> restart

DEPARTAMENTO DE MATEMATICAS APLICADAS
1325_24-2_1FA_672
ECUACIONES DIFERENCIALES
PRIMER EXAMEN FINAL COLEGIADO

> restart

1)

> $Ecua := (x^2 + y^2 + x) + (x \cdot y) \cdot y' = 0$

$$Ecua := x^2 + y(x)^2 + x + x\,y(x)\left(\frac{d}{dx}\,y(x)\right) = 0 \quad (1)$$

RESULTADO

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[\_rational, \_Bernoulli] \quad (2)$$

> $FactInt := intfactor(Ecua)$

$$FactInt := x \quad (3)$$

> $EcuaExacta := expand(FactInt \cdot lhs(Ecua) = 0)$

$$EcuaExacta := x^3 + x\,y(x)^2 + x^2 + x^2\,y(x)\left(\frac{d}{dx}\,y(x)\right) = 0 \quad (4)$$

> $odeadvisor(EcuaExacta)$

$$[\_exact, \_rational, \_Bernoulli] \quad (5)$$

> $M := x^3 + x\,y^2 + x^2$

$$M := x^3 + y^2\,x + x^2 \quad (6)$$

> $N := x^2\,y$

$$N := y\,x^2 \quad (7)$$

> $IntMx := int(M, x)$

$$IntMx := \frac{1}{4}\,x^4 + \frac{1}{2}\,x^2\,y^2 + \frac{1}{3}\,x^3 \quad (8)$$

> $SolGral := IntMx + int((N - diff(IntMx, y)), y) = \_C1$

$$SolGral := \frac{1}{4}\,x^4 + \frac{1}{2}\,x^2\,y^2 + \frac{1}{3}\,x^3 = \_C1 \quad (9)$$

> $SolFinal := \frac{1}{4}\,x^4 + \frac{1}{2}\,x^2\,y(x)^2 + \frac{1}{3}\,x^3 = \_C1$

$$SolFinal := \frac{x^4}{4} + \frac{x^2\,y(x)^2}{2} + \frac{x^3}{3} = \_C1 \quad (10)$$

> $DerSolFinal := simplify(isolate(diff(SolFinal, x), diff(y(x), x)))$

$$DerSolFinal := \frac{d}{dx}\,y(x) = \frac{-x^2 - y(x)^2 - x}{x\,y(x)} \quad (11)$$

> $DerEcua := isolate(Ecua, diff(y(x), x))$

(12)

$$DerEcua := \frac{d}{dx}\, y(x) = \frac{-x^2 - y(x)^2 - x}{x\, y(x)} \quad \textbf{(12)}$$

> $Comprobar := rhs(DerEcua) - rhs(DerSolFinal) = 0$

$$Comprobar := 0 = 0 \quad \textbf{(13)}$$

> $restart$

2)

> $Ecua := y'' - 2\, y' + y = x^{(-2)} \cdot \exp(x)$

$$Ecua := \frac{d^2}{dx^2}\, y(x) - 2\, \frac{d}{dx}\, y(x) + y(x) = \frac{e^x}{x^2} \quad \textbf{(14)}$$

RESULTADO

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\, y(x) - 2\, \frac{d}{dx}\, y(x) + y(x) = 0 \quad \textbf{(15)}$$

> $Q := rhs(Ecua)$

$$Q := \frac{e^x}{x^2} \quad \textbf{(16)}$$

> $EcuaAlg := m^2 - 2 \cdot m + 1 = 0$

$$EcuaAlg := m^2 - 2\, m + 1 = 0 \quad \textbf{(17)}$$

> $Raiz := solve(EcuaAlg)$

$$Raiz := 1, 1 \quad \textbf{(18)}$$

Caso II

> $yy[1] := \exp(Raiz[1] \cdot x);\, yy[2] := x \cdot \exp(Raiz[1] \cdot x)$

$$yy_1 := e^x$$

$$yy_2 := x\, e^x \quad \textbf{(19)}$$

> $SolGralHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolGralHom := y(x) = \_C1\, e^x + \_C2\, x\, e^x \quad \textbf{(20)}$$

> $SolGralNoHom := y(x) = AA \cdot yy[1] + BB \cdot yy[2]$

$$SolGralNoHom := y(x) = AA\, e^x + BB\, x\, e^x \quad \textbf{(21)}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^x & x\, e^x \\ e^x & e^x + x\, e^x \end{bmatrix} \quad \textbf{(22)}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \frac{e^x}{x^2} \end{bmatrix} \quad \textbf{(23)}$$

> $Para := simplify(linsolve(WW, BB))$

(24)

$$Para := \begin{bmatrix} -\frac{1}{x} & \frac{1}{x^2} \end{bmatrix} \quad (24)$$

> $Aprima := Para[1]; Bprima := Para[2]$

$$Aprima := -\frac{1}{x}$$

$$Bprima := \frac{1}{x^2} \quad (25)$$

> $AA := int(Aprima, x) + \_C1; BB := int(Bprima, x) + \_C2$

$$AA := -\ln(x) + \_C1$$

$$BB := -\frac{1}{x} + \_C2 \quad (26)$$

> $SolFinal := expand(SolGralNoHom)$

$$SolFinal := y(x) = \_C2\, x\, e^x - e^x \ln(x) + \_C1\, e^x - e^x \quad (27)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolFinal), Ecua)))$

$$Comprobar := \frac{e^x}{x^2} = \frac{e^x}{x^2} \quad (28)$$

> $restart$

3)

> $Ecua := diff(y(t), t) + y(t) = \exp(2\,t) \cdot \mathrm{Heaviside}(t-2)$

$$Ecua := \frac{d}{dt}\, y(t) + y(t) = e^{2\,t}\, \mathrm{Heaviside}(t-2) \quad (29)$$

> $CondIni := y(0) = 0$

$$CondIni := y(0) = 0 \quad (30)$$

RESULTADO

> $with(inttrans):$

> $EcuaTL := subs(CondIni, laplace(Ecua, t, s))$

$$EcuaTL := s\, \mathcal{L}(y(t), t, s) + \mathcal{L}(y(t), t, s) = \frac{e^{4-2\,s}}{s-2} \quad (31)$$

> $SolTL := isolate(EcuaTL, laplace(y(t), t, s))$

$$SolTL := \mathcal{L}(y(t), t, s) = \frac{e^{4-2\,s}}{(s-2)\,(s+1)} \quad (32)$$

> $SolPart := invlaplace(SolTL, s, t)$

$$SolPart := y(t) = \frac{\mathrm{Heaviside}(t-2)\,\left(-e^{6-t} + e^{2\,t}\right)}{3} \quad (33)$$

> $Comprobar := simplify(eval(subs(y(t) = rhs(SolPart), Ecua)))$

$$Comprobar := e^{2\,t}\, \mathrm{Heaviside}(t-2) = e^{2\,t}\, \mathrm{Heaviside}(t-2) \quad (34)$$

> $ComprobarDos := simplify(subs(t = 0, SolPart))$

$$ComprobarDos := y(0) = 0 \quad (35)$$

> $restart$

4)

> $Sistema := diff(x(t), t) = -3 \cdot x(t) + \mathrm{Dirac}(t-1), diff(y(t), t) = -2 \cdot x(t) - 3 \cdot y(t) + 5 \cdot \mathrm{Dirac}(t) : Sistema[1]; Sistema[2]$

$$\frac{d}{dt} x(t) = -3\, x(t) + \mathrm{Dirac}(t-1)$$

$$\frac{d}{dt} y(t) = -2\, x(t) - 3\, y(t) + 5\, \mathrm{Dirac}(t) \qquad \textbf{(36)}$$

> $CondIni := x(0) = 1, y(0) = -2$

$$CondIni := x(0) = 1, y(0) = -2 \qquad \textbf{(37)}$$

RESPUESTA

> $with(inttrans) :$

> $SistTLUno := subs(CondIni, laplace(Sistema[1], t, s))$

$$SistTLUno := s\, \mathcal{L}(x(t), t, s) - 1 = -3\, \mathcal{L}(x(t), t, s) + e^{-s} \qquad \textbf{(38)}$$

> $SolTLUno := isolate(SistTLUno, laplace(x(t), t, s))$

$$SolTLUno := \mathcal{L}(x(t), t, s) = \frac{1 + e^{-s}}{s+3} \qquad \textbf{(39)}$$

> $SolUno := invlaplace(SolTLUno, s, t)$

$$SolUno := x(t) = e^{-3\,t} + \mathrm{Heaviside}(t-1)\, e^{-3\,t+3} \qquad \textbf{(40)}$$

> $SistDos := subs(x(t) = rhs(SolUno), Sistema[2])$

$$SistDos := \frac{d}{dt} y(t) = -2\, e^{-3\,t} - 2\, \mathrm{Heaviside}(t-1)\, e^{-3\,t+3} - 3\, y(t) + 5\, \mathrm{Dirac}(t) \qquad \textbf{(41)}$$

> $SistTLDos := subs(CondIni, laplace(SistDos, t, s))$

$$SistTLDos := s\, \mathcal{L}(y(t), t, s) + 2 = \frac{13 - 2\, e^{-s} + 5\, s}{s+3} - 3\, \mathcal{L}(y(t), t, s) \qquad \textbf{(42)}$$

> $SolTLDos := simplify(isolate(SistTLDos, laplace(y(t), t, s)))$

$$SolTLDos := \mathcal{L}(y(t), t, s) = \frac{3\, s + 7 - 2\, e^{-s}}{(s+3)^2} \qquad \textbf{(43)}$$

> $SolDos := invlaplace(SolTLDos, s, t)$

$$SolDos := y(t) = -2\, \mathrm{Heaviside}(t-1)\, (t-1)\, e^{-3\,t+3} - e^{-3\,t}\, (2\,t-3) \qquad \textbf{(44)}$$

> $SolUno; SolDos$

$$x(t) = e^{-3\,t} + \mathrm{Heaviside}(t-1)\, e^{-3\,t+3}$$

$$y(t) = -2\, \mathrm{Heaviside}(t-1)\, (t-1)\, e^{-3\,t+3} - e^{-3\,t}\, (2\,t-3) \qquad \textbf{(45)}$$

>

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5)

> $Ecua := diff(u(x, t), x\$2) - u(x, t) = diff(u(x, t), t)$

$$Ecua := \frac{\partial^2}{\partial x^2} u(x, t) - u(x, t) = \frac{\partial}{\partial t} u(x, t) \qquad \textbf{(46)}$$

RESPUESTA

> $EcuaSeparable := eval(subs(u(x, t) = F(x) \cdot G(t), Ecua))$

$$EcuaSeparable := \left(\frac{d^2}{dx^2} F(x)\right) G(t) - F(x)\, G(t) = F(x) \left(\frac{d}{dt} G(t)\right) \quad \textbf{(47)}$$

> $EcuaSep := simplify\left(\frac{lhs(EcuaSeparable)}{F(x) \cdot G(t)}\right) = \frac{rhs(EcuaSeparable)}{F(x) \cdot G(t)}$

$$EcuaSep := \frac{-F(x) + \frac{d^2}{dx^2} F(x)}{F(x)} = \frac{\frac{d}{dt} G(t)}{G(t)} \quad \textbf{(48)}$$

> $EcuaX := lhs(EcuaSep) = 0$

$$EcuaX := \frac{-F(x) + \frac{d^2}{dx^2} F(x)}{F(x)} = 0 \quad \textbf{(49)}$$

> $EcuaT := rhs(EcuaSep) = 0$

$$EcuaT := \frac{\frac{d}{dt} G(t)}{G(t)} = 0 \quad \textbf{(50)}$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = c_1 e^{-x} + c_2 e^{x} \quad \textbf{(51)}$$

> $SolT := dsolve(EcuaT)$

$$SolT := G(t) = c_1 \quad \textbf{(52)}$$

> $SolFinal := uu(x, t) = rhs(SolX) \cdot rhs(subs(c_1 = 1, SolT))$

$$SolFinal := uu(x, t) = c_1 e^{-x} + c_2 e^{x} \quad \textbf{(53)}$$

> *restart*

FIN EXAMEN

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