0.85 t} \end{bmatrix}$$
 (78)
[=
> BB := array([0, 0, 0, Q])
      BB :=  $\begin{bmatrix} 0 & 0 & 0 & 5 e^{-3t} \cos(2t) \end{bmatrix}$  (79)
[=
> Para := simplify(linsolve(WW, BB)) : evalf(%, 2)
       $[0.31 \sin(2.4 t) e^{-3 \cdot t} \cos(2. t), -0.31 \cos(2.4 t) e^{-3 \cdot t} \cos(2. t), 0.46 e^{-3.8 t} \cos(2. t),$ 
       $-0.46 e^{-2.2 t} \cos(2. t)]$  (80)

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$$\begin{aligned}
& \text{Aprima} := \text{Para}[1] : \text{evalf}(\%, 2); \text{Bprima} := \text{Para}[2] : \text{evalf}(\%, 2); \text{Dprima} := \text{Para}[3] : \\
& \quad \text{evalf}(\%, 2); \text{Eprima} := \text{Para}[4] : \text{evalf}(\%, 2) \\
& \quad 0.31 \sin(2.4 t) e^{-3.t} \cos(2. t) \\
& \quad -0.31 \cos(2.4 t) e^{-3.t} \cos(2. t) \\
& \quad 0.46 e^{-3.8 t} \cos(2. t) \\
& \quad -0.46 e^{-2.2 t} \cos(2. t)
\end{aligned} \tag{81}$$

$$\begin{aligned}
& \text{AA} := \text{simplify}(\text{int}(\text{Aprima}, t) + _C10) : \text{evalf}(\%, 2); \text{BB} := \text{simplify}(\text{int}(\text{Bprima}, t) + _C20) : \\
& \quad \text{evalf}(\%, 2); \text{DD} := \text{simplify}(\text{int}(\text{Dprima}, t) + _C30) : \text{evalf}(\%, 2); \text{EE} \\
& \quad := \text{simplify}(\text{int}(\text{Eprima}, t) + _C40) : \text{evalf}(\%, 2) \\
& -0.0069 \cos(0.40 t) e^{-3.t} - 0.025 \cos(4.4 t) e^{-3.t} - 0.017 \sin(4.4 t) e^{-3.t} \\
& \quad - 0.054 \sin(0.40 t) e^{-3.t} + 1.0 _C10 \\
& -0.0072 \sin(0.40 t) e^{-3.t} - 0.026 \sin(4.4 t) e^{-3.t} + 1.0 _C20 + 0.054 \cos(0.40 t) e^{-3.t} \\
& \quad + 0.018 \cos(4.4 t) e^{-3.t} \\
& \quad -0.091 e^{-3.8 t} \cos(2. t) + 0.050 e^{-3.8 t} \sin(2. t) + 1.0 _C30 \\
& \quad 0.12 e^{-2.2 t} \cos(2. t) - 0.11 e^{-2.2 t} \sin(2. t) + 1.2 _C40
\end{aligned} \tag{82}$$

$$\begin{aligned}
& \text{SolGralNoHom} : \text{evalf}(\%, 2) \\
& y(t) = -0.0000049 (1400. \cos(0.40 t) e^{-3.t} + 5100. \cos(4.4 t) e^{-3.t} + 3500. \sin(4.4 t) e^{-3.t} \\
& \quad + 11000. \sin(0.40 t) e^{-3.t} - 2.1 10^5 _C10) \cos(2.4 t) + 0.0000050 (\\
& \quad -1400. \sin(0.40 t) e^{-3.t} - 5100. \sin(4.4 t) e^{-3.t} + 2.1 10^5 _C20 + 11000. \cos(0.40 t) e^{-3.t} \\
& \quad + 3500. \cos(4.4 t) e^{-3.t}) \sin(2.4 t) - 0.00038 (240. e^{-3.8 t} \cos(2. t) - 130. e^{-3.8 t} \sin(2. t) \\
& \quad - 2600. _C30) e^{0.85 t} - 0.00084 (-140. e^{-2.2 t} \cos(2. t) + 130. e^{-2.2 t} \sin(2. t) \\
& \quad - 1400. _C40) e^{-0.85 t}
\end{aligned} \tag{83}$$

$$\begin{aligned}
& \text{EcuaUno} := \text{subs}(t=0, \text{rhs}(\text{SolGralNoHom})) = -2 : \text{evalf}(\%, 2) \\
& \quad 1.0 _C10 + 1.0 _C30 + 1.2 _C40 = -2.
\end{aligned} \tag{84}$$

$$\begin{aligned}
& \text{EcuaDos} := \text{subs}(t=0, \text{rhs}(\text{diff}(\text{SolGralNoHom}, t))) = 0 : \text{evalf}(\%, 2) \\
& \quad -0.01 + 2.5 _C20 + 0.84 _C30 - 1.0 _C40 = 0.
\end{aligned} \tag{85}$$

$$\begin{aligned}
& \text{EcuaTres} := \text{subs}(t=0, \text{rhs}(\text{diff}(\text{SolGralNoHom}, t\$2))) = 7 : \text{evalf}(\%, 2) \\
& \quad 0.28 - 5.8 _C10 + 0.73 _C30 + 0.85 _C40 = 7.
\end{aligned} \tag{86}$$

$$\begin{aligned}
& \text{EcuaCuatro} := \text{subs}(t=0, \text{rhs}(\text{diff}(\text{SolGralNoHom}, t\$3))) = -5 : \text{evalf}(\%, 2) \\
& \quad -1.5 - 15. _C20 + 0.63 _C30 - 0.75 _C40 = -5.
\end{aligned} \tag{87}$$

$$\begin{aligned}
& \text{Param} := \text{solve}(\{\text{EcuaUno}, \text{EcuaDos}, \text{EcuaTres}, \text{EcuaCuatro}\}) : \text{evalf}(\%, 2) \\
& \quad \{ _C10 = -1.3, _C20 = 0.27, _C30 = -1.1, _C40 = -0.28 \}
\end{aligned} \tag{88}$$

$$\begin{aligned}
& \text{SolPart} := \text{simplify}(\text{subs}(_C10 = \text{rhs}(\text{Param}[1]), _C20 = \text{rhs}(\text{Param}[2]), _C30 \\
& \quad = \text{rhs}(\text{Param}[3]), _C40 = \text{rhs}(\text{Param}[4]), \text{SolGralNoHom})) : \text{evalf}(\%, 2) \\
& y(t) = -0.020 \cos(0.40 t) e^{-3.t} \cos(2.4 t) - 0.020 e^{-3.t} \cos(4.4 t) \cos(2.4 t) \\
& \quad - 0.054 e^{-3.t} \sin(4.4 t) \cos(2.4 t) - 0.054 e^{-3.t} \sin(0.40 t) \cos(2.4 t) \\
& \quad - 0.015 e^{-3.t} \sin(4.4 t) \sin(2.4 t) - 0.020 e^{-3.t} \sin(0.40 t) \sin(2.4 t) \\
& \quad + 0.049 \cos(0.40 t) e^{-3.t} \sin(2.4 t) + 0.059 e^{-3.t} \cos(4.4 t) \sin(2.4 t) - 0.49 e^{0.85 t} \\
& \quad + 0.49 e^{-0.85 t} + 0.24 \sin(2.4 t) - 1.5 \cos(2.4 t) - 0.049 e^{-3.t} \sin(2. t)
\end{aligned} \tag{89}$$

[illegible]

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(92)

OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN NO HOMOGÉNEA UTILIZANDO EXCLUSIVAMENTE EL MÉTODO DE VARIACIÓN DE PARÁMETROS (sin utilizar dsolve)

$$> -2 y(x) + \left(\frac{d^2}{dx^2} y(x) \right) x^2 + 2 x \left(\frac{d}{dx} y(x) \right) = 32 x^2$$

$$-2 y(x) + \left(\frac{d^2}{dx^2} y(x) \right) x^2 + 2 x \left(\frac{d}{dx} y(x) \right) = 32 x^2 \quad (93)$$

>

SOLUCIÓN 5

$$> SolGral := y(x) = \frac{C_1}{x^2} + C_2 x$$

$$SolGral := y(x) = \frac{C_1}{x^2} + C_2 x \quad (94)$$

$$> EcuaHom := -2 y(x) + \left(\frac{d^2}{dx^2} y(x) \right) x^2 + 2 x \left(\frac{d}{dx} y(x) \right) = 0$$

$$EcuaHom := -2 y(x) + \left(\frac{d^2}{dx^2} y(x) \right) x^2 + 2 x \left(\frac{d}{dx} y(x) \right) = 0 \quad (95)$$

$$> EcuaNoHom := -2 y(x) + \left(\frac{d^2}{dx^2} y(x) \right) x^2 + 2 x \left(\frac{d}{dx} y(x) \right) = 32 x^2$$

$$EcuaNoHom := -2 y(x) + \left(\frac{d^2}{dx^2} y(x) \right) x^2 + 2 x \left(\frac{d}{dx} y(x) \right) = 32 x^2 \quad (96)$$

$$> EcuaHomStandard := expand\left(\frac{lhs(EcuaHom)}{x^2}\right) = 0$$

$$EcuaHomStandard := -\frac{2 y(x)}{x^2} + \frac{d^2}{dx^2} y(x) + \frac{2 \left(\frac{d}{dx} y(x) \right)}{x} = 0 \quad (97)$$

$$> EcuaNoHomStandard := expand\left(\frac{lhs(EcuaNoHom)}{x^2}\right) = expand\left(\frac{rhs(EcuaNoHom)}{x^2}\right)$$

$$EcuaNoHomStandard := -\frac{2 y(x)}{x^2} + \frac{d^2}{dx^2} y(x) + \frac{2 \left(\frac{d}{dx} y(x) \right)}{x} = 32 \quad (98)$$

$$> Q := rhs(EcuaNoHomStandard)$$

$$Q := 32 \quad (99)$$

$$> ComprobarUno := simplify(eval(subs(y(x) = rhs(SolGral), EcuaHomStandard)))$$

$$ComprobarUno := 0 = 0 \quad (100)$$

$$> SolNoHom := y(x) = \frac{AA}{x^2} + BB \cdot x$$

$$SolNoHom := y(x) = \frac{AA}{x^2} + BB x \quad (101)$$

$$\triangleright yy[1] := \frac{1}{x^2}; yy[2] := x$$

$$yy_1 := \frac{1}{x^2}$$

$$yy_2 := x$$

(102)

> with(linalg) :

```
> WW := wronskian([yy[1], yy[2]], x)
```

$$WW:=\begin{bmatrix} \frac{1}{x^2} & x \\ -\frac{2}{x^3} & 1 \end{bmatrix}$$

(103)

```
> BB := array([0, Q])
```

$$BB := \begin{bmatrix} 0 & 32 \end{bmatrix}$$

(104)

```
> Para := simplify( linsolve( WW, BB) )
```

$$Para := \left[\begin{array}{cc} -\frac{32}{3}x^3 & \frac{32}{3} \end{array} \right]$$

(105)

```
> Aprima := Para[1]; Bprima := Para[2]
```

$$Aprima := -\frac{32}{3} x^3$$

$$B_{prima} := \frac{32}{3}$$

(106)

```
> AA := simplify(int(Aprima, x) + _C1)
```

$$AA := -\frac{8}{3}x^4 + _CI$$

(107)

```
> BB := simplify(int(Bprima, x) + _C2)
```

$$BB := \frac{32}{3} x + _C2$$

(108)

```
> SolFinal := expand(SolNoHom)
```

$$SolFinal := y(x) = 8 x^2 + \frac{C1}{x^2} + x_C2$$

(109)

```
> comprobarDos := simplify(eval(subs(y(x) = rhs(SolFinal), lhs(EcuaNoHom)
    - rhs(EcuaNoHom) = 0)))
```

$$comprobarDos := 0 = 0$$

(110)

>

[illegible]

>

FINAL SERIE 2

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