>

>

>

FACULTAD DE INGENIERÍA
DIVISIÓN DE CIENCIAS BÁSICAS
COORDINACIÓN DE CIENCIAS APLICADAS
MATEMÁTICAS APLICADAS
ECUACIONES DIFERENCIALES
SEMESTRE 2025 − 1 GRUPO 13
SEGUNDO EXAMEN PARCIAL
TEMAS 2 & 3
SOLUCIÓN

Octubre 19 *de* 2024

> $restart$

1)

> $EcuaHom := x^2 \cdot y'' - x \cdot y' + y = 0$

$$EcuaHom := x^2 \left(\frac{d^2}{dx^2}\, y(x)\right) - x\left(\frac{d}{dx}\, y(x)\right) + y(x) = 0 \quad \textbf{(1)}$$

> $EcuaHomNormal := expand\left(\frac{EcuaHom}{x^2}\right)$

$$EcuaHomNormal := \frac{d^2}{dx^2}\, y(x) - \frac{\frac{d}{dx}\, y(x)}{x} + \frac{y(x)}{x^2} = 0 \quad \textbf{(2)}$$

> $yy[1] := x; yy[2] := x \cdot \log(x)$

$$yy_1 := x$$

$$yy_2 := x \ln(x) \quad \textbf{(3)}$$

> $ComprobarUno := simplify(eval(subs(y(x) = yy[1], EcuaHomNormal)))$

$$ComprobarUno := 0 = 0 \quad \textbf{(4)}$$

> $ComprobarDos := simplify(eval(subs(y(x) = yy[2], EcuaHomNormal)))$

$$ComprobarDos := 0 = 0 \quad \textbf{(5)}$$

> $EcuaNoHom := lhs(EcuaHom) = 4 \cdot x \cdot \log(x)$

$$EcuaNoHom := x^2 \left(\frac{d^2}{dx^2}\, y(x)\right) - x\left(\frac{d}{dx}\, y(x)\right) + y(x) = 4\, x \ln(x) \quad \textbf{(6)}$$

> $EcuaNoHomNormal := expand\left(\frac{EcuaNoHom}{x^2}\right)$

$$EcuaNoHomNormal := \frac{d^2}{dx^2}\, y(x) - \frac{\frac{d}{dx}\, y(x)}{x} + \frac{y(x)}{x^2} = \frac{4 \ln(x)}{x} \quad \textbf{(7)}$$

> $Q := rhs(EcuaNoHomNormal)$

$$Q := \frac{4 \ln(x)}{x} \quad \textbf{(8)}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} x & x\ln(x) \\ 1 & \ln(x)+1 \end{bmatrix} \tag{9}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \frac{4\ln(x)}{x} \end{bmatrix} \tag{10}$$

> $ParaVar := linsolve(WW, BB)$

$$ParaVar := \begin{bmatrix} -\frac{4\ln(x)^2}{x} & \frac{4\ln(x)}{x} \end{bmatrix} \tag{11}$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := -\frac{4\ln(x)^2}{x}$$

$$Bprima := \frac{4\ln(x)}{x} \tag{12}$$

> $SolGral := y(x) = expand((int(Aprima, x) + \_C1)\cdot yy[1] + (int(Bprima, x) + \_C2)\cdot yy[2])$

$$SolGral := y(x) = \frac{2x\ln(x)^3}{3} + x\_C1 + x\ln(x)\_C2 \tag{13}$$

> $EcuaNoHom$

$$x^2\left(\frac{d^2}{dx^2}y(x)\right) - x\left(\frac{d}{dx}y(x)\right) + y(x) = 4x\ln(x) \tag{14}$$

> $comprobarTres := simplify(eval(subs(y(x) = rhs(SolGral), lhs(EcuaNoHom) - rhs(EcuaNoHom) = 0)))$

$$comprobarTres := 0 = 0 \tag{15}$$

>

> $restart$

2)

> $Ecua := y'' + 2\cdot y' + 2\cdot y = \exp(-x)\cdot\sec(x)$

$$Ecua := \frac{d^2}{dx^2}y(x) + 2\frac{d}{dx}y(x) + 2y(x) = e^{-x}\sec(x) \tag{16}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}y(x) + 2\frac{d}{dx}y(x) + 2y(x) = 0 \tag{17}$$

> $Q := rhs(Ecua)$

$$Q := e^{-x}\sec(x) \tag{18}$$

> $EcuaCarac := m^2 + 2\cdot m + 2 = 0$

$$EcuaCarac := m^2 + 2m + 2 = 0 \tag{19}$$

> $Raiz := solve(EcuaCarac)$

(20)

$$Raiz := -1 + \mathrm{I}, -1 - \mathrm{I} \quad \textbf{(20)}$$

> $yy[1] := \exp(\mathrm{Re}(Raiz[1]) \cdot x) \cdot \cos(\mathrm{Im}(Raiz[1]) \cdot x); yy[2] := \exp(\mathrm{Re}(Raiz[1]) \cdot x) \cdot \sin(\mathrm{Im}(Raiz[1]) \cdot x)$

$$yy_1 := \mathrm{e}^{-x} \cos(x)$$

$$yy_2 := \mathrm{e}^{-x} \sin(x) \quad \textbf{(21)}$$

> $with(linalg) :$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \mathrm{e}^{-x} \cos(x) & \mathrm{e}^{-x} \sin(x) \\ -\mathrm{e}^{-x} \cos(x) - \mathrm{e}^{-x} \sin(x) & -\mathrm{e}^{-x} \sin(x) + \mathrm{e}^{-x} \cos(x) \end{bmatrix} \quad \textbf{(22)}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \mathrm{e}^{-x} \sec(x) \end{bmatrix} \quad \textbf{(23)}$$

> $Para := simplify(linsolve(WW, BB))$

$$Para := \begin{bmatrix} -\tan(x) & 1 \end{bmatrix} \quad \textbf{(24)}$$

> $Aprima := Para[1]; Bprima := Para[2]$

$$Aprima := -\tan(x)$$

$$Bprima := 1 \quad \textbf{(25)}$$

> $SolGralNoHom := y(x) = expand(simplify((int(Aprima, x) + \_C1) \cdot yy[1] + (int(Bprima, x) + \_C2) \cdot yy[2]))$

$$SolGralNoHom := y(x) = \frac{\sin(x)\ \_C2}{\mathrm{e}^{x}} + \frac{\sin(x)\ x}{\mathrm{e}^{x}} + \frac{\cos(x)\ \ln(\cos(x))}{\mathrm{e}^{x}} + \frac{\cos(x)\ \_C1}{\mathrm{e}^{x}} \quad \textbf{(26)}$$

> $Ecua$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\ y(x) + 2\ \frac{\mathrm{d}}{\mathrm{d}x}\ y(x) + 2\ y(x) = \mathrm{e}^{-x} \sec(x) \quad \textbf{(27)}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGralNoHom), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad \textbf{(28)}$$

> $restart$

3)

> $Sistema := diff(y[1](t), t) = 2 \cdot y[1](t) + 2 \cdot y[2](t), diff(y[2](t), t) = 3 \cdot y[1](t) + 3 \cdot y[2](t) + 2 : Sistema[1]; Sistema[2]$

$$\frac{\mathrm{d}}{\mathrm{d}t}\ y_1(t) = 2\ y_1(t) + 2\ y_2(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\ y_2(t) = 3\ y_1(t) + 3\ y_2(t) + 2 \quad \textbf{(29)}$$

> $CondIni := y[1](0) = 1, y[2](0) = 2$

$$CondIni := y_1(0) = 1, y_2(0) = 2 \quad \textbf{(30)}$$

> $Xcero := array([1, 2])$

$$Xcero := \begin{bmatrix} 1 & 2 \end{bmatrix} \quad \textbf{(31)}$$

> $BB := array([0, 2])$

$$BB := \begin{bmatrix} 0 & 2 \end{bmatrix} \quad (32)$$

> $AA := array([[2, 2], [3, 3]])$

$$AA := \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} \quad (33)$$

> $with(linalg):$

> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} \frac{3}{5} + \frac{2\,e^{5\,t}}{5} & \frac{2\,e^{5\,t}}{5} - \frac{2}{5} \\ \frac{3\,e^{5\,t}}{5} - \frac{3}{5} & \frac{2}{5} + \frac{3\,e^{5\,t}}{5} \end{bmatrix} \quad (34)$$

> $SolHom := evalm(MatExp \&* Xcero) : y[1](t) = SolHom[1]; y[2](t) = SolHom[2]$

$$y_1(t) = -\frac{1}{5} + \frac{6\,e^{5\,t}}{5}$$

$$y_2(t) = \frac{9\,e^{5\,t}}{5} + \frac{1}{5} \quad (35)$$

> $MatExpTau := map(rcurry(eval, t='t - \tau'), MatExp)$

$$MatExpTau := \begin{bmatrix} \frac{3}{5} + \frac{2\,e^{5\,t - 5\,\tau}}{5} & \frac{2\,e^{5\,t - 5\,\tau}}{5} - \frac{2}{5} \\ \frac{3\,e^{5\,t - 5\,\tau}}{5} - \frac{3}{5} & \frac{2}{5} + \frac{3\,e^{5\,t - 5\,\tau}}{5} \end{bmatrix} \quad (36)$$

> $BBtau := map(rcurry(eval, t='\tau'), BB)$

$$BBtau := \begin{bmatrix} 0 & 2 \end{bmatrix} \quad (37)$$

> $ProdTau := evalm(MatExpTau \&* BBtau) : ProdTau[1]; ProdTau[2]$

$$\frac{4\,e^{5\,t - 5\,\tau}}{5} - \frac{4}{5}$$

$$\frac{4}{5} + \frac{6\,e^{5\,t - 5\,\tau}}{5} \quad (38)$$

> $SolNoHom := map(int, ProdTau, \tau = 0 .. t) : y[1](t) = SolNoHom[1]; y[2](t) = SolNoHom[2];$

$$y_1(t) = -\frac{4}{25} + \frac{4\,e^{5\,t}}{25} - \frac{4\,t}{5}$$

$$y_2(t) = -\frac{6}{25} + \frac{6\,e^{5\,t}}{25} + \frac{4\,t}{5} \quad (39)$$

> $SolFinal := evalm(SolHom + SolNoHom) : y[1](t) = SolFinal[1]; y[2](t) = SolFinal[2]$

$$y_1(t) = -\frac{9}{25} + \frac{34\,e^{5\,t}}{25} - \frac{4\,t}{5}$$

$$y_2(t) = \frac{51\, e^{5t}}{25} - \frac{1}{25} + \frac{4t}{5} \tag{40}$$

> *CondIni*

$$y_1(0) = 1, y_2(0) = 2 \tag{41}$$

> $ComprobarUno := y[1](0) = simplify(subs(t=0, SolFinal[1]))$

$$ComprobarUno := y_1(0) = 1 \tag{42}$$

> $ComprobarDos := y[2](0) = simplify(subs(t=0, SolFinal[2]))$

$$ComprobarDos := y_2(0) = 2 \tag{43}$$

> $Sistema[1]; Sistema[2]$

$$\frac{d}{dt}\, y_1(t) = 2\, y_1(t) + 2\, y_2(t)$$

$$\frac{d}{dt}\, y_2(t) = 3\, y_1(t) + 3\, y_2(t) + 2 \tag{44}$$

> $ComprobarTres := simplify(eval(subs(y[1](t) = SolFinal[1], y[2](t) = SolFinal[2], lhs(Sistema[1]) - rhs(Sistema[1]) = 0)))$

$$ComprobarTres := 0 = 0 \tag{45}$$

> $ComprobarCuatro := simplify(eval(subs(y[1](t) = SolFinal[1], y[2](t) = SolFinal[2], lhs(Sistema[2]) - rhs(Sistema[2]) = 0)))$

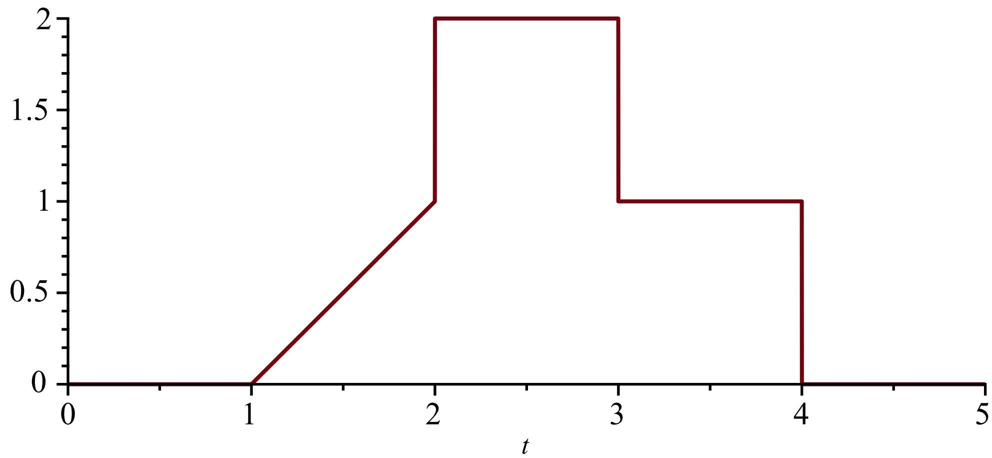
$$ComprobarCuatro := 0 = 0 \tag{46}$$

> *restart*

4)

a) Expresar f en términos de rampa y escalón unitario

> $f := (t-1)\cdot \mathrm{Heaviside}(t-1) - (t-2)\cdot \mathrm{Heaviside}(t-2) + \mathrm{Heaviside}(t-2) - \mathrm{Heaviside}(t-3) - \mathrm{Heaviside}(t-4) : plot(f, t = 0..5, scaling = CONSTRAINED)$



>

b) Obtener la transformada de Laplace de f

> *with*(*inttrans*) :

> $F := laplace(f, t, s)$

$$F := \frac{e^{-s} + (-1+s)\,e^{-2s}}{s^2} - \frac{e^{-3s} + e^{-4s}}{s} \quad \textbf{(47)}$$

> *restart*

5)

> $Ecua := f(t) = 3 \cdot t^2 - \exp(-t) - int(f(\mathrm{tau}) \cdot \exp(t - \mathrm{tau}), \mathrm{tau} = 0\,..t)$

$$Ecua := f(t) = 3\,t^2 - e^{-t} - \left(\int_0^t f(\tau)\,e^{t-\tau}\,d\tau\right) \quad \textbf{(48)}$$

> *with*(*inttrans*) :

> $EcuaTL := laplace(Ecua, t, s)$

$$EcuaTL := \mathcal{L}(f(t), t, s) = \frac{6}{s^3} - \frac{1}{1+s} - \frac{\mathcal{L}(f(t), t, s)}{s-1} \quad \textbf{(49)}$$

> $SolTL := isolate(EcuaTL, laplace(f(t), t, s))$

(50)

$$SolTL := \mathcal{L}(f(t), t, s) = \frac{\frac{6}{s^3} - \frac{1}{1+s}}{1 + \frac{1}{s-1}} \tag{50}$$

> $Sol := invlaplace(SolTL, s, t)$

$$Sol := f(t) = 3\,t^2 - 2\,\mathrm{e}^{-t} - t^3 + 1 \tag{51}$$

> $SolTau := subs(t = \mathrm{tau}, Sol)$

$$SolTau := f(\tau) = 3\,\tau^2 - 2\,\mathrm{e}^{-\tau} - \tau^3 + 1 \tag{52}$$

> $ProdTau := subs\left(f(\mathrm{tau}) = rhs(SolTau), f(\tau)\,\mathrm{e}^{t-\tau}\right)$

$$ProdTau := \left(3\,\tau^2 - 2\,\mathrm{e}^{-\tau} - \tau^3 + 1\right)\mathrm{e}^{t-\tau} \tag{53}$$

> $IntTau := int(ProdTau, \mathrm{tau} = 0\,..t)$

$$IntTau := t^3 - 1 + \mathrm{e}^{-t} \tag{54}$$

> $EcuaDos := f(t) = 3\,t^2 - \mathrm{e}^{-t} - IntTau$

$$EcuaDos := f(t) = 3\,t^2 - 2\,\mathrm{e}^{-t} - t^3 + 1 \tag{55}$$

> $Comprobar := subs(f(t) = rhs(EcuaDos), lhs(EcuaDos) - rhs(EcuaDos) = 0)$

$$Comprobar := 0 = 0 \tag{56}$$

> $restart$

>