> restart

UNAM
FACULTAD DE INGENIERÍA
DIVISIÓN DE CIENCIAS BÁSICAS
ECUACIONES DIFERENCIALES
GRUPO 11 SEMESTRE 2024-2
SEGUNDOEXAMEN PARCIAL Temas 3 & 4
SOLUCIÓN

2024-05-23

> restart

1)

> $AA := array([[5,-1],[3,1]])$

$$AA := \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} \quad (1)$$

> $BB := array([0,t])$

$$BB := \begin{bmatrix} 0 & t \end{bmatrix} \quad (2)$$

> $Xcero := array([2,1])$

$$Xcero := \begin{bmatrix} 2 & 1 \end{bmatrix} \quad (3)$$

Respuesta

> $with(linalg):$

> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} -\frac{e^{2t}}{2} + \frac{3\,e^{4t}}{2} & -\frac{e^{4t}}{2} + \frac{e^{2t}}{2} \\ \frac{3\,e^{4t}}{2} - \frac{3\,e^{2t}}{2} & \frac{3\,e^{2t}}{2} - \frac{e^{4t}}{2} \end{bmatrix} \quad (4)$$

> $SolHom := evalm(MatExp \&* Xcero) : x[1](t) = SolHom[1]; x[2](t) = SolHom[2]$

$$x_1(t) = -\frac{e^{2t}}{2} + \frac{5\,e^{4t}}{2}$$

$$x_2(t) = \frac{5\,e^{4t}}{2} - \frac{3\,e^{2t}}{2} \quad (5)$$

> $MatExpTau := map(rcurry(eval, t = 't - tau'), MatExp)$

$$MatExpTau := \begin{bmatrix} -\frac{e^{2t-2\tau}}{2} + \frac{3\,e^{4t-4\tau}}{2} & -\frac{e^{4t-4\tau}}{2} + \frac{e^{2t-2\tau}}{2} \\ \frac{3\,e^{4t-4\tau}}{2} - \frac{3\,e^{2t-2\tau}}{2} & \frac{3\,e^{2t-2\tau}}{2} - \frac{e^{4t-4\tau}}{2} \end{bmatrix} \quad (6)$$

> $BBtau := map(rcurry(eval, t = 'tau'), BB)$

$$BBtau := \begin{bmatrix} 0 & \tau \end{bmatrix} \quad (7)$$

> $ProdTau := evalm(MatExpTau \&* BBtau):$

> $SolNoHom := map(int, ProdTau, \mathrm{tau}=0..t) : x[1](t) = SolNoHom[1]; x[2](t) = SolNoHom[2];$

$$x_1(t) = -\frac{3}{32} - \frac{e^{4t}}{32} + \frac{e^{2t}}{8} - \frac{t}{8}$$

$$x_2(t) = -\frac{11}{32} - \frac{e^{4t}}{32} + \frac{3\,e^{2t}}{8} - \frac{5\,t}{8} \quad (8)$$

> $SolFinal := evalm(SolHom + SolNoHom) : x[1](t) = SolFinal[1]; x[2](t) = SolFinal[2];$

$$x_1(t) = -\frac{3\,e^{2t}}{8} + \frac{79\,e^{4t}}{32} - \frac{3}{32} - \frac{t}{8}$$

$$x_2(t) = \frac{79\,e^{4t}}{32} - \frac{9\,e^{2t}}{8} - \frac{11}{32} - \frac{5\,t}{8} \quad (9)$$

> $ComprobarUno := x[1](0) = eval(subs(t=0, SolFinal[1]))$

$$ComprobarUno := x_1(0) = 2 \quad (10)$$

> $ComprobarDos := x[1](0) = eval(subs(t=0, SolFinal[2]))$

$$ComprobarDos := x_1(0) = 1 \quad (11)$$

> $Sistema := diff(x[1](t), t) = 5\cdot x[1](t) - x[2](t), diff(x[2](t), t) = 3\cdot x[1](t) + x[2](t) + t : Sistema[1]; Sistema[2]$

$$\frac{d}{dt}\, x_1(t) = 5\, x_1(t) - x_2(t)$$

$$\frac{d}{dt}\, x_2(t) = 3\, x_1(t) + x_2(t) + t \quad (12)$$

> $ComprobarTres := simplify(eval(subs(x[1](t) = SolFinal[1], x[2](t) = SolFinal[2], lhs(Sistema[1]) - rhs(Sistema[1]) = 0)))$

$$ComprobarTres := 0 = 0 \quad (13)$$

> $ComprobarTres := simplify(eval(subs(x[1](t) = SolFinal[1], x[2](t) = SolFinal[2], lhs(Sistema[2]) - rhs(Sistema[2]) = 0)))$

$$ComprobarTres := 0 = 0 \quad (14)$$

> $restart$

2)

> $Sistema := diff(x(t), t\$2) + diff(y(t), t\$2) = \mathrm{Heaviside}(t), diff(x(t), t\$2) - diff(y(t), t\$2) = 4\cdot t : Sistema[1]; Sistema[2]$

$$\frac{d^2}{dt^2}\, x(t) + \frac{d^2}{dt^2}\, y(t) = \mathrm{Heaviside}(t)$$

$$\frac{d^2}{dt^2}\, x(t) - \frac{d^2}{dt^2}\, y(t) = 4\, t \quad (15)$$

> $CondIni := x(0) = 0, D(x)(0) = 0, y(0) = 0, D(y)(0) = 0$

$$CondIni := x(0) = 0, D(x)(0) = 0, y(0) = 0, D(y)(0) = 0 \quad (16)$$

RESPUESTA

> $with(inttrans):$

> $TLSistema := subs(CondIni, laplace(Sistema[1], t, s)), subs(CondIni, laplace(Sistema[2], t, s)) : TLSistema[1]; TLSistema[2]$

$$s^2 \mathcal{L}(x(t), t, s) + s^2 \mathcal{L}(y(t), t, s) = \frac{1}{s}$$

$$s^2 \mathcal{L}(x(t), t, s) - s^2 \mathcal{L}(y(t), t, s) = \frac{4}{s^2} \quad \textbf{(17)}$$

> $TLSol := solve([TLSistema], [laplace(x(t), t, s), laplace(y(t), t, s)])$

$$TLSol := \left[\left[\mathcal{L}(x(t), t, s) = \frac{s+4}{2\, s^4}, \mathcal{L}(y(t), t, s) = \frac{s-4}{2\, s^4}\right]\right] \quad \textbf{(18)}$$

> $Sol[1] := invlaplace(TLSol[1, 1], s, t)$

$$Sol_1 := x(t) = \frac{t^2\, (4\, t+3)}{12} \quad \textbf{(19)}$$

> $Sol[2] := invlaplace(TLSol[1, 2], s, t)$

$$Sol_2 := y(t) = -\frac{t^2\, (4\, t-3)}{12} \quad \textbf{(20)}$$

> $ComprobarUno := simplify(eval(subs(x(t) = rhs(Sol[1]), y(t) = rhs(Sol[2]), Sistema[1])))$

$$ComprobarUno := 1 = \mathrm{Heaviside}(t) \quad \textbf{(21)}$$

> $ComprobarDos := simplify(eval(subs(x(t) = rhs(Sol[1]), y(t) = rhs(Sol[2]), Sistema[2])))$

$$ComprobarDos := 4\, t = 4\, t \quad \textbf{(22)}$$

> $restart$

3)

> $Ecua := diff(y(t), t\$2) + 4 \cdot y(t) = \mathrm{Dirac}(t - 2 \cdot \mathrm{Pi})$

$$Ecua := \frac{d^2}{dt^2}\, y(t) + 4\, y(t) = \mathrm{Dirac}(t - 2\, \pi) \quad \textbf{(23)}$$

> $CondIni := y(0) = 0, \mathrm{D}(y)(0) = 0$

$$CondIni := y(0) = 0, \mathrm{D}(y)(0) = 0 \quad \textbf{(24)}$$

RESPUESTA

> $with(inttrans) :$

> $EcuaTL := subs(CondIni, laplace(Ecua, t, s))$

$$EcuaTL := s^2 \mathcal{L}(y(t), t, s) + 4\, \mathcal{L}(y(t), t, s) = \mathrm{e}^{-2\, s\, \pi} \quad \textbf{(25)}$$

> $SolTL := isolate(EcuaTL, laplace(y(t), t, s))$

$$SolTL := \mathcal{L}(y(t), t, s) = \frac{\mathrm{e}^{-2\, s\, \pi}}{s^2 + 4} \quad \textbf{(26)}$$

> $Sol := invlaplace(SolTL, s, t)$

$$Sol := y(t) = \frac{\mathrm{Heaviside}(t - 2\, \pi)\, \sin(2\, t)}{2} \quad \textbf{(27)}$$

> $Comprobar := simplify(eval(subs(y(t) = rhs(Sol), Ecua)))$

$$Comprobar := \mathrm{Dirac}(t - 2\, \pi) = \mathrm{Dirac}(t - 2\, \pi) \quad \textbf{(28)}$$

> $restart$

4)

> $Ecua := diff(x(y,z), y\$2, z) + 2\cdot diff(x(y,z), y, z\$2) = 0$

$$Ecua := \frac{\partial^3}{\partial y^2 \partial z} x(y,z) + 2 \frac{\partial^3}{\partial y \partial z^2} x(y,z) = 0 \quad \textbf{(29)}$$

> $EcuaSeparable := eval(subs(x(y,z) = F(y)\cdot G(z), Ecua))$

$$EcuaSeparable := \left(\frac{d^2}{dy^2} F(y)\right)\left(\frac{d}{dz} G(z)\right) + 2\left(\frac{d}{dy} F(y)\right)\left(\frac{d^2}{dz^2} G(z)\right) = 0 \quad \textbf{(30)}$$

> $EcuaSeparada := \frac{\left(lhs(EcuaSeparable) - 2\left(\frac{d}{dy} F(y)\right)\left(\frac{d^2}{dz^2} G(z)\right)\right)}{2\cdot\left(\frac{d}{dz} G(z)\right)\cdot\left(\frac{d}{dy} F(y)\right)} = \frac{\left(rhs(EcuaSeparable) - 2\left(\frac{d}{dy} F(y)\right)\left(\frac{d^2}{dz^2} G(z)\right)\right)}{2\cdot\left(\frac{d}{dz} G(z)\right)\cdot\left(\frac{d}{dy} F(y)\right)}$

$$EcuaSeparada := \frac{\frac{d^2}{dy^2} F(y)}{2\left(\frac{d}{dy} F(y)\right)} = -\frac{\frac{d^2}{dz^2} G(z)}{\frac{d}{dz} G(z)} \quad \textbf{(31)}$$

> $EcuaY := lhs(EcuaSeparada) = -3$

$$EcuaY := \frac{\frac{d^2}{dy^2} F(y)}{2\left(\frac{d}{dy} F(y)\right)} = -3 \quad \textbf{(32)}$$

> $EcuaZ := rhs(EcuaSeparada) = -3$

$$EcuaZ := -\frac{\frac{d^2}{dz^2} G(z)}{\frac{d}{dz} G(z)} = -3 \quad \textbf{(33)}$$

> $SolY := dsolve(EcuaY)$

$$SolY := F(y) = c_1 + c_2 e^{-6y} \quad \textbf{(34)}$$

> $SolZ := dsolve(EcuaZ)$

$$SolZ := G(z) = c_1 + c_2 e^{3z} \quad \textbf{(35)}$$

> $SolFinal := x(y,z) = rhs(SolY)\cdot subs(c_1 = c_3, c_2 = c_4, rhs(SolZ))$

$$SolFinal := x(y,z) = \left(c_1 + c_2 e^{-6y}\right)\left(c_3 + c_4 e^{3z}\right) \quad \textbf{(36)}$$

> $Comprobar := simplify(eval(subs(x(y,z) = rhs(SolFinal), Ecua)))$

$$Comprobar := 0 = 0 \quad \textbf{(37)}$$

> $restart$

5) Obtener la serie coseno

> $f := \text{Pi} - x$

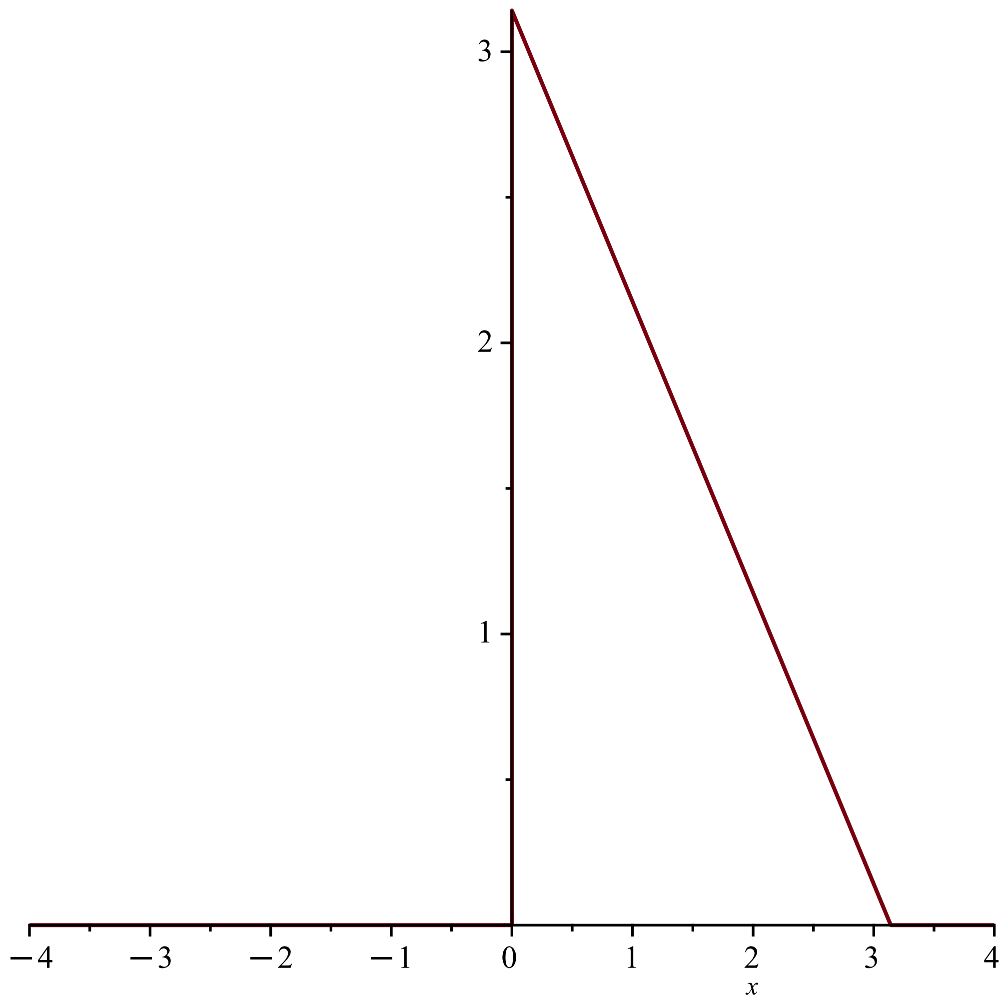
$$f := \pi - x \tag{38}$$

> $L := 4$

$$L := 4 \tag{39}$$

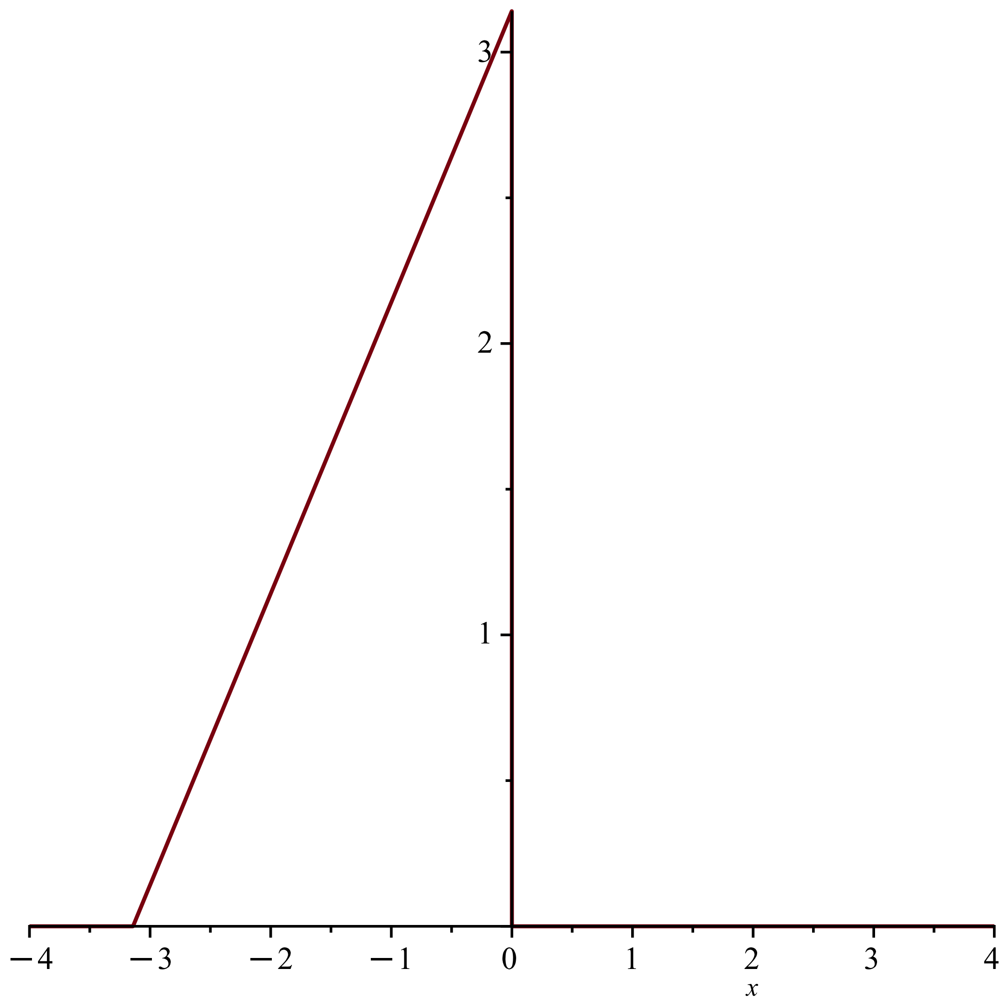
> $g := \text{Heaviside}(x) \cdot (\text{Pi} - x) - \text{Heaviside}(x - \text{Pi}) \cdot (\text{Pi} - x);\ plot(g, x = -4 ..4)$

$$g := \text{Heaviside}(x)\ (\pi - x) - \text{Heaviside}(x - \pi)\ (\pi - x)$$

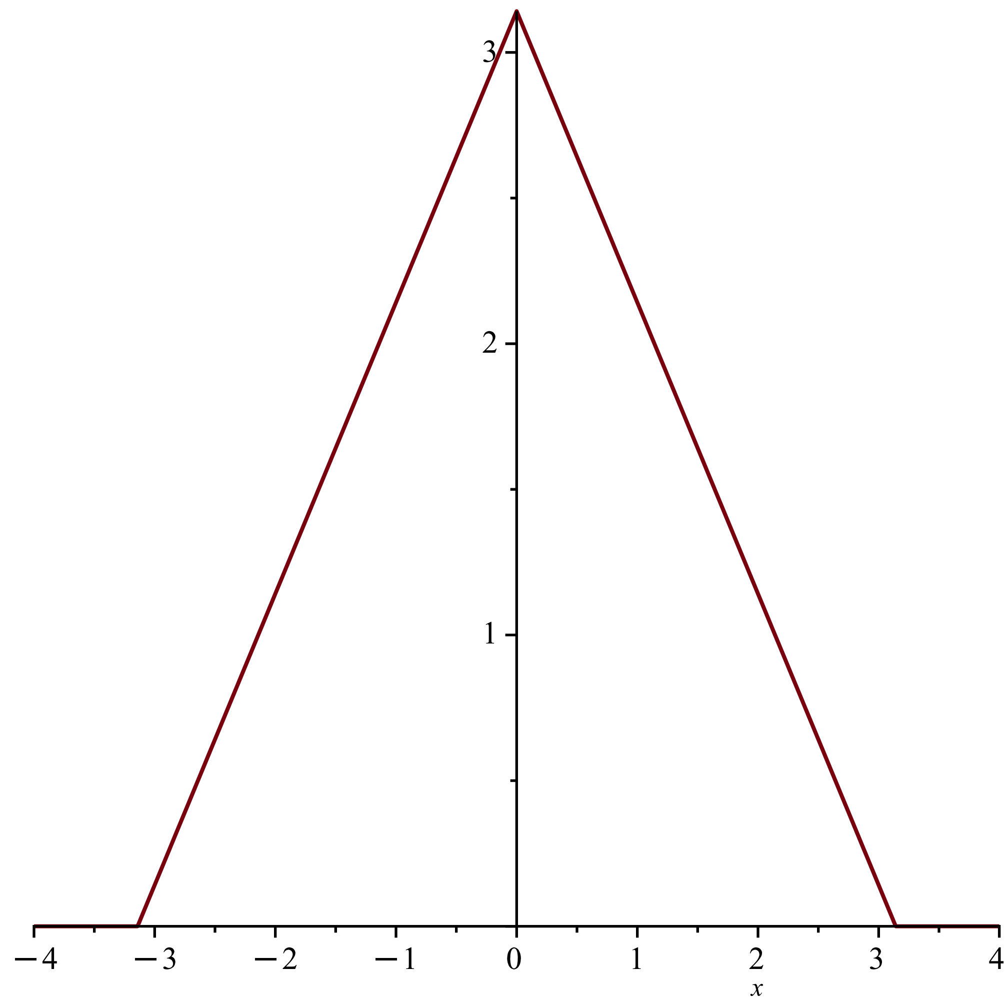


> $h := \text{Heaviside}(\text{Pi} + x) \cdot (\text{Pi} + x) - \text{Pi} \cdot \text{Heaviside}(x) - \text{Heaviside}(x) \cdot x;\ plot(h, x = -4 ..4)$

$$h := \text{Heaviside}(\pi + x)\ (\pi + x) - \pi\,\text{Heaviside}(x) - \text{Heaviside}(x)\ x$$



> $j := g + h : plot(j, x = -4..4)$



> $with(inttrans):$

> $b[n] := simplify\left(\frac{1}{L} \cdot int\left(j \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), x = -L..L\right)\right)$

$$b_n := 0 \tag{40}$$

> $a[0] := \frac{1}{L} \cdot int(j, x = -L..L)$

$$a_0 := \frac{\pi^2}{4} \tag{41}$$

> $a[n] := simplify\left(\frac{1}{L} \cdot int\left(j \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), x = -L..L\right)\right)$

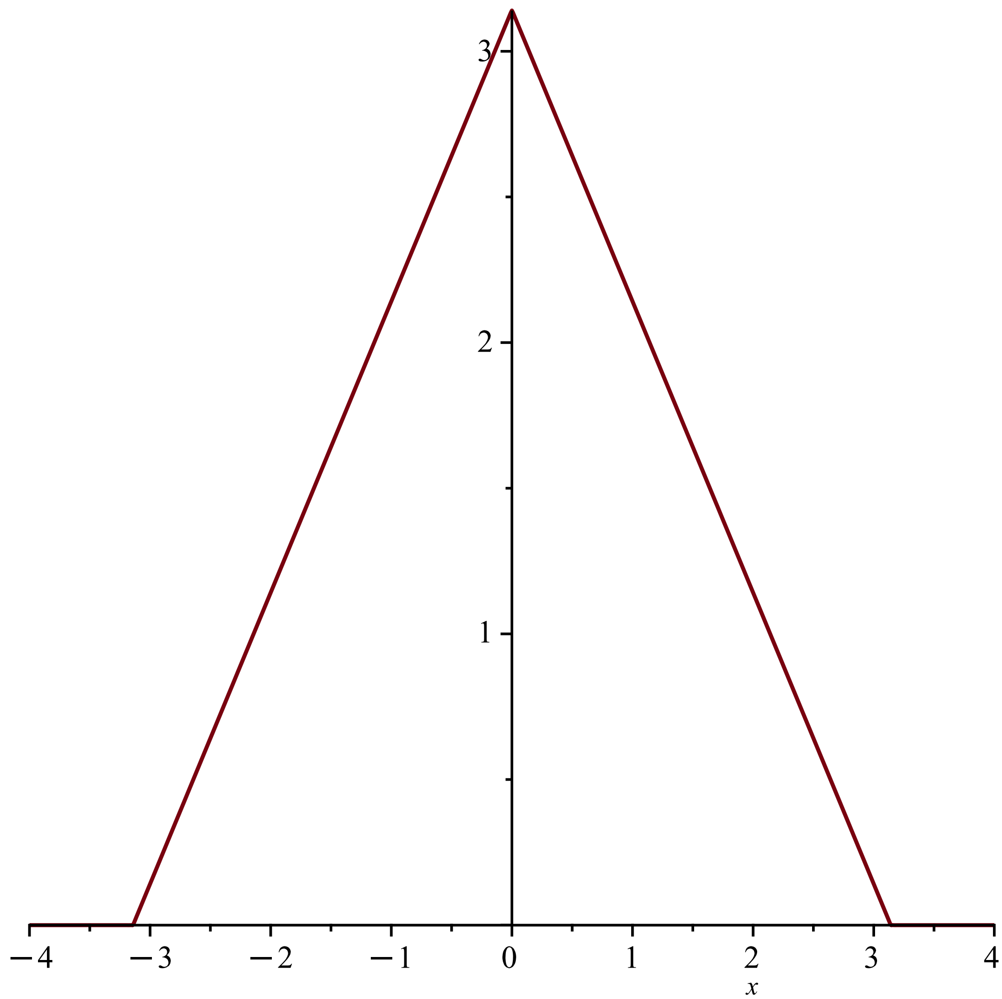
$$a_n := \frac{-8 \cos\left(\frac{n\,\pi^2}{4}\right) + 8}{n^2\,\pi^2} \tag{42}$$

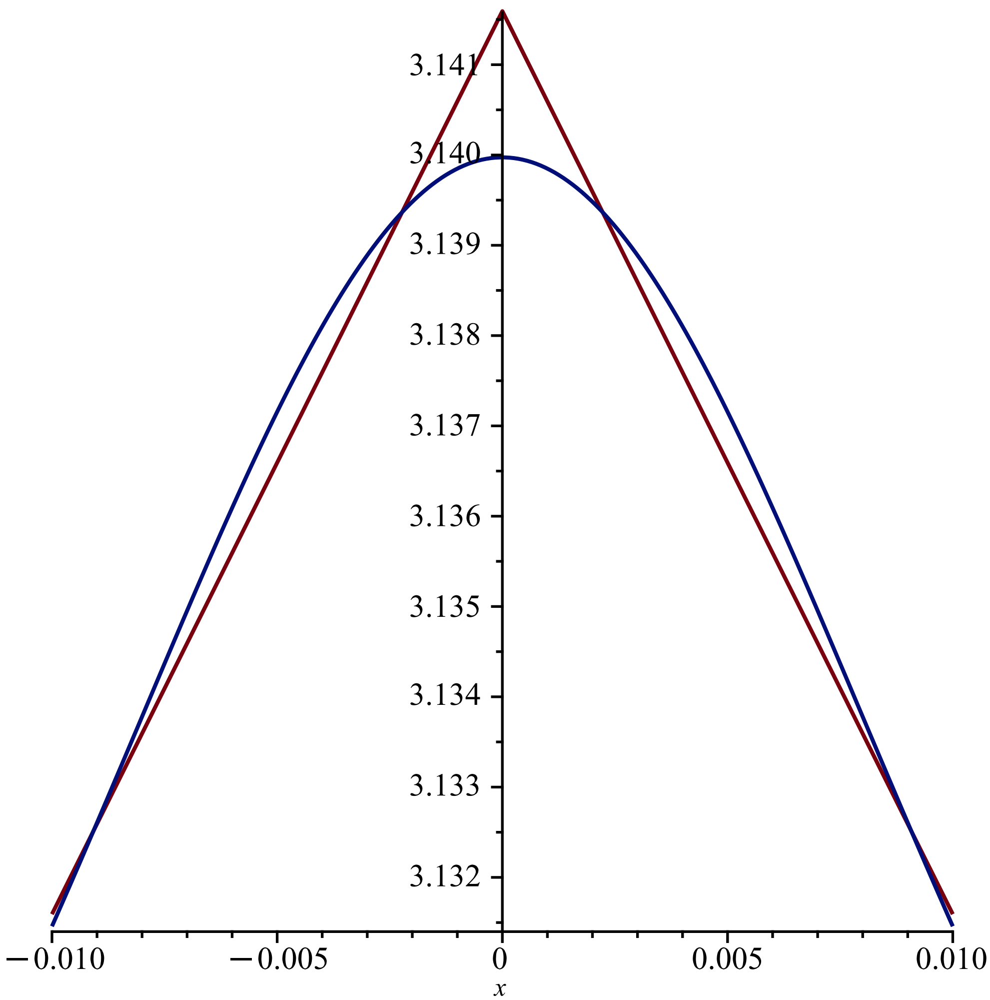
> $STFcos := \frac{a[0]}{2} + Sum\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), n = 1..\mathrm{infinity}\right)$

(43)

$$STFcos := \frac{\pi^2}{8} + \left(\sum_{n=1}^{\infty} \frac{\left(-8\cos\left(\frac{n\,\pi^2}{4}\right) + 8\right)\cos\left(\frac{n\,\pi\,x}{4}\right)}{n^2\,\pi^2} \right) \tag{43}$$

> $STF500 := \frac{a[0]}{2} + sum\left(a[n]\cdot\cos\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot x\right), n=1..500\right) : plot(STF500, x=-L..L);$
> $plot(\{j, STF500\}, x=-0.01..0.01); plot(\{j, STF500\}, x=3.13..3.16)$




3.141
3.140
3.139
3.138
3.137
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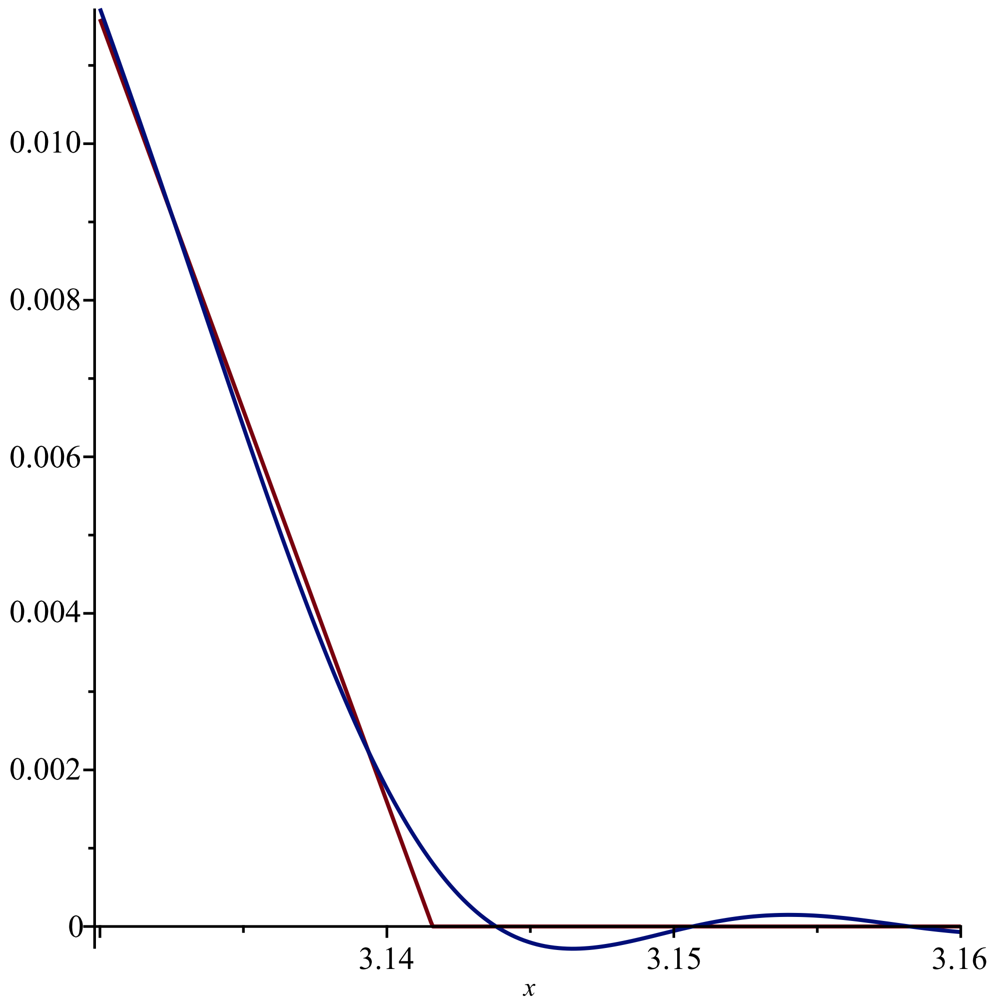
3.135
3.134
3.133
3.132
−0.010
−0.005
0
0.005
0.010
x



0.010
0.008
0.006
0.004
0.002
0
3.14
3.15
3.16
x

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