

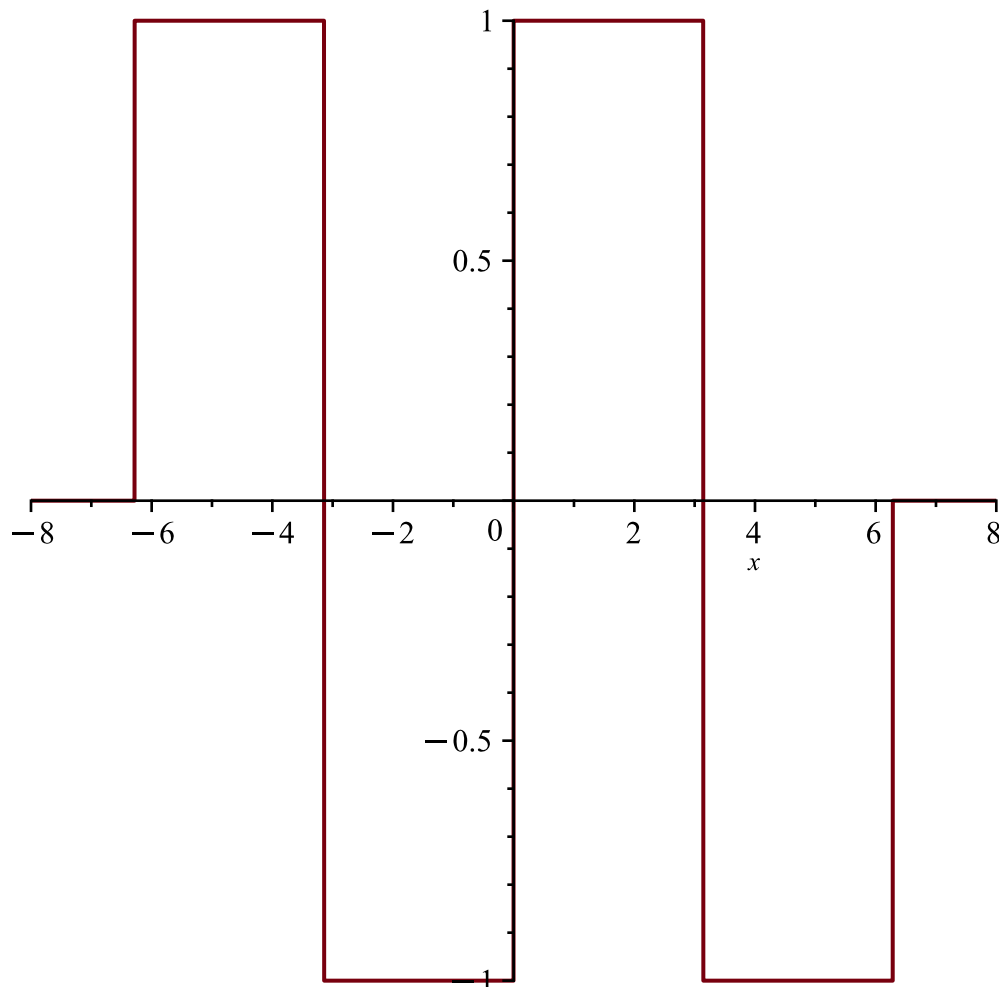
FACULTAD DE INGENIERÍA
DIVISIÓN DE CIENCIAS BÁSICAS
COORDINACIÓN DE CIENCIAS APLICADAS
MATEMÁTICAS APLICADAS
ECUACIONES DIFERENCIALES
SEMESTRE 2025 – 1 GRUPO 13
TERCER EXAMEN PARCIAL
TEMAS 4
SOLUCIÓN (2)

Noviembre de 2024

1) Obtenga los coeficientes de la Serie de Fourier $\pi < x < \pi$

> restart

> $f := \text{Heaviside}(x + 2\pi) - 2 \cdot \text{Heaviside}(x + \pi) + 2 \cdot \text{Heaviside}(x) - 2 \cdot \text{Heaviside}(x - \pi) + \text{Heaviside}(x - 2\pi) : \text{plot}(f, x = -8..8)$



> $L := \pi$

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(1)

> $a[0] := \frac{1}{L} \cdot \text{int}(f, x = -L..L)$

$a_0 := 0$

(2)

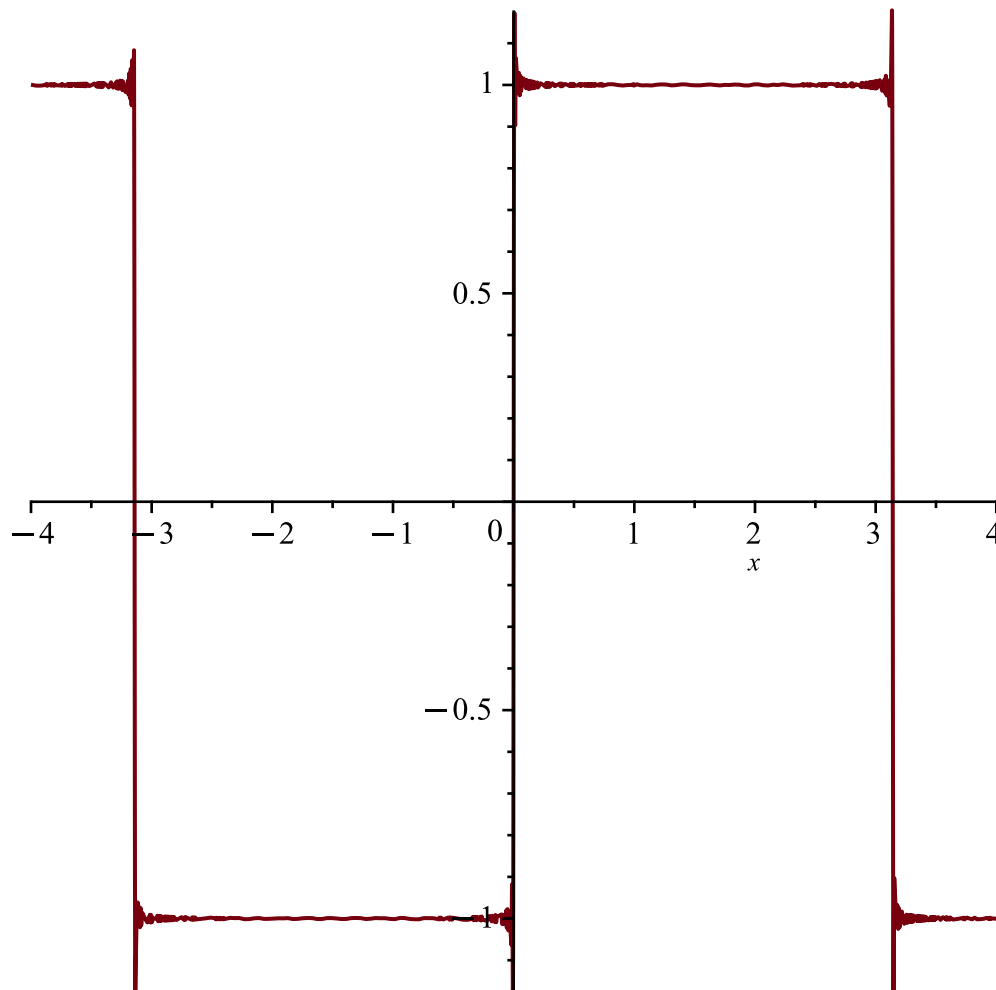
$$\begin{aligned} &> a[n] := \text{simplify}\left(\frac{1}{L} \cdot \text{int}\left(f \cdot \cos\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), x = -L..L\right)\right) \\ &\qquad\qquad\qquad a_n := 0 \end{aligned} \tag{3}$$

$$\begin{aligned} &> b[n] := \text{simplify}\left(\text{subs}\left(\cos(n \cdot \text{Pi}) = (-1)^n, \frac{1}{L} \cdot \text{int}\left(f \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), x = -L..L\right)\right)\right) \\ &\qquad\qquad\qquad b_n := \frac{-2(-1)^n + 2}{n \pi} \end{aligned} \tag{4}$$

$$\begin{aligned} &> STF := \text{Sum}\left(b[n] \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), n = 1..infinity\right) \\ &\qquad\qquad\qquad STF := \sum_{n=1}^{\infty} \frac{(-2(-1)^n + 2) \sin(nx)}{n \pi} \end{aligned} \tag{5}$$

$$> STF500 := \text{sum}\left(b[n] \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), n = 1..500\right) :$$

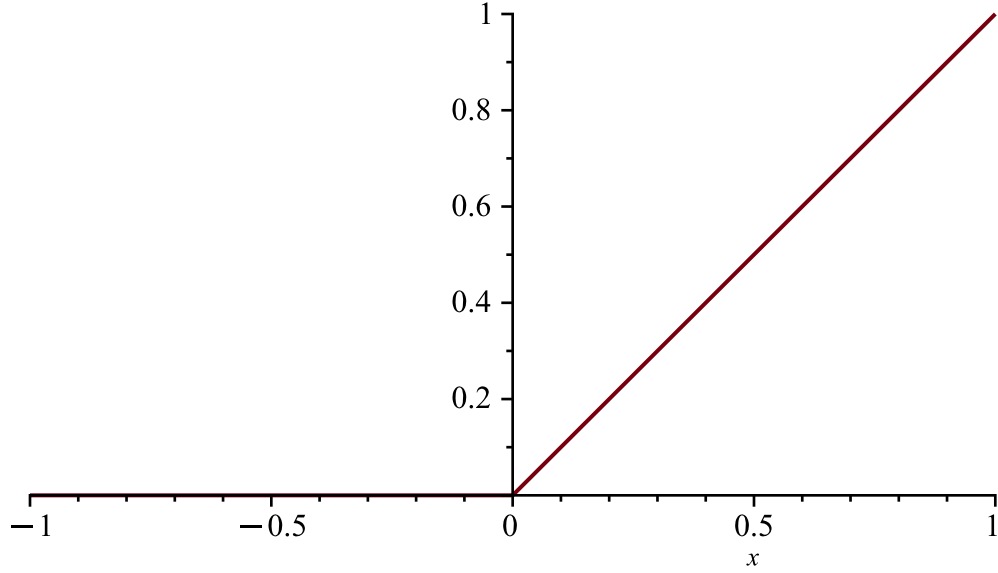
$$> \text{plot}(STF500, x = -4..4)$$



$$> \text{restart}$$

2) Determinar la Serie de Fourier

$$> f := (x) \cdot \text{Heaviside}(x) : \text{plot}(f, x = -1..1, \text{scaling} = \text{CONSTRAINED})$$



$$\begin{aligned} &> L := 1 \\ &L := 1 \end{aligned} \tag{6}$$

$$\begin{aligned} &> a[0] := \frac{1}{L} \cdot \text{int}(f, x = -L..L) \\ &a_0 := \frac{1}{2} \end{aligned} \tag{7}$$

$$\begin{aligned} &> a[n] := \text{subs}\left(\sin(n \cdot \text{Pi}) = 0, \cos(n \cdot \text{Pi}) = (-1)^n, \frac{1}{L} \cdot \text{int}\left(f \cdot \cos\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), x = -L..L\right)\right) \\ &a_n := \frac{-1 + (-1)^n}{n^2 \pi^2} \end{aligned} \tag{8}$$

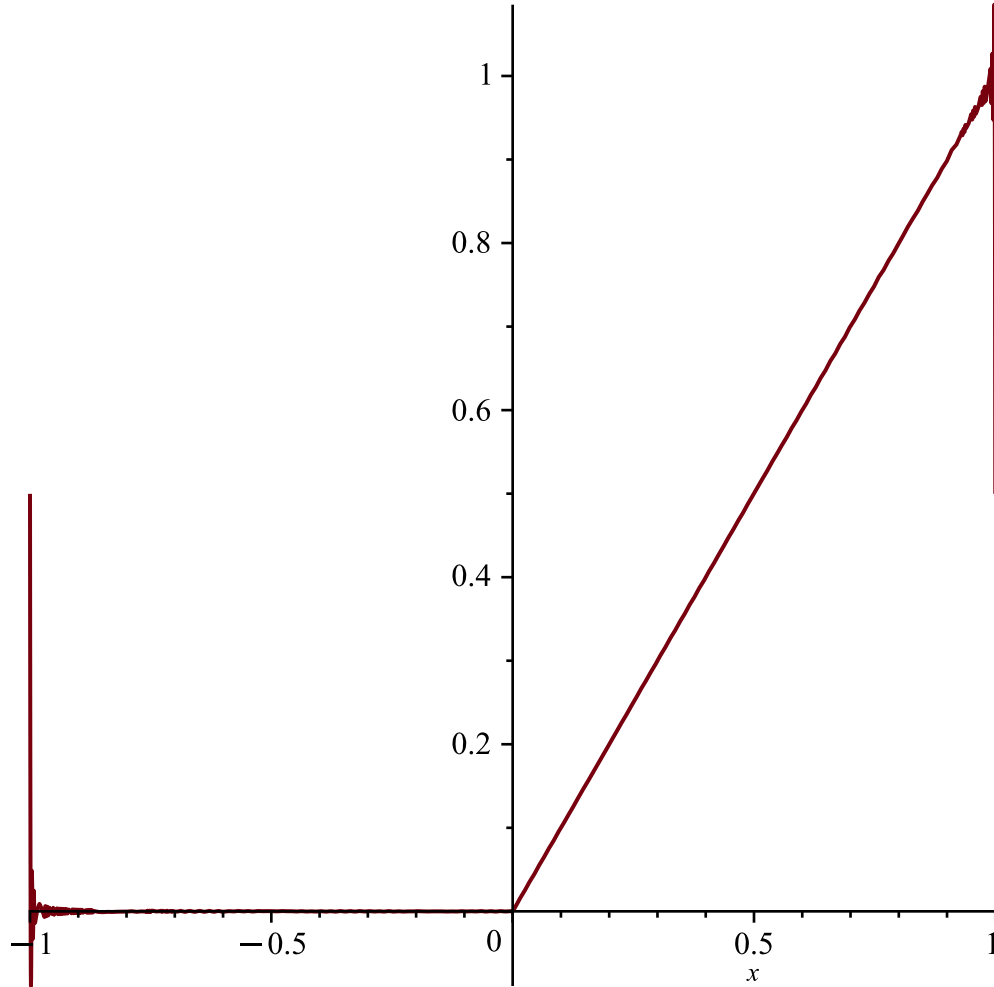
$$\begin{aligned} &> b[n] := \text{subs}\left(\sin(n \cdot \text{Pi}) = 0, \cos(n \cdot \text{Pi}) = (-1)^n, \frac{1}{L} \cdot \text{int}\left(f \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), x = -L..L\right)\right) \\ &b_n := -\frac{(-1)^n}{n \pi} \end{aligned} \tag{9}$$

$$\begin{aligned} &> STF := \frac{a[0]}{2} + \text{Sum}\left(a[n] \cdot \cos\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right) + b[n] \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), n = 1..infinity\right) \end{aligned} \tag{10}$$

$$STF := \frac{1}{4} + \sum_{n=1}^{\infty} \left(\frac{(-1 + (-1)^n) \cos(n \pi x)}{n^2 \pi^2} - \frac{(-1)^n \sin(n \pi x)}{n \pi} \right) \quad (10)$$

> $STF500 := \frac{a[0]}{2} + \text{sum}\left(a[n] \cdot \cos\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right) + b[n] \cdot \sin\left(\frac{n \cdot \text{Pi}}{L} \cdot x\right), n = 1 \dots 500\right) :$

> $\text{plot}(STF500, x = -L \dots L)$



> restart

3) Resuelva para constante = 5

> $Ecua := \text{diff}(u(x, t), x) = \frac{3}{x} \cdot \text{diff}(u(x, t), t)$

$$Ecua := \frac{\partial}{\partial x} u(x, t) = \frac{3 \left(\frac{\partial}{\partial t} u(x, t) \right)}{x} \quad (11)$$

> $EcuaDos := \text{lhs}(Ecua) \cdot x = \text{rhs}(Ecua) \cdot x$

$$EcuaDos := \left(\frac{\partial}{\partial x} u(x, t) \right) x = 3 \frac{\partial}{\partial t} u(x, t) \quad (12)$$

> $EcuaSep := \text{eval}(\text{subs}(u(x, t) = F(x) \cdot G(t), EcuaDos))$

$$EcuaSep := \left(\frac{d}{dx} F(x) \right) G(t) x = 3 F(x) \left(\frac{d}{dt} G(t) \right) \quad (13)$$

$$\begin{aligned} &> \text{EcuaSeparada} := \frac{\text{lhs}(\text{EcuaSep})}{3 \cdot F(x) \cdot G(t)} = \frac{\text{rhs}(\text{EcuaSep})}{3 \cdot F(x) \cdot G(t)} \\ &\quad \text{EcuaSeparada} := \frac{\left(\frac{d}{dx} F(x) \right) x}{3 F(x)} = \frac{\frac{d}{dt} G(t)}{G(t)} \end{aligned} \quad (14)$$

$$\begin{aligned} &> \text{EcuaX} := \text{lhs}(\text{EcuaSeparada}) = 5 \\ &\quad \text{EcuaX} := \frac{\left(\frac{d}{dx} F(x) \right) x}{3 F(x)} = 5 \end{aligned} \quad (15)$$

$$\begin{aligned} &> \text{EcuaT} := \text{rhs}(\text{EcuaSeparada}) = 5 \\ &\quad \text{EcuaT} := \frac{\frac{d}{dt} G(t)}{G(t)} = 5 \end{aligned} \quad (16)$$

$$\begin{aligned} &> \text{SolX} := \text{dsolve}(\text{EcuaX}) \\ &\quad \text{SolX} := F(x) = c_1 x^{15} \end{aligned} \quad (17)$$

$$\begin{aligned} &> \text{SolT} := \text{dsolve}(\text{EcuaT}) \\ &\quad \text{SolT} := G(t) = c_1 e^{5t} \end{aligned} \quad (18)$$

$$\begin{aligned} &> \text{SolGral} := u(x, t) = \text{rhs}(\text{SolX}) \cdot \text{subs}(c_1 = 1, \text{rhs}(\text{SolT})) \\ &\quad \text{SolGral} := u(x, t) = c_1 x^{15} e^{5t} \end{aligned} \quad (19)$$

$$\begin{aligned} &> \text{Comprobar} := \text{simplify}(\text{eval}(\text{subs}(u(x, t) = \text{rhs}(\text{SolGral}), \text{lhs}(\text{Ecua}) - \text{rhs}(\text{Ecua}) = 0))) \\ &\quad \text{Comprobar} := 0 = 0 \end{aligned} \quad (20)$$

> restart

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