

**FACULTAD DE INGENIERÍA  
DIVISIÓN DE CIENCIAS BÁSICAS  
COORDINACIÓN DE CIENCIAS APLICADAS  
MATEMÁTICAS APLICADAS  
ECUACIONES DIFERENCIALES  
SEMESTRE 2025 – 1 GRUPO 13  
PRIMER EXAMEN PARCIAL  
TIPO "A"  
SOLUCIÓN**

> restart

1)

> Ecua := exp(y) · (1 + x<sup>2</sup>) · y' - 2 · x · (1 + exp(y)) = 0

$$Ecua := e^{y(x)} (x^2 + 1) \left( \frac{d}{dx} y(x) \right) - 2x (1 + e^{y(x)}) = 0 \quad (1)$$

> CondicionInicial := y(1) = 0

$$CondicionInicial := y(1) = 0 \quad (2)$$

RESPUESTA

> with(DEtools) :

> odeadvisor(Ecua)

$$[_separable] \quad (3)$$

LA ECUACIÓN ES DE VARIABLES SEPARABLES

> M := - 2 x (1 + e<sup>y</sup>)

$$M := -2x (1 + e^y) \quad (4)$$

> N := e<sup>y</sup> (x<sup>2</sup> + 1)

$$N := e^y (x^2 + 1) \quad (5)$$

> P := -2 · x; Q := (1 + e<sup>y</sup>); R := (x<sup>2</sup> + 1); S := e<sup>y</sup>

$$P := -2x$$

$$Q := 1 + e^y$$

$$R := x^2 + 1$$

$$S := e^y$$

(6)

> SolGral := int(  $\frac{P}{R}, x$  ) + int(  $\frac{S}{Q}, y$  ) = \_C1

$$SolGral := -\ln(x^2 + 1) + \ln(1 + e^y) = _C1 \quad (7)$$

> SolGralDos := simplify(exp(lhs(SolGral))) = \_C1

$$SolGralDos := \frac{1 + e^y}{x^2 + 1} = _C1 \quad (8)$$

> Para := simplify(subs(x = 1, y = 0, SolGralDos))

$$Para := 1 = _C1$$

(9)

> SolPart := subs(\_C1 = lhs(Para), SolGralDos)

$$SolPart := \frac{1 + e^y}{x^2 + 1} = 1 \quad (10)$$

$$> SolPartFinal := \frac{1 + e^{y(x)}}{x^2 + 1} = 1$$

$$SolPartFinal := \frac{1 + e^{y(x)}}{x^2 + 1} = 1 \quad (11)$$

$$> DerEcua := isolate(Ecua, diff(y(x), x))$$

$$DerEcua := \frac{d}{dx} y(x) = \frac{2x(1 + e^{y(x)})}{e^{y(x)}(x^2 + 1)} \quad (12)$$

$$> DerSolPart := isolate(diff(SolPartFinal, x), diff(y(x), x))$$

$$DerSolPart := \frac{d}{dx} y(x) = \frac{2x(1 + e^{y(x)})}{e^{y(x)}(x^2 + 1)} \quad (13)$$

$$> Comprobar := simplify(rhs(DerEcua) - rhs(DerSolPart)) = 0$$

$$Comprobar := 0 = 0 \quad (14)$$

> restart

(2)

$$> Ecua := 4 \cdot x^2 + x \cdot y - 3 \cdot y^2 + y' \cdot (-5 \cdot x^2 + 2 \cdot x \cdot y + y^2) = 0$$

$$Ecua := 4x^2 + xy(x) - 3y(x)^2 + \left(\frac{d}{dx} y(x)\right)(-5x^2 + 2xy(x) + y(x)^2) = 0 \quad (15)$$

RESPUESTA

> with(DEtools):

$$> odeadvisor(Ecua) \quad [[_homogeneous, class A], _rational, _dAlembert] \quad (16)$$

LA ECUACIÓN ES DE COEFICIENTES HOMOGÉNEOS

$$> EcuaDos := simplify(isolate(eval(subs(y(x) = u(x) \cdot x, Ecua)), diff(u(x), x)))$$

$$EcuaDos := \frac{d}{dx} u(x) = \frac{-u(x)^3 + u(x)^2 + 4u(x) - 4}{(-5 + 2u(x) + u(x)^2)x} \quad (17)$$

$$> odeadvisor(EcuaDos)$$

$$[_separable] \quad (18)$$

$$> EcuaTres := lhs(EcuaDos) \cdot x = rhs(EcuaDos) \cdot x$$

$$EcuaTres := \left(\frac{d}{dx} u(x)\right)x = \frac{-u(x)^3 + u(x)^2 + 4u(x) - 4}{-5 + 2u(x) + u(x)^2} \quad (19)$$

$$> M := -\left(\frac{-u^3 + u^2 + 4u - 4}{-5 + 2u + u^2}\right)$$

$$M := -\frac{-u^3 + u^2 + 4u - 4}{u^2 + 2u - 5} \quad (20)$$

$$> N := x$$

$$N := x \quad (21)$$

>  $P := 1; Q := M; R := N; S := 1$

$$\begin{aligned} P &:= 1 \\ Q &:= -\frac{-u^3 + u^2 + 4u - 4}{u^2 + 2u - 5} \\ R &:= x \\ S &:= 1 \end{aligned} \quad (22)$$

>  $SolGralUno := \int \left( \frac{P}{R}, x \right) + \int \left( \frac{S}{Q}, u \right) = \_CI$

$$SolGralUno := \ln(x) + \frac{3 \ln(u-2)}{4} - \frac{5 \ln(u+2)}{12} + \frac{2 \ln(u-1)}{3} = \_CI \quad (23)$$

>  $SolGralDos := \text{simplify}(\exp(\text{lhs}(SolGralUno))) = \_CI$

$$SolGralDos := \frac{x(u-2)^{3/4}(u-1)^{2/3}}{(u+2)^{5/12}} = \_CI \quad (24)$$

>  $SolGralTres := \text{simplify}\left(\text{subs}\left(u = \frac{y(x)}{x}, SolGralDos\right)\right)$

$$SolGralTres := \frac{x \left( \frac{y(x)-2x}{x} \right)^{3/4} \left( \frac{y(x)-x}{x} \right)^{2/3}}{\left( \frac{y(x)+2x}{x} \right)^{5/12}} = \_CI \quad (25)$$

>  $DerSolGral := \text{simplify}(\text{isolate}(\text{diff}(SolGralTres, x), \text{diff}(y(x), x)))$

$$DerSolGral := \frac{d}{dx} y(x) = \frac{-4x^2 - xy(x) + 3y(x)^2}{-5x^2 + 2xy(x) + y(x)^2} \quad (26)$$

>  $DerEcua := \text{isolate}(Ecua, \text{diff}(y(x), x))$

$$DerEcua := \frac{d}{dx} y(x) = \frac{-4x^2 - xy(x) + 3y(x)^2}{-5x^2 + 2xy(x) + y(x)^2} \quad (27)$$

>  $Comprobar := \text{simplify}(\text{rhs}(DerEcua) - \text{rhs}(DerSolGral)) = 0$

$$Comprobar := 0 = 0 \quad (28)$$

> restart

3)

>  $Ecua := (3 \cdot x^2 + 6 \cdot x \cdot y^2) + (6 \cdot x^2 \cdot y + 4 \cdot y^3) \cdot y' = 0$

$$Ecua := 3x^2 + 6xy(x)^2 + (6x^2y(x) + 4y(x)^3) \left( \frac{d}{dx} y(x) \right) = 0 \quad (29)$$

>  $CondIni := y(2) = 4$

$$CondIni := y(2) = 4 \quad (30)$$

RESPUESTA

> with(DEtools):

> odeadvisor(Ecua)

$$[_{exact}, _{rational}] \quad (31)$$

>  $M := 3x^2 + 6xy^2$

(32)

$$M := 6 x y^2 + 3 x^2 \quad (32)$$

$$> N := 6 x^2 y + 4 y^3$$

$$N := 6 x^2 y + 4 y^3 \quad (33)$$

$$> \text{diff}(M, y) = \text{diff}(N, x)$$

$$12 x y = 12 x y \quad (34)$$

ES EXACTA: SOLUCIÓN UNO

$$> \text{IntMx} := \text{int}(M, x)$$

$$\text{IntMx} := 3 x^2 y^2 + x^3 \quad (35)$$

$$> \text{SolGral} := \text{IntMx} + \text{int}((N - \text{diff}(\text{IntMx}, y)), y) = \_C1$$

$$\text{SolGral} := 3 x^2 y^2 + y^4 + x^3 = \_C1 \quad (36)$$

$$> \text{Para} := \text{subs}(x=2, y=4, \text{SolGral})$$

$$\text{Para} := 456 = \_C1 \quad (37)$$

$$> \text{SolPartUno} := \text{subs}(\_C1 = \text{lhs}(\text{Para}), \text{SolGral})$$

$$\text{SolPartUno} := 3 x^2 y^2 + y^4 + x^3 = 456 \quad (38)$$

$$> \text{SolPartFinal} := 3 x^2 y(x)^2 + y(x)^4 + x^3 = 456$$

$$\text{SolPartFinal} := 3 x^2 y(x)^2 + y(x)^4 + x^3 = 456 \quad (39)$$

$$> \text{DerSolPart} := \text{isolate}(\text{diff}(\text{SolPartFinal}, x), \text{diff}(y(x), x))$$

$$\text{DerSolPart} := \frac{d}{dx} y(x) = \frac{-3 x^2 - 6 x y(x)^2}{6 x^2 y(x) + 4 y(x)^3} \quad (40)$$

$$> \text{DerEcua} := \text{isolate}(\text{Ecua}, \text{diff}(y(x), x))$$

$$\text{DerEcua} := \frac{d}{dx} y(x) = \frac{-3 x^2 - 6 x y(x)^2}{6 x^2 y(x) + 4 y(x)^3} \quad (41)$$

$$> \text{ComprobarUno} := \text{simplify}(\text{rhs}(\text{DerEcua}) - \text{rhs}(\text{DerSolPart})) = 0$$

$$\text{ComprobarUno} := 0 = 0 \quad (42)$$

$$> \text{ComprobarDos} := \text{subs}(x=2, y=4, \text{SolPart})$$

$$\text{ComprobarDos} := \text{SolPart} \quad (43)$$

SOLUCIÓN DOS

$$> \text{IntNy} := \text{int}(N, y)$$

$$\text{IntNy} := \frac{(3 x^2 + 2 y^2)^2}{4} \quad (44)$$

$$> \text{SolGral} := \text{IntNy} + \text{int}((M - \text{diff}(\text{IntNy}, x)), x) = \_C1$$

$$\text{SolGral} := \frac{(3 x^2 + 2 y^2)^2}{4} - \frac{9 x^4}{4} + x^3 = \_C1 \quad (45)$$

$$> \text{Para} := \text{subs}(x=2, y=4, \text{SolGral})$$

$$\text{Para} := 456 = \_C1 \quad (46)$$

$$> \text{SolPartDos} := \text{subs}(\_C1 = \text{lhs}(\text{Para}), \text{SolGral})$$

$$\text{SolPartDos} := \frac{(3 x^2 + 2 y^2)^2}{4} - \frac{9 x^4}{4} + x^3 = 456 \quad (47)$$

$$\begin{aligned} &> \text{SolPartTres} := \text{expand}(\text{SolPartDos}) \\ &\qquad\qquad\qquad \text{SolPartTres} := 3x^2y^2 + y^4 + x^3 = 456 \end{aligned} \quad (48)$$

$$\begin{aligned} &> \text{SolPartUno} \\ &\qquad\qquad\qquad 3x^2y^2 + y^4 + x^3 = 456 \end{aligned} \quad (49)$$

$\text{restart}$

4)

$$\begin{aligned} &> \text{Ecua} := (x^4 \cdot \log(x) - 2 \cdot x \cdot y^3) + 3 \cdot x^2 \cdot y^2 \cdot y' = 0 \\ &\qquad\qquad\qquad \text{Ecua} := x^4 \ln(x) - 2xy(x)^3 + 3x^2y(x)^2 \left( \frac{d}{dx} y(x) \right) = 0 \end{aligned} \quad (50)$$

$\text{RESPUESTA}$

$\text{with}(\text{DEtools}) :$

$$\begin{aligned} &> \text{odeadvisor}(\text{Ecua}) \\ &\qquad\qquad\qquad [_{\text{Bernoulli}}] \end{aligned} \quad (51)$$

$$\begin{aligned} &> \text{FI} := \text{intfactor}(\text{Ecua}) \\ &\qquad\qquad\qquad \text{FI} := \frac{1}{x^4} \end{aligned} \quad (52)$$

$$\begin{aligned} &> \text{M} := x^4 \ln(x) - 2xy^3 \\ &\qquad\qquad\qquad \text{M} := x^4 \ln(x) - 2xy^3 \end{aligned} \quad (53)$$

$$\begin{aligned} &> \text{N} := 3x^2y^2 \\ &\qquad\qquad\qquad \text{N} := 3y^2x^2 \end{aligned} \quad (54)$$

$$\begin{aligned} &> \text{Comprobar} := \text{diff}(\text{M}, y) \neq \text{diff}(\text{N}, x) \\ &\qquad\qquad\qquad \text{Comprobar} := -6y^2x \neq 6y^2x \end{aligned} \quad (55)$$

NO ES EXACTA

$$\begin{aligned} &> \text{MM} := \text{expand}(\text{M} \cdot \text{FI}) \\ &\qquad\qquad\qquad \text{MM} := \ln(x) - \frac{2y^3}{x^3} \end{aligned} \quad (56)$$

$$\begin{aligned} &> \text{NN} := \text{N} \cdot \text{FI} \\ &\qquad\qquad\qquad \text{NN} := \frac{3y^2}{x^2} \end{aligned} \quad (57)$$

$$\begin{aligned} &> \text{ComprobarDos} := \text{diff}(\text{MM}, y) = \text{diff}(\text{NN}, x) \\ &\qquad\qquad\qquad \text{ComprobarDos} := -\frac{6y^2}{x^3} = -\frac{6y^2}{x^3} \end{aligned} \quad (58)$$

$$\begin{aligned} &> \text{EcuaDos} := \ln(x) - \frac{2y(x)^3}{x^3} + \frac{3y(x)^2}{x^2} \cdot \text{diff}(y(x), x) = 0 \\ &\qquad\qquad\qquad \text{EcuaDos} := \ln(x) - \frac{2y(x)^3}{x^3} + \frac{3y(x)^2 \left( \frac{d}{dx} y(x) \right)}{x^2} = 0 \end{aligned} \quad (59)$$

$\text{odeadvisor}(\text{EcuaDos})$

$$[_{exact}, _{Bernoulli}] \quad (60)$$

>

$$\text{IntMMx} := \text{int}(\text{MM}, x)$$

$$\text{IntMMx} := \frac{y^3}{x^2} + x \ln(x) - x \quad (61)$$

$$\text{SolGral} := \text{IntMMx} + \text{int}((\text{NN} - \text{diff}(\text{IntMMx}, y)), y) = \_CI$$

$$\text{SolGral} := \frac{y^3}{x^2} + x \ln(x) - x = \_CI \quad (62)$$

$$\text{SolGralFinal} := \frac{y(x)^3}{x^2} + x \ln(x) - x = \_CI$$

$$\text{SolGralFinal} := \frac{y(x)^3}{x^2} + x \ln(x) - x = \_CI \quad (63)$$

$$\text{DerEcua} := \text{expand}(\text{isolate}(\text{Ecua}, \text{diff}(y(x), x)))$$

$$\text{DerEcua} := \frac{d}{dx} y(x) = -\frac{x^2 \ln(x)}{3 y(x)^2} + \frac{2 y(x)}{3 x} \quad (64)$$

$$\text{DerSolGral} := \text{expand}(\text{isolate}(\text{diff}(\text{SolGralFinal}, x), \text{diff}(y(x), x)))$$

$$\text{DerSolGral} := \frac{d}{dx} y(x) = -\frac{x^2 \ln(x)}{3 y(x)^2} + \frac{2 y(x)}{3 x} \quad (65)$$

$$\text{ComprobarUno} := \text{simplify}(\text{rhs}(\text{DerEcua}) - \text{rhs}(\text{DerSolGral})) = 0$$

$$\text{ComprobarUno} := 0 = 0 \quad (66)$$

> restart

5)

$$\text{Ecua} := 2 x \cdot y' - y = 3 \cdot x^2$$

$$\text{Ecua} := 2 x \left( \frac{d}{dx} y(x) \right) - y(x) = 3 x^2 \quad (67)$$

>

RESPUESTA

$$\text{EcuaDos} := \text{expand}\left(\frac{\text{lhs}(\text{Ecua})}{2 x}\right) = \frac{\text{rhs}(\text{Ecua})}{2 x}$$

$$\text{EcuaDos} := \frac{d}{dx} y(x) - \frac{y(x)}{2 x} = \frac{3 x}{2} \quad (68)$$

$$p := -\frac{1}{2 x}$$

$$p := -\frac{1}{2 x} \quad (69)$$

$$q := \frac{3 x}{2}$$

$$q := \frac{3 x}{2} \quad (70)$$

$$\begin{aligned} & \text{SolGral} := y(x) = \_C1 \cdot \exp(-\text{int}(p, x)) + \exp(-\text{int}(p, x)) \cdot \text{int}(\exp(\text{int}(p, x)) \cdot q, x) \\ & \text{SolGral} := y(x) = \_C1 \sqrt{x} + x^2 \end{aligned} \quad (71)$$

$$\begin{aligned} & \text{Comprobar} := \text{simplify}(\text{eval}(\text{subs}(y(x) = \text{rhs}(\text{SolGral}), \text{lhs}(\text{Ecua}) - \text{rhs}(\text{Ecua}) = 0))) \\ & \text{Comprobar} := 0 = 0 \end{aligned} \quad (72)$$

$\text{restart}$